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TECHNICAL PAPERS

AND

DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

LATERAL EARTH AND CONCRETE PRESSURES

By Lazarus White[1] and George Paaswell,[2]
Members, Am. Soc. C. E.

Synopsis

The former Committee of the Society on Earths and Foundations, in conjunction with its studies of the distribution of vertical pressures beneath foundations, assigned to the writers, as a Sub-Committee, the study of lateral earth pressures. This paper is a result of such a study. It has long been recognized that the classic methods of analyzing lateral pressures as developed by Coulomb and Rankine were extremely limited in practical application and many attempts have been made to reshape the classic analysis to bring the theory into better consonance with practice. The writers feel that it is better to discard the classic method entirely, and start from the fundamental laws of the theory of elasticity, as had been done with recognized success in the study of the distribution of vertical pressures. The mathematical work and resulting equations are not as simple as the writers would desire, but it is hoped that further study and application of the methods outlined herein will lead to simplifications without loss of rigor. The labor of using the somewhat involved equations is lightened by the tables presented herewith. It is believed that not only are more correct values determined, but a much better understanding may be had of the phenomena attendant upon the three phases of the problem of lateral earth pressures.

Introduction

The science of soil mechanics has made rapid strides in the decade, 1928 to 1938, in the analysis of foundation problems. The formulas developed by Boussinesq[3] have proved to be extremely valuable in this connection and the writers have used them in the study of lateral pressures upon earth-sustaining

Note.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted November 15, 1938.

[1] Pres., Spencer, White & Prentis, Inc., New York, N. Y.

[2] Secy. and Treas., Spencer & Ross, Inc., Detroit, Mich.

[3] See Progress Rept., Special Committee on Earths and Foundations, *Proceedings*, Am. Soc. C. E., May, 1933, p. 779.

structures. For more than a century many attempts have been made to reconcile actual problems to formulas based upon artificial and non-existent soil types. The writers feel that these attempts are futile and that the engineering terminology identified with them is no longer of real significance.

Soils possess sufficient elastic properties to permit the application of the theory of elasticity in solids to lateral earth pressures. There are three fundamental phases of the analysis of lateral pressure: (I) The lateral pressures induced upon a sustaining structure of sufficient rigidity to prevent fracture of the bank; (II) the lateral pressures induced upon a wall when movement of the wall is sufficient to permit the formation of fracture surfaces in the bank; and (III) the lateral pressures induced upon a wall due to a surface loading. It is seen that only for Phase II have the classic theories of Coulomb and Rankine had any coherence and practical application. Charles Terzaghi, M. Am. Soc. C. E., has given an interesting discussion on this aspect of the problem.[4]

The writers feel that the ever-important question of the pressures developed in forms due to concrete may find a solution along the lines developed for lateral pressures. Although the mathematical treatment may seem complex to those who seek a simple "rule-of-thumb" expression, the formulas as derived are comparatively simple.

Notation.—The symbols used in this paper are defined where they first appear and are assembled for reference in the Appendix.

Phase I

In Phase I it is assumed that the wall is sufficiently rigid so that the sustained earth bank will not develop fracture surfaces. When an earth mass exerts a lateral pressure upon a sustaining structure this structure is distorted. Since the earth mass itself possesses elasticity, the yielding of the sustaining structure permits a lateral yielding of the earth mass toward the wall or sheeting. This lateral yielding is accompanied by a reduction in the lateral pressures. If the wall is a composite one, comprising members of varied rigidity, the less rigid will yield more, with release of pressure upon them, the more rigid ones finally taking the greater part of the pressure. The distortion of the structure may be small, and yet the re-arrangement and reduction in pressures may be very marked. In the illuminating series of experiments conducted by Professor Terzaghi, at the Massachusetts Institute of Technology, it was found that a movement of the experimental wall as little as 1 mm caused a reduction of 75% in the pressures induced upon the wall. If the movement of the wall persists so that fracture surfaces are developed, it has been found that the general shape of such fracture is as shown in Fig. 1. The surface of fracture is generally recognized as a shear failure surface. At some future time the writers hope to present the development of the pressures due to this shear failure based upon the assumption that the failure surface is a cosine curve. It is important to remember that this characteristic failure surface with the emergent break at the surface, approximately distant from the wall one-half the depth of wall, is independent of the type of ground, the only requisite for its appearance being a

[4] *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., *Paper J-3*, Vol. 1, Harvard Univ., June 22, 1936.

sufficiently flexible wall, or a yielding foundation under the wall. The general ground movement due to a shear failure in saturated soils such as clay and silts is a separate phenomenon not discussed in this paper.

Sheeting and bracing, when properly designed and installed, come within Phase I. The system is one of varied rigidity—the sheeting being the most flexible, the wales next, and, finally, the braces the most rigid. The relative yielding of these component parts of the sustaining structure produces an accumulation of stress zones at the more rigid points. The pressures induced upon these component parts may be viewed as derived from soils of three characteristic properties each producing a system of lateral pressures of increasing intensity going from the sheeting to the wales and finally to the braces or tie-rods.

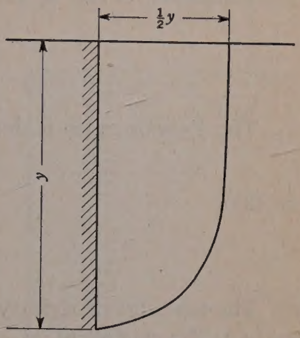


FIG. 1.—TYPICAL SECTION OF A FRACTURE SURFACE

THEORY OF DESIGN—SHEETING AND BRACING

For an elastic solid subject to its own weight alone, the total lateral pressure at any depth, y, below the surface is given by,

$$F = -w\,y\,\frac{\mu}{1-\mu} \qquad (1)$$

in which μ is the Poisson ratio. The negative sign in Equation (1) indicates a compression. In accord with elastic theory, μ may have any value between 0.5 and -1. If it were greater than 0.5, the material would not resist distortion; and if it were less than -1, it would not resist compression. As a matter of fact it is positive for all known structural materials. For a true liquid, $\mu = 0.5$, and Equation (1) reduces to the hydrostatic law, $F = -w\,y$.

As the theory of lateral pressures involves Poisson's ratio, it may be well to outline, briefly, its relation to the other elastic constants. Assume that a load, P, is applied along the X-axis of a unit cube and is the only force acting upon this solid. Let the resulting strains induced in this solid by the load along the X, Y, and Z-axes be ϵ_e, ϵ_f, and ϵ_g, respectively. If m and n are two, at present, undefined constants, the following equations exist:

$$P = (m + 2\,n)\,\epsilon_e + n\,(\epsilon_f + \epsilon_g) \qquad (2)$$

$$0 = (m + 2\,n)\,\epsilon_f + n\,(\epsilon_g + \epsilon_e) \qquad (3)$$

and,

$$0 = (m + 2\,n)\,\epsilon_g + n\,(\epsilon_e + \epsilon_f) \qquad (4)$$

From these three equations of equilibrium,

$$P = \frac{n\,(3\,m + 2\,n)}{m + n}\,\epsilon_e \qquad (5)$$

and,

$$\epsilon_f = \epsilon_g = -\frac{m\,\epsilon_e}{2\,(m + n)} \qquad (6)$$

The modulus of elasticity is defined as $E = \frac{P}{\epsilon_e}$, giving a relation between the constants, m and n, and the modulus, E, equal to:

$$E = \frac{n\,(3\,m + 2\,n)}{m + n} \quad \text{(7)}$$

The Poisson ratio is defined by the ratio between axial and lateral strain,

$$\epsilon_f = \epsilon_g = -\,\mu\,\epsilon_e \quad \text{(8)}$$

so that,

$$\mu = \frac{0.5\,m}{m + n} \quad \text{(9)}$$

The modulus of rigidity or shear (usually denoted by the symbol, G) is the same as the coefficient, n. The three elastic constants, E, G, and μ, are related by:

$$E = 2\,G\,(1 + \mu) \quad \text{(10)}$$

For soils it has been found that μ may vary between $\frac{3}{8}$ and $\frac{1}{6}$, the lower values applying to loose-textured soils and the higher values to clays and, possibly, dense sands. It should be noted that a variation in the value of μ changes only the scale, and not the manner, of the distribution of lateral pressure. A general form of expression for lateral pressure against a sheeted cut may be given as,

$$F = -\,K\,w\,y \quad \text{(11)}$$

Referring to Equation (1) K may receive a maximum value of 0.6 and a minimum value of 0.2. A rational method of designing sheeting and bracing would be to use the maximum value of K for the more rigid members of the supporting system and a much smaller one for the flexible sheeting.

Permanent structures, such as retaining walls, sub-surface structures, abutments, etc., require careful study as to possible movement before determining the maximum design pressures. Walls of uniform rigidity will move under the initial pressure. A drop in pressure ensues as the material supported by the wall expands elastically behind the wall. The pressure may again build up slowly, causing a further movement of the wall with another cycle of elastic expansion, until sufficient movement has occurred to cause fracture surfaces and possible slides. An analysis of these movements with their possible limitations by design of wall and foundation may indicate economies of design by using lower pressure coefficients. A further complication may enter should the material be saturated and should the foundation itself be wet, particularly if such hydraulic conditions may permit a loss of sustaining power due to the upward flow of water beneath the foundation.

Lateral Pressures Due to Surface Loads

The characteristic distribution of lateral pressures along a vertical surface due to a surface load is shown in Fig. 2. The maximum intensity is at the surface, diminishing rapidly with the depth. Any effect of the introduction of a

free vertical surface upon the analysis is ignored. It is assumed that the neglect of this factor introduces no serious error.

The simplest problem is that of a single load, analyzed by Boussinesq.[5] (The formulas usually attributed to Boussinesq are more properly termed the Boussinesq-Hertz formulas.) As loads are generally spread over large areas it becomes necessary to partition the area into smaller ones, treating each as a single load and adding the total to secure the composite effect. However, it is possible, by an analysis developed by A. E. H. Love,[6] to obtain, directly, the effect of a rectangularly distributed surface load. The formulas are more cumbersome, but are in a usable form entailing less work than the partitioning process; and they remove the uncertainty involved in a failure to partition the area into small enough units. When the loaded area is long and narrow, the strip method may be used—that is, the method of plane sections. Fairly simple formulas result for this case. To complete the analysis for strip loading, equations developed by Professor N. M. Guersevanoff for irregular types are added.[7]

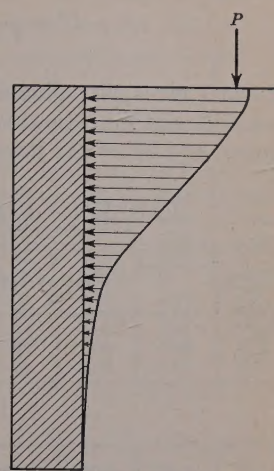


Fig. 2.—Typical Distribution of Lateral Pressures Against a Vertical Face Due to a Surface Load

Point Load.—Referring to Fig. 3(*a*),

$$F = -k\frac{P}{\rho^2} \qquad (12)$$

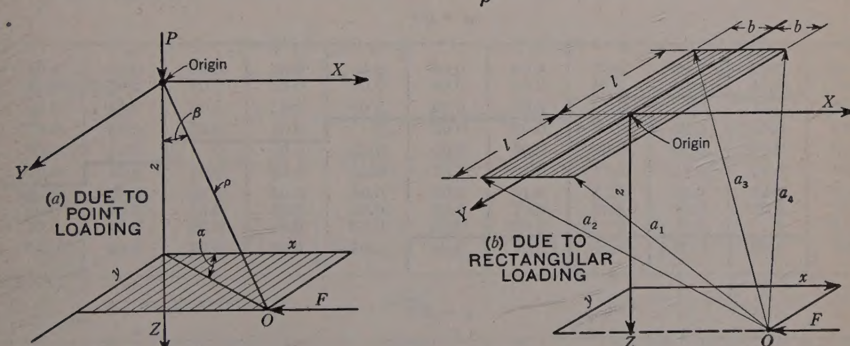


Fig. 3.—Co-Ordinate System for Lateral Pressures

in which P is the intensity of surface loads, and k is a constant, expressed by:

$$k = \frac{1}{2\pi}\left[3\sin^2\beta\cos\beta\cos^2\alpha - (1 - 2\mu)(\cos\beta - U)\right] \qquad (13a)$$

[5] See "Applications des Potentials," by Boussinesq.

[6] *Philosophical Transactions*, Series A, Vol. 228, 1929.

[7] These equations developed by Prof. Guersevanoff were taken from a brochure by him sent to the writers. The brochure was lost in a fire and the writers are unable to refer to the exact title and date of publication.

and, U is a substitution factor:

$$U = \frac{1}{1 + \cos\beta} - \frac{\sin^2\beta\,(2 + \cos\beta)}{(1 + \cos\beta)^2}\cos^2\alpha \quad \text{..........(13b)}$$

Table 1 gives the value of k for a range of values of the co-ordinate angles, α and β, and for several values of the Poisson ratio, μ. Values of k above the upper, and below the lower, heavy lines denote tensile stresses.

TABLE 1.—VALUES OF k FOR SOLVING EQUATIONS (13)*

Values of Angle β (see Fig. 3(a))	VALUES OF k FOR THE FOLLOWING VALUES OF ANGLE α (SEE FIG. 3(a))									
	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
					$\mu = 0.1$					
0°	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
10°	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06
20°	0.02	0.01	0.01	0.02	0.03	0.04	0.05	0.05	0.06	0.06
30°	0.04	0.04	0.03	0.02	0	0.02	0.02	0.04	0.04	0.04
40°	0.09	0.08	0.07	0.05	0.03	0.01	0	0.01	0.01	0.02
50°	0.10	0.09	0.09	0.08	0.06	0.04	0.03	0	0.01	0.01
60°	0.09	0.09	0.09	0.08	0.06	0.05	0.04	0.03	0.03	0.02
70°	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
80°	0.03	0.02	0.02	0	0.03	0.04	0.06	0.07	0.08	0.09
90°	0.12	0.12	0.10	0.06	0.01	0.03	0.06	0.10	0.12	0.13
					$\mu = 0.2$					
0°	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
10°	0.04	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05
20°	0.01	0	0	0.01	0.02	0.03	0.03	0.03	0.04	0.04
30°	0.05	0.05	0.04	0.03	0.02	0	0.01	0.03	0.03	0.03
40°	0.10	0.10	0.08	0.06	0.05	0.02	0.01	0	0.01	0.02
50°	0.12	0.12	0.11	0.10	0.07	0.05	0.03	0.01	0.01	0.01
60°	0.12	0.11	0.11	0.10	0.07	0.06	0.04	0.03	0.02	0.02
70°	0.07	0.07	0.07	0.07	0.06	0.05	0.05	0.04	0.04	0.04
80°	0	0	0	0.02	0.03	0.03	0.05	0.06	0.06	0.07
90°	0.09	0.09	0.07	0.05	0.01	0.02	0.05	0.07	0.09	0.10
					$\mu = 0.3$					
0°	0.03	0.03	0.03	0.03	0.03	0.03	0.03	−0.03	0.03	0.03
10°	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03
20°	0.02	0.02	0.02	0.01	0	0.01	0.02	0.02	0.03	0.03
30°	0.07	0.07	0.05	0.05	0.03	0.01	0.01	0.01	0.02	0.02
40°	0.11	0.11	0.10	0.08	0.06	0.04	0.02	0	0.01	0.01
50°	0.14	0.13	0.12	0.11	0.09	0.05	0.04	0.01	0.01	0
60°	0.14	0.13	0.12	0.11	0.08	0.06	0.04	0.02	0.02	0.01
70°	0.10	0.10	0.09	0.08	0.06	0.06	0.05	0.03	0.02	0.02
80°	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.04
90°	0.06	0.06	0.05	0.03	0	0.01	0.03	0.05	0.06	0.06

* Values above and below the two heavy lines denote tensile stresses.

TABLE 1.—*Continued*

Values of Angle β (see Fig. 3(*a*))	Values of k for the Following Values of Angle α (see Fig. 3(*a*))									
	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
					$\mu = 0.4$					
0°	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
10°	0	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02
20°	0.04	0.03	0.03	0.02	0.01	0	0	0	0.01	0.01
30°	0.09	0.08	0.07	0.06	0.05	0.03	0.02	0	0.01	0.01
40°	0.13	0.13	0.11	0.09	0.08	0.05	0.03	0.01	0	0.01
50°	0.16	0.15	0.14	0.13	0.10	0.06	0.04	0.02	0.01	0
60°	0.16	0.15	0.14	0.13	0.09	0.07	0.04	0.02	0.01	0.01
70°	0.12	0.12	0.11	0.10	0.07	0.06	0.04	0.02	0.01	0.01
80°	0.05	0.05	0.05	0.05	0.04	0.03	0.03	0.02	0.02	0.02
90°	0.03	0.03	0.02	0.02	0	0.01	0.02	0.02	0.03	0.03
					$\mu = 0.5$					
0°	0	0	0	0	0	0	0	0	0	0
10°	0.01	0.01	0.01	0.01	0.01	0.01	0	0	0	0
20°	0.05	0.05	0.05	0.04	0.03	0.02	0.01	0	0	0
30°	0.10	0.10	0.09	0.08	0.06	0.04	0.03	0.01	0	0
40°	0.15	0.15	0.13	0.11	0.09	0.06	0.04	0.02	0.01	0
50°	0.18	0.17	0.16	0.14	0.11	0.08	0.05	0.02	0.01	0
60°	0.18	0.17	0.16	0.14	0.10	0.07	0.04	0.02	0.01	0
70°	0.14	0.14	0.13	0.11	0.08	0.06	0.04	0.02	0	0
80°	0.08	0.08	0.07	0.06	0.05	0.03	0.02	0.01	0	0
90°	0	0	0	0	0	0	0	0	0	0

Rectangular Loading.—Referring to Fig. 3(*b*),

$$F = \frac{P}{2\,\pi}\left[2\,\mu\,A - (1 - 2\,\mu)\,B - z\,C\right] \quad \text{.............. (14)}$$

in which,

$$A = -\left[2\,\pi - (q_1 + q_2 + q_3 + q_4)\right] \quad \text{.............. (15a)}$$

$$B = (m_1 + m_2 + m_3 + m_4) - (n_1 + n_2 + n_3 + n_4) \quad \text{....... (15b)}$$

and,

$$C = -\left\{\frac{x_1}{k_1^2}\left[\frac{y_1}{a_1} + \frac{y_2}{a_4}\right] + \frac{x_2}{k_4^2}\left[\frac{y_1}{a_2} + \frac{y_2}{a_3}\right]\right. \quad \text{.......... (15c)}$$

$$x_1 = b - x \quad \text{....... (16a)} \qquad y_1 = l - y \quad \text{....... (16c)}$$

$$x_2 = b + x \quad \text{....... (16b)} \qquad y_2 = l + y \quad \text{....... (16d)}$$

and,

$$k_1^2 = x_1^2 + z^2 \quad \text{...... (17a)} \qquad k_3^2 = y_2^2 + z^2 \quad \text{...... (17c)}$$

$$k_2^2 = y_1^2 + z^2 \quad \text{...... (17b)} \qquad k_4^2 = x_2^2 + z^2 \quad \text{...... (17d)}$$

and,

$$\cos q_1 = \frac{x_1\,y_1}{k_1\,k_2} \quad \text{...... (18a)} \qquad \cos q_3 = \frac{x_2\,y_1}{k_2\,k_4} \quad \text{...... (18c)}$$

$$\cos q_2 = \frac{x_1\,y_2}{k_1\,k_3} \quad \text{...... (18b)} \qquad \cos q_4 = \frac{x_2\,y_2}{k_3\,k_4} \quad \text{...... (18d)}$$

If x or y is greater than b or l so that $\cos q$ is negative, substitute for q, in Equation (15a), the value, $\pi - q$. All angular values are in circular measure:

$$\tan m_1 = \frac{y_1}{x_1} \ldots\ldots (19a) \qquad \tan m_3 = \frac{y_1}{x_2} \ldots\ldots (19c)$$

$$\tan m_2 = \frac{y_2}{x_1} \ldots\ldots (19b) \qquad \tan m_4 = \frac{y_2}{x_2} \ldots\ldots (19d)$$

and,

$$a_1^2 = k_1^2 + y_1^2 \ldots\ldots (20a) \qquad a_3^2 = k_4^2 + y_2^2 \ldots\ldots (20c)$$

$$a_2^2 = k_4^2 + y_1^2 \ldots\ldots (20b) \qquad a_4^2 = k_1^2 + y_2^2 \ldots\ldots (20d)$$

and,

$$\beta_1 = \frac{z}{a_1} \ldots\ldots (21a) \qquad \beta_3 = \frac{z}{a_3} \ldots\ldots (21c)$$

$$\beta_2 = \frac{z}{a_2} \ldots\ldots (21b) \qquad \beta_4 = \frac{z}{a_4} \ldots\ldots (21d)$$

and,

$$\tan n_1 = \beta_1 \tan m_1 \ldots (22a) \qquad \tan n_3 = \beta_2 \tan m_3 \ldots (22c)$$

$$\tan n_2 = \beta_4 \tan m_2 \ldots (22b) \qquad \tan n_4 = \beta_3 \tan m_4 \ldots (22d)$$

If $\tan m$ or $\tan n$ appears negative, use negative values for these angles as derived from Equations (22).

This analysis for rectangular loads fails for points directly below the corners of the load. Determinate values may be found, however, for neighboring points and should serve the purpose.

TABLE 2.—LATERAL PRESSURES AT VARIOUS POINTS BELOW A LOADED AREA

($P = 1$; $b = 1$; $l = 2$; and the area $= 8$)

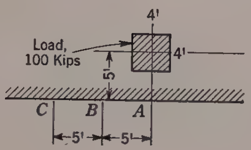
Values of Poisson ratio, μ	(a) $x=0$; $y=0$; and $z=1$				(b) $x=0$; $y=0$; and $z=0.5$			(c) $x=2$; $y=2$; and $z=1$			
	Rec-tangular values, Equation (14)	Values by Equation (12) for the following number of sub-divisions:			Rec-tangular values	Values by Equation (12) for the following number of sub-divisions:		Rec-tangular values	Values by Equation (12) for the following number of sub-divisions:		
		2	4	16		4	16		1	4	16
0.1	0.04	−0.16	0.0	0.03	0.26	0.28	0.24	0.06	0.04	0.06	0.06
0.2	0.08	−0.13	0.10	0.08	0.31	0.20	0.32	0.07	0.04	0.08	0.07
0.3	0.11	−0.06	0.16	0.11	0.35	0.20	0.36	0.08	0.05	0.09	0.08
0.4	0.14	−0.03	0.26	0.12	0.40	0.16	0.40	0.09	0.06	0.09	0.09
0.5	0.18	0	0.32	0.16	0.44	0.16	0.40	0.10	0.06	0.11	0.10

In applying the Boussinesq equations to spread footings, it is not easy, for points close to the loaded surface, to arrive at a proper sub-division for a fairly accurate result. Equations (14) to (22) give the correct theoretical result. For comparative purposes, the data in Table 2 show the values of the lateral pressures at certain points below a loaded area as derived by Equation (14) and

then show the derived pressures as obtained by using the Boussinesq equations for single and subdivided loads for the same area. The applied loading is taken as unit intensity.

Table 3 shows the distribution of lateral pressures under a pier footing. The total load on the pier is 100 kips, giving a load of 6.250 kips per sq ft.

TABLE 3.—DISTRIBUTION OF LATERAL PRESSURES, IN KIPS* PER SQUARE FOOT UNDER A PIER FOOTING



Distance below base of footing, in feet	Interval, in feet	Point *A*		Point *B*		Point *C*	
		Pressures for the Following Values of Poisson's Ratio, μ:					
		0.3	0.4	0.3	0.4	0.3	0.4
0		....	....	....	....	....	....
	2						
2		0.38	0.45	0.09	0.11	0.03	0.02
	3						
5		0.26	0.30	0.09	0.12	0.03	0.02
	3						
8		0.04	0.06	0.05	0.06	0.01	0.01
	5						
13		0.01	0.01	0	0.02	0	0
	5						
18		0	0	0	0.01	0	0

* 1 kip = 1 000 lb.

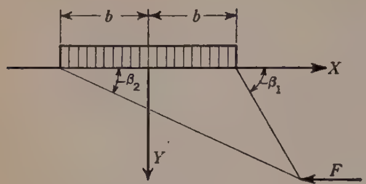
The lateral pressures have been found on the assumption of a single concentration at the center of the pier. As a check, using Equation (14) for Point $x = 5$; $y = 0$; $z = 2$, with $\mu = 0.3$; and $F = 0.38$. For $\mu = 0.4$, $F = 0.47$, which shows that the assumption of a single concentration is accurate.

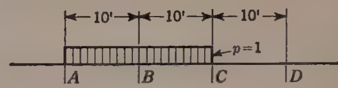
Strip Loading.—For a continuous strip of loaded area, such as a wall, railroad track, etc., a typical section may be taken, and the lateral pressure from this section is given by:

$$F = -\frac{p}{2\,\pi}\left[2\,(\beta_1 - \beta_2) + \sin 2\,\beta_1 - \sin 2\,\beta_2\right] \qquad (23)$$

The angles are in circular measure. Table 4 shows the distribution of pressures under the center, edge, and a distance of 10 ft away from a strip, 20 ft wide, loaded with unit loading. This type of loading is important in the analysis of the effect of neighboring foundation loads upon sub-surface structures and the use of Equation (23) obviates the application of "rule-of-thumb" methods of determining the zone of disturbance and calculation of lateral pressures from equivalent "surcharge" effects.

TABLE 4.—DISTRIBUTION OF LATERAL PRESSURES, IN KIPS* PER SQUARE FOOT, UNDER STRIP LOADING





Distance below base of footing (1)	Interval, in feet (2)	Point B (3)	Point C (4)	Point D (5)	Distance below base of footing (1)	Interval, in feet (2)	Point B (3)	Point C (4)	Point D (5)	Distance below base of footing (1)	Interval, in feet (2)	Point B (3)	Point C (4)	Point D (5)
0		....	....	....	5		0.45	0.35	0.17	30		0.01	0.04	0.09
1	1	0.88	0.47	0.04	10	5	0.18	0.23	0.22	40	10	0	0.02	0.05
2	1	0.75	0.44	0.08	15	5	0.08	0.14	0.18	50	10	0	0.01	0.03
3	1	0.64	0.41	0.12	20	5	0.04	0.09	0.15	60	10	0	0	0.02
4	1	0.52	0.37	0.15	25	5	0.02	0.06	0.11	70	10	0	0	0
5	1	0.45	0.35	0.17	30	5	0.01	0.04	0.09	80	10	....	....	....

* 1 kip = 1 000 lb.

Point Load.—If the strip is narrow, or points are selected at a distance from the strip (see Fig. 4(*a*)) a simple formula gives the lateral pressure,

$$F = \frac{2\,P\,y\,x^2}{\pi\,\rho^4} \qquad (24)$$

Trapezoidal Loading.—For the loading shown in Fig. 4(*b*):

$$F = \frac{2\,y}{\pi}\,(b_1\,M + b_2\,N) + \frac{1}{\pi}\,(b_1\,x + p_0)\,\tan^{-1}\frac{2\,b_2\,y}{u^2} \qquad (25a)$$

$$p_0 = \frac{1}{2}\,(p_a + p_b) \qquad (25b)$$

$$b_1 = \frac{1}{2\,b_2}\,(p_b - p_a) \qquad (25c)$$

$$M = \log_e \frac{\rho_1}{\rho_2} \qquad (25d)$$

$$u^2 = x^2 - b_2^2 - y^2 \qquad (25e)$$

and,

$$N = \frac{2\,b_1\,x\,y^2 + b_1\,x + p_0)\,u^2}{u^4 + 4\,x^2\,y^2} \qquad (25f)$$

Triangular loading, a special case of trapezoidal loading (see Fig. 4(*c*)), is expressed by,

$$F = -\frac{j}{\pi}\left[y\,Q + x\left(\tan^{-1}\frac{y}{x-t} - \tan^{-1}\frac{y}{x}\right) + y\,t\,\frac{x-t}{(x-t)^2+y^2}\right] \quad (26a)$$

and,

$$Q = \log_e \frac{(x-t)^2 + y^2}{x^2 + y^2} \quad (26b)$$

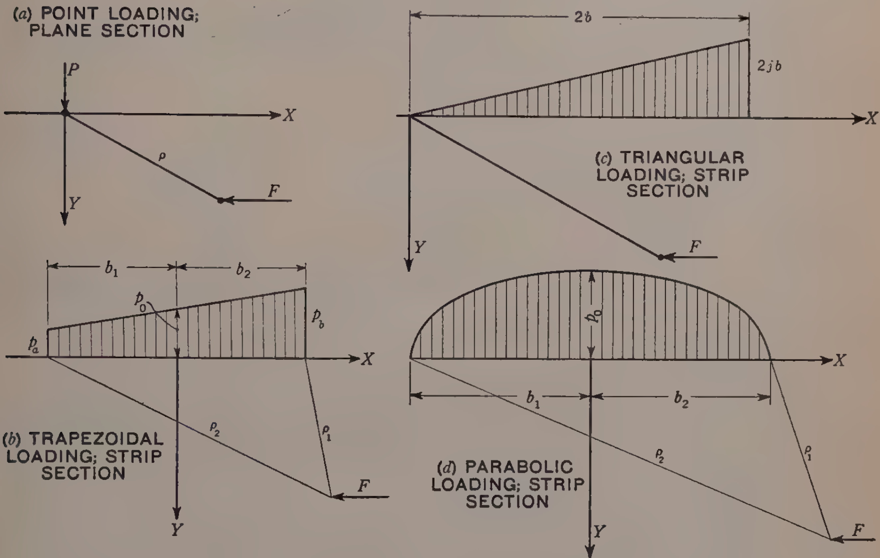


Fig. 4.—Lateral Pressures: Various Types of Loading

Parabolic Loading.—For the loading shown in Fig. 4(*d*):

$$F = -\frac{p_0}{\pi\,b_2^2}\left[(3\,y^2 + b_2^2 - x^2)\tan^{-1}\frac{2\,b_2\,y}{u^2} - 4\,x\,y\,M - 2\,b_2\,y\right] \quad (27)$$

in which u^2 and M are as defined by Equations (25*d*) and (25*e*).

The lateral pressures, as given by Equations (23) to (27), are assumed to be distributed along rigid supports yielding no more than the elastic material which it supports. Should the supporting structure be a composite one, such as temporary sheeting and bracing, the same cycles of elastic expansion and loading take place as described herein under the heading, "Theory of Sheeting and Bracing," and the more rigid parts of the sustaining system gradually take up the bulk of the loading.

Concrete Pressures

The construction of high concrete piers for the Alton (Ill.) Dam, on the Mississippi River, afforded an opportunity to make a series of readings to ascertain pressures exerted by green concrete on forms. The pressures were

computed from extensometer readings taken on tie-rods supporting the forms. The tie-rods were free to slip inside boiler-tubes which acted as separators for the form panels. Two pressure curves, derived from the extensometer readings, are shown in Fig. 5. The translation of extensometer readings to pressures is not given herein and is based upon an assumed distribution of pressure from sheeting panel to stud and wales and, in turn, to tie-rod.

The experimental curves have suggested to the writers that the methods outlined previously for the determination of lateral pressures may prove serviceable in analyzing concrete pressures.

A form of fairly standard construction is shown in Fig. 6. The problem is to trace the concrete pressures at some elevation, Plane AA, as the newly

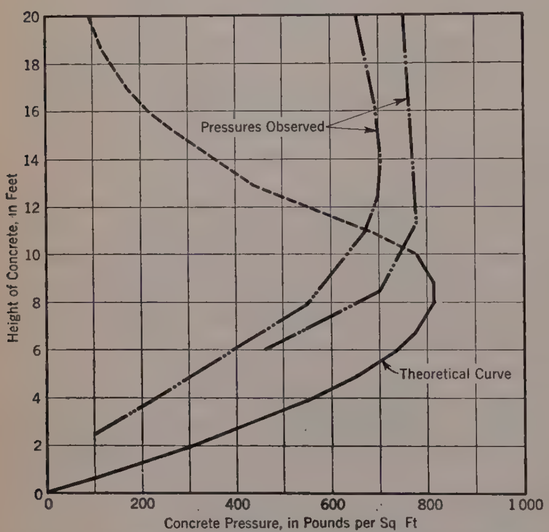


FIG. 5.—PRESSURE EXERTED BY NEWLY FORMED CONCRETE

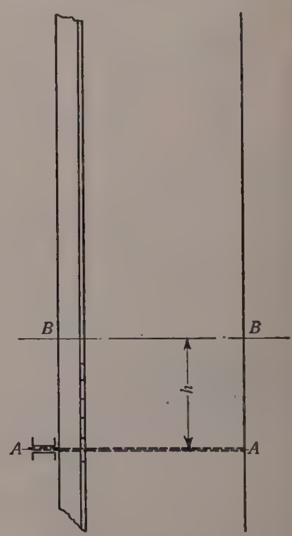


FIG. 6

poured concrete rises in the form, above the level, AA. From Point A to some height, h, the pressures at Point A are due to the weight of the concrete mass as expressed by Equation (6). The value of the Poisson ratio, μ, decreases from that of a pure liquid at Point A to the value of concrete when set. At the height, h, it is assumed that the concrete at Point A has taken a set and the pressures at that point are then due to a superimposed load upon a free surface which itself rises as the concrete takes a set above Level A. As the pressures gradually decrease in accordance with this condition, the theoretical curves should follow that shown in Fig. 5. Actually, however, the setting of the concrete prevents the tie-rod from returning to its initial length; it can simply recover the distortion it has undergone since the concrete passed Level h. The curves of pressure obtained experimentally take this shape and the foregoing analysis is an attempt to explain the result.

To give mathematical form to the foregoing reasoning it may be assumed that the change in the Poisson ratio occurring in the first stage of the concrete

pressures is given by an empirical relation,

$$\mu = \mu_0 + c\,t^2 \qquad (28)$$

in which t is the time the concrete has been in the form at Elevation AA (Fig. 6). If μ is assumed to have the liquid value at time, $t = 0$; and if it finally attains a value of 0.25 when the set has occurred (or, for time, $t = t_0$), the relation becomes,

$$\mu = \frac{1}{2} - \frac{t^2}{6\,t_0^2} \qquad (29)$$

and Equation (6) may be written in the form,

$$F = w\,h\,\frac{3 - U\,h^2}{3 + U\,h^2} \qquad (30)$$

in which,

$$U = \frac{1}{H^2\,t_0^2} \qquad (31)$$

since $h = H\,t$. The maximum pressure occurs for a value of $h = 0.84\,H\,t_0$ and, at this point, the maximum pressure becomes,

$$F = 0.52\,w\,H\,t_0 \qquad (32)$$

The determination of the pressures when the concrete at Elevation AA has set is taken from the formula for strip loading (Equation (23)). Since the condition is expressed by Fig. 7, the equation may be simplified for this case so that,

$$F = \frac{P_B}{2\,\pi}\,(2\,\beta_0 - \sin 2\,\beta_0) \qquad (33)$$

in which P_B is the weight of the concrete above Plane BB (Fig. 6). Angle β_0 is determined by the width of the form and the distance, z. The weight of concrete, P, will be constant and the variable will be the distance, z. In Fig. 5 the shape of this curve of decreasing pressures is shown dotted.

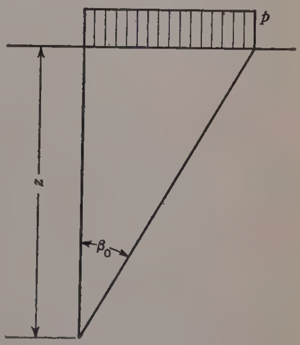


FIG. 7

In the experiments at Alton the weight of the concrete was about 155 lb per cu ft and the level rose in the form at a rate of 2 ft per hr. The time of set was about 5 hr. In the foregoing analysis, therefore, $H = 2$; and, $X_0\,t_0 = 5$. The height for maximum pressure is then 8.4 ft and the maximum pressure, 806 lb. To complete the curve (not of practical significance herein) take $B = 1\,550$ lb and use Equation (33).

Obviously, better fit may be made to the actual observed curve of pressure by "juggling" the empirical relation given by Equation (28), but this mathematical manipulation is not warranted by the conditions of the problem, and the similarity of shape of the curves indicates that the theory possesses some

validity and, therefore, seems a better guide to the determination of concrete pressures than the usual assumptions of modified hydrostatic relations.

Conclusions

The analysis of lateral earth pressures resolves itself into a study of three phases of this problem. The first phase is that of the distribution of pressures along a wall sufficiently rigid or unyielding so that no fracture surfaces appear in the ground. In this case the pressures cause a slight yielding of the wall and a redistribution of pressures appears, tending to concentrate around the more unyielding portions of the supporting system. The pressure systems may be then separated for each system of support using a smaller intensity of pressures for the more yielding portion of the system such as the sheeting and to some extent the wales, and the maximum intensity of pressures for the braces or tie-rods. To accomplish this analysis it is suggested that the soil be assumed to possess a variable Poisson's ratio, the smaller for the sheeting and the highest for the braces. Although this may seem an arbitrary assumption as to physical constants of the soil, it seems at present the most suitable device to use.

The second phase, that of pressures developed when a fracture surface is permitted to develop is discussed very briefly. Only in this phase has the analysis of Rankine and Coulomb any significance. The writers hope to present some numerical examples of pressures developed in actual work in this phase at some later date.

The third phase, that of the distribution of pressures due to a surface load, is covered in extensive mathematical detail. It is hoped that the results given will supersede the usual rule-of-thumb method for determining pressures due to surcharges. It is important to note that the intensity of pressures due to a surface load is a maximum quite near the surface and diminishes rapidly in intensity, a fact quite in contradiction to the usual method of analyzing surcharge loads.

The writers have purposely avoided introducing the terms, "coherent," "granular," "non-coherent," and, finally, "angle-of-repose" or "internal friction." For a long period they have recognized that these terms are misleading and of no significance in discussing lateral pressures. The successful application of the principles of the theory of elasticity to soils, both in theory and in their own practice has convinced them that the road to correct determination of soil phenomena lay in such application and not in futile attempts to extend the classic formulas by modifications of coefficients, and the like.

Finally, it should be stated that almost ten years have elapsed since the writers started the preparation of this paper. Experiments have been performed and observations made on lateral pressures in this period which the writers trust will be brought forth in the course of discussion which may clarify or even refute some of the theory presented.

APPENDIX

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering and Testing Materials[8] compiled by a committee of the American Standards Association with Society representation, and approved by the Association in 1932:

A = a substitution factor defined by Equation (15*a*);
a = a distance; a_1, a_2, a_3, and a_4 (see Fig. 3(*b*)) are distances to the corners of a rectangular load;
B = a substitution factor defined by Equation (15*b*);
b = breadth; one-half the width of a rectangular load; b_1 and b_2 are defined where they are introduced;
C = a substitution factor defined by Equation (15*c*);
E = modulus of elasticity;
e = base of Naperian logarithms;
F = total lateral pressure;
G = modulus of elasticity in shear; modulus of rigidity;
g = acceleration due to gravity;
H = head;
h = height;
I = rectangular moment of inertia;
j = a ratio in Fig. 4(*c*);
K = hydrostatic pressure ratio;
k = a substitution coefficient defined by Equation (13*a*);
L = length;
l = one-half the length of a rectangular load;
M = a substitution factor defined by Equation (25*d*);
m = a constant; m_1, m_2, m_3, and m_4 refer to the corners of a rectangular load;
N = a substitution factor defined by Equation (25*f*);
n = a constant (equal to G); n_1, n_2, n_3, and n_4 refer to the corners of a rectangular load;
P = a load; P_B = the weight of the concrete above Plane BB;
p = pressure per unit area; p_H = intensity of lateral pressure; p_V = intensity of vertical pressure;
Q = a substitution factor defined by Equation (26*b*);
q = a substitution factor defined by Equations (18);
t = setting time for concrete;
U = a substitution factor defined by Equation (31);
u = a substitution factor defined by Equation (25*e*);
w = load per unit volume;
x = distances parallel to the X-axis; x_1, x_2, x_3, and x_4 are distances, x, to the corners of a rectangular load;

[8] A.S.A.—*Z* 10*A*—1932.

y = deflection; also distances measured parallel to the Y-axis; y_1, y_2, y_3, and y_4 are distances, y, to the corners of a rectangular load;
z = distances parallel to the Z-axis measured from the base of a footing; z_1, z_2, z_3, and z_4 are distances, z, to the corners of a rectangular load;
α = angular distance to the point of application of Force F, in the XY-plane;
β = half-angle formed under a load, P, by the radius, ρ (Fig. 3(a));
δ = unit elongation;
ϵ = strain; for example, ϵ_e, ϵ_f, and ϵ_g in Equations (2) to (6);
θ = angular distance in polar co-ordinates;
μ = Poisson's ratio;
ρ = radial distance in polar co-ordinates.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

WIND FORCES ON A TALL BUILDING

By J. Charles Rathbun,[1] M. Am. Soc. C. E.

Synopsis

Since the construction of the Empire State Building in New York, N. Y., considerable data have been collected pertaining to the intensity and distribution of the wind pressure during storms, as observed by instruments placed in this structure. At the same time the movement of the top of the building and the stresses in some of the members of the frame have been observed.

A model of the steel frame was constructed and measurements of its deformation under horizontal load, as well as its period of vibration, were noted. From these observations the actual lateral loads on the building were estimated under certain assumptions. The ratio between the stiffness of the building and its frame (acting alone) was then computed to give an indication of the percentage of the load that is carried by the frame. The plastic as well as the elastic action of the structure is indicated in the data.

The following studies were contemplated when this set of experiments was undertaken:

(*a*) The relation between the velocity of the wind and the pressure produced;

(*b*) The variation of stresses due to wind across a bent;

(*c*) A check, or refutation, of the basic principles of the portal and cantilever methods of analysis; and

(*d*) An estimate of the proper horizontal load that should be used in designing a tall building to insure proper rigidity and stability.

Because of the erratic nature of the data, definite conclusions have not been reached in the solution of these problems.

The instruments installed in the building for the special purposes of the observations were: 1 anemometer; 30 manometers; 28 cameras with operating mechanism; 22 extensometers; 1 collimator and target; and 1 plumb-bob.

Note.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted **November 15, 1938.**

[1] Associate Prof., Civ. Eng., Coll. of the City of New York, New York, N. Y.

The paper is divided into three parts. Part I contains a discussion of: (*a*) The nature of the wind and the pressure produced by it; and also (*b*) the deflection and stresses in the building due to wind action. Part II gives an estimate of the stiffening effect of the masonry on the steel and an approximate evaluation of the uniform load on the structure due to the wind. A model, with its equipment, was tested in the laboratory for the investigation described in Part II. Finally, Part III cites nine concise conclusions.

PART I.—DESCRIPTIVE

Introduction

The Technical Research Committee of the American Institute of Steel Construction by arrangement with the owners, architects, consulting engineer, and contractors for the Empire State Building equipped that building during its construction with instruments, by the use of which data were later obtained as to the effect of wind and the resulting stresses and strains.

With the exception of the work of Clyde T. Morris, M. Am. Soc. C. E., on the American Insurance Union Building[2] this is the first opportunity known to the writer in which engineers have been permitted to conduct an extensive study of the action of a large building over a period of several years, with a view to increasing their knowledge of the action of a structure subjected to wind loads. The data obtained, therefore, are of considerable value inasmuch as they tend to clarify the very intricate problem of determining the nature of wind pressure on large areas and permit the study of the plastic and elastic action of the building itself.

By the nature of the problem a high degree of accuracy was not necessary; nor was it obtainable. There were many factors beyond the control of the observers, and many data that were necessary for a complete solution of the problem could not be obtained. From the beginning of the project in April, 1931, until the removal of the instruments in the summer of 1936 a considerable number of observers gave their time to the task of making those records. Many of these data were fragmentary but as a whole they furnish a picture of the phenomena being observed.

Description of Building

The Empire State Building is the tallest in the world to date (1938); and it is not, like many tall buildings, simply a tower surmounting a comparatively low building. Its breadth is sufficient to present a very extensive surface to the wind (see Fig. 1). Its plan dimensions are 197.5 ft by 424.5 ft from the ground to the sixth floor. It has several setbacks, but from the thirtieth floor to the seventy-third, it covers approximately one-fourth the area of the ground floor. From the seventy-third floor it is gradually reduced in area to the roof on the eighty-sixth floor. A mast which supports the anemometer for measuring the wind velocity extends about 200 ft above the roof. All observations were made below the eighty-sixth floor. The design is almost ideally simple from a

[2] "Practical Design of Wind Bracing," by Clyde T. Morris, *Proceedings*, Am. Inst. of Steel Construction, Inc., October, 1927.

structural point of view. All the floors are symmetrical about two axes, with but few minor exceptions, which have little or no effect on the action of the building. The total height above the street floor, including the mast, is 1 248 ft.

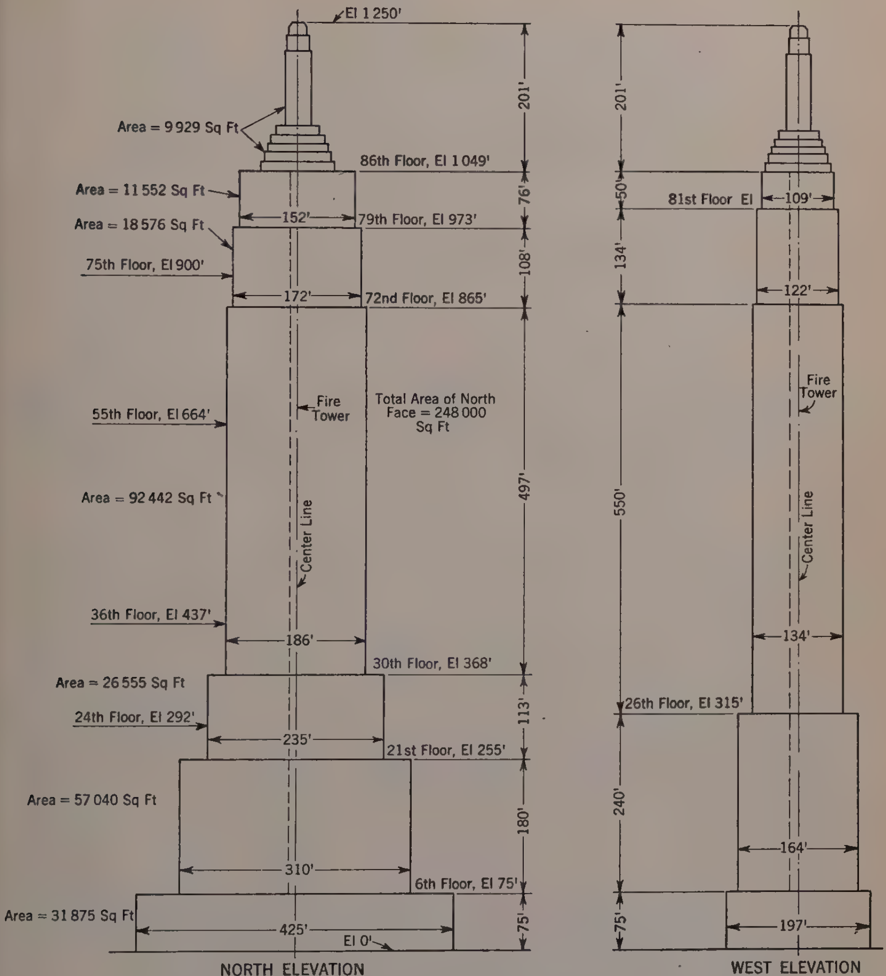


FIG. 1.—SIDE VIEWS OF EMPIRE STATE BUILDING

Structurally, the building consists of steel columns and girders, the latter being fastened to the columns by the shallow wind-brace connections of the type shown in Fig. 2. In addition, the central bays around the elevator shafts have a system of knee-braces which extend from approximately the third point on the girder to the point where the corresponding girder on the floor below joins the column. The system of knee-braces in any set of bays between two columns, with the accompanying columns and floor-beams, forms a vertical

truss which materially stiffens and strengthens the frame of the building. These are shown with their form of bracing in Fig. 3 which gives an outline of the frame of the building in the narrow direction. As explained in Part II, there are three typical frames in the narrow direction of the building.

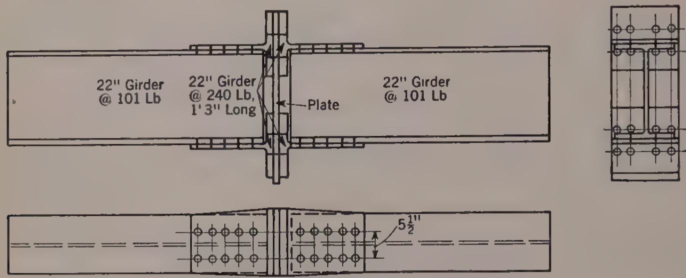


FIG. 2.—TYPICAL COLUMN AND GIRDER JOINT

The floors are of cinder concrete slab construction supported by steel beams approximately 8 ft apart. The outside partitions are covered with chrome-nickel steel and Indiana limestone. Until a floor was rented it had no interior partitions surrounding the elevators and utility rooms. Fig. 4 is a typical layout.

With the exception of one offset for the main entrance the columns are in rows in both directions and each column extends from the foundation to the roof. The symmetry and regularity of the building have aided greatly in the study of its action.

Surrounding the steel, and acting with it, are other materials of construction, such as concrete floors, tile and plaster partitions, etc. Much of this material is not elastic and does not follow Hooke's law. As a result, when it is subjected to loads, the building shows a predominance of plastic action; that is, when it is deflected by a horizontal force it will not return exactly to a definite fixed position. A study of the readings of the building deflections reveals this phenomenon clearly.

THE VICINITY

The buildings in the vicinity of the structure are low compared to those a few blocks farther north, where one group of the tall buildings of the city is located. The buildings along Forty-Second Street present a solid wall of high structures which tend to break up the air currents at elevations above, as well as below, their tops. There are a number of tall buildings to the west as well as to the north; and, to the south and the southeast, there are few obstructions. Fig. 5, a view of the city near the building, affords some idea of the character of obstructions which, it is easily seen, are of a type that would disturb the air currents and interfere with any tendency to be streamlined.

Fig. 6 is a map giving the approximate elevations, in feet, of the roofs of those buildings that are high enough, and close enough, to affect the air currents seriously. Fig. 7 gives the heights, in number of stories, of the buildings in the immediate neighborhood.

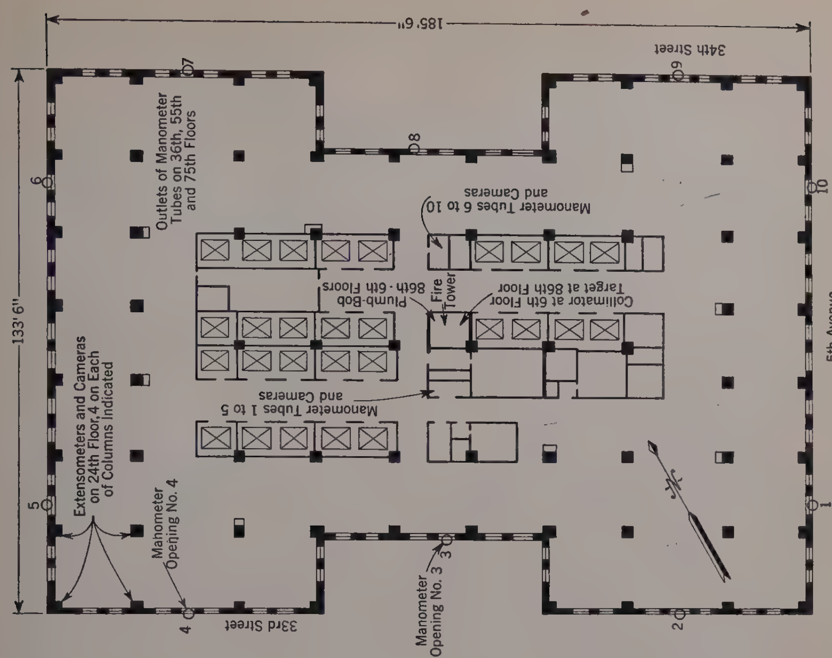


FIG. 4.—TYPICAL FLOOR LAYOUT. NUMBERS DENOTE OBSERVATION STATIONS

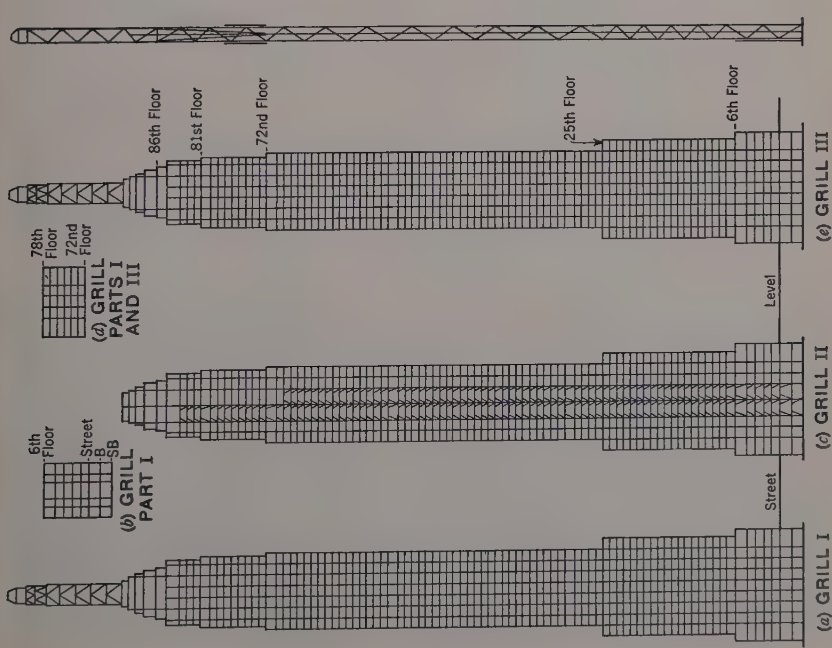


FIG. 3.—OUTLINE OF MODEL AND OF STEEL FRAME OF PROTOTYPE, NARROW DIRECTION

Several methods were used in making the survey for the maps in Figs. 6 and 7, after which the public records for the heights of the more important buildings were looked up as a check. Accuracy sufficient for the purpose was obtained by these methods although they could not be relied upon where great care was necessary. The first method used was to observe the heights with a hand-level from the several floors of the Empire State Building. This method proved inaccurate for all except the closest structures.

An ingenious method was suggested and used by one of the student observers, which proved quite successful. Photographs were taken from the seventy-fifth, fifty-fifth, and thirty-sixth floors using windows in the same vertical line. With the known distance between floors as a base the heights of the buildings could be computed from the photographs by triangulation. The distances to the buildings were known from the published maps so that checks on this work were obtained.

From Figs. 5, 6, and 7, it is quite evident that the structure is many times higher than most of its neighbors and even those that compare with it in height (more than one-half as high) are isolated towers a considerable distance away. The air currents at the lower elevations (to a height that is extremely problematical) are very much disturbed.

Velocity of the Wind

The velocity and direction of the wind were obtained from an anemometer installed approximately 15 ft above the top of the mast, or 1 263 ft above the street. This instrument relies for its operation on a 32-blade rotor which does not spin (as does the cup anemometer). The dial (not self-recording), was read just before and just after the deflection readings. The hands often fluctuated over a range of about 15 miles per hr during a severe storm. The average velocity was taken and not the maximum due to this swing. In one instance, a fluctuation of 20 miles per hr on each side of a mean of about 50 miles per hr was noted.

There is some question as to the accuracy of the anemometer for recording the velocity of the storm. Although the instrument may register, correctly, the velocity at the mast head at the time noted, the air currents are doubtless influenced by the presence of the building itself, as well as by the other tall buildings in the vicinity.

There are three Weather Bureaus on Manhattan Island, all within less than five miles of the Empire State Building. In order to have as many data as possible on the wind movement during the observations the records of these several Weather Bureaus were consulted. Table 1 gives the direction and velocity of the wind at each station, at the time of the observation indicated. In the case of the anemometer at the Empire State Building, the readings were taken as near as possible to the time of taking the other data and at the time indicated in Column (2). If the velocity needle was fluctuating the mean reading was taken. In the case of the three neighboring stations, the average velocity and direction during the hour of the readings were recorded. In addition, the maximum velocity for a 5-min period attained by the wind during

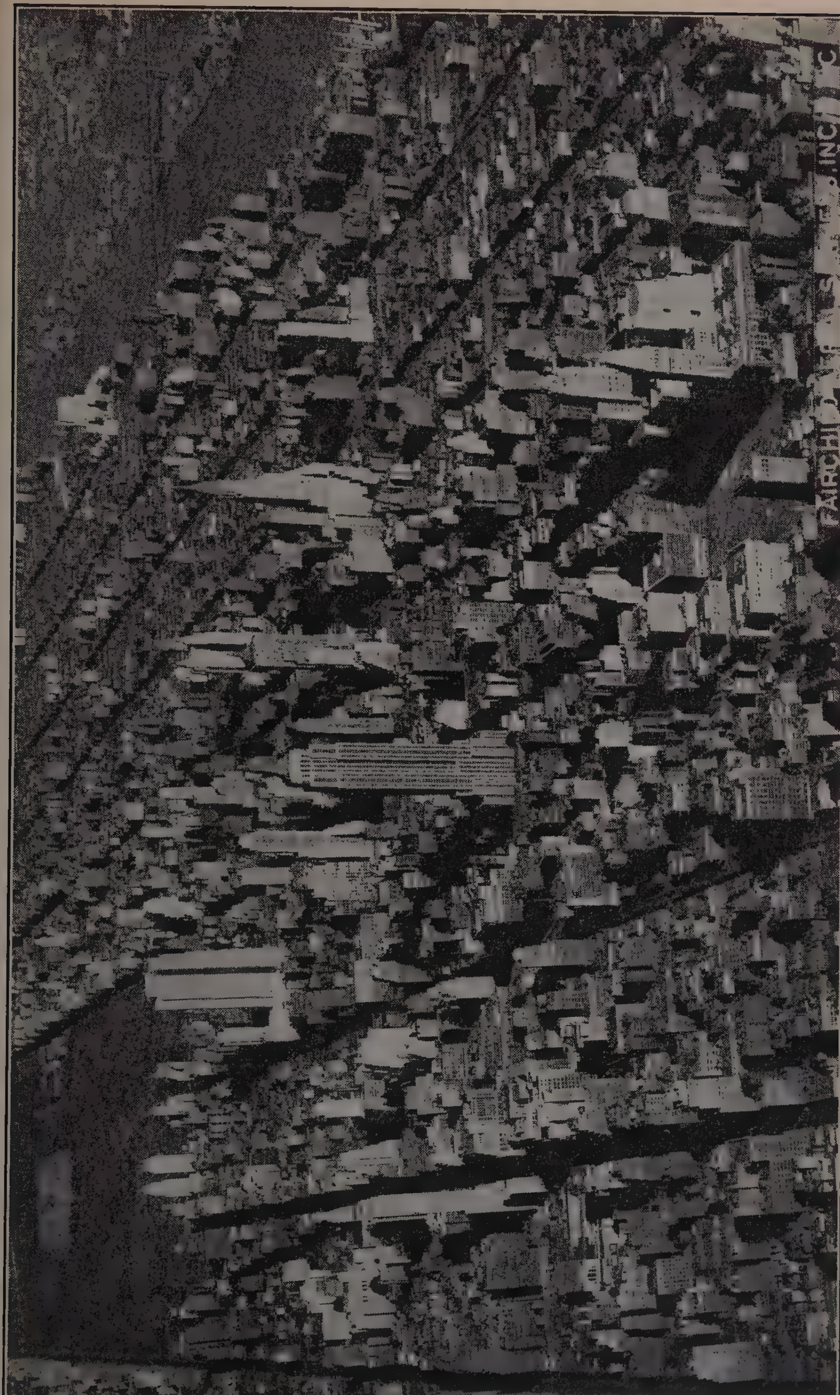


FIG. 5.—BUILDINGS IN VICINITY OF THE EMPIRE STATE BUILDING; VIEW FACING NORTHEAST

TABLE 1.—WIND RECORDS, NEW YORK, N. Y.

Item No.	Date*	Eastern Standard Time, in hours and minutes past midnight	Empire State Building		New York City Meteorological Observatory		*Daily News* Building		United States Weather Bureau Observatory			
									Hourly reading		Maximum	
			Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Velocity, in miles per hour	Time, in hours and minutes past midnight
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Height of anemometer, in feet		....		62†		504‡		454§			
	Maximum velocities (5-min period); 3/22/36...		....	....	....	34	....	....	....	....	65	....
	Extreme velocities		....	102	....	54.4	....	....	....	....	71	....
1	2/10/32	15:00	SW	5	NNE	4	SW	2	NW	5	....	....
2	2/10/32	17:00	N	4	ENE	6	NE	5	NW	3	....	....
3	2/11/32	13:00	S	15	ENE	9	NE	9	NE	4	....	....
4	2/12/32	16:00	W	35	WSW	11	W	14	SW	14	....	....
5	3/ 9/32	14:30	W	20	WNW	12	NW	15	W	21	....	....
6	3/ 9/32	16:00	W	25	WSW	13	NW	18	W	21	....	....
7	3/10/32	15:00	WNW	45	NW	18	NW	17	NW	30	....	....
8	3/11/32	14:30	WNW	40	NW	16	NW	14	NW	27	30	14:50
9	3/25/32	14:30	SE	19–22	SSW	12	SE	15	S	17	....	....
10	4/25/32	14:30	SE	18–22	ESE	9	SE	11	SE	12	....	....
11	4/27/32	14:20	NW	29–38	WNW	18	NW	18	NW	32	46	14:36
12	4/29/32	14:00	SSW	12–18	S	12	SE	18	S	18	....	....
13	5/ 2/32	14:15	NW	28	NNW	12	NW	12	NW	21	....	....
14	5/ 3/32	14:04	WNW	8–11	NNW	10	NW	8	NW	14	....	....
15	5/ 4/32	13:55	SSW	12	WNW	7	S	8	SW	12	....	....
16	5/ 9/32	14:55	E	11–14	ESE	9	SE	6	SE	13	....	....
17	5/18/32	14:00	N	8	NNW	7	NE	3	NW	7	....	....
18	5/18/32	14:45	NNE	5	NNW	7	NE	3	NW	7	....	....
19	5/19/32	14:00	SSE	13–17	SSW	10	S	10	S	12	....	....
20	5/19/32	15:25	SSE	15	SSW	11	SE	12	S	14	....	....
21	5/23/32	11:15	W	15	WNW	9	NW	10	SW	14	....	....
22	5/23/32	12:00	W	12	WNW	9	NW	10	SW	15	....	....
23	5/24/32	11:00	SSW	8	SSW	6	S	8	SW	10	....	....
24	5/25/32	10:00	WSW	18	WSW	9	SW	11	SW	12	....	....
25	5/25/32	14:50	SSW	22	WSW	12	SW	11	SW	16	....	....
26	5/26/32	14:40	SSW	30	WSW	14	SW	19	SW	24	25	14:40
27	5/26/32	15:50	SSW	35	WSW	14	SW	20	SW	21	....	....
28	5/31/32	14:40	SSW	18	SSW	15	S	18	S	16	....	....
29	6/ 3/32	15:15	SSW	16	S	11	S	13	S	14	....	....
30	6/ 6/32	15:35	NW	22–24	NW	10	N	9	NW	17	....	....
31	6/ 7/32	16:45	N	45	NNE	18	N	19	N	26	31	16:40
32	6/ 8/32	13:25	WNW	25	NW	13	NW	11	NW	22	27	11:54
33	6/ 8/32	15:40	WNW	25	NW	14	NW	9	NW	21	26	15:29
34	6/ 9/32	14:40	WSW	30	WNW	13	SW	13	W	22	....	....
35	6/13/32	11:50	NE	15	ENE	12	NE	16	E	6	....	....
36	6/14/32	15:30	NE	2	E	6	SE	3	S	9	....	....
37	6/15/32	17:30	S	25	SSW	10	S	14	E	18	....	....
38	6/16/32	14:00	NNE	20	ESE	9	S	7	S	8	....	....
39	6/16/32	14:25	SSE	12	ESE	9	S	7	S	8	....	....
40	6/17/32	13:00	N	15	ENE	12	NE	13	E	8	....	....
41	6/17/32	13:20	N	20	ENE	12	NE	13	E	8	....	....
42	6/17/32	14:20	N	25	ENE	12	NE	19	NE	10	....	....
43	6/22/32	17:50	W	30	WNW	12	NW	17	NW	22	28	16:04
44	6/23/32	11:55	WNW	30	NW	16	NW	13	NW	25	29	10:00
45	6/23/32	15:25	W	32	WNW	15	NW	17	NW	29	36	14:25
46	6/24/32	11:15	WNW	25	NNW	14	NW	10	NW	24	28	11:09
47	6/24/32	13:50	WNW	40	NW	15	NW	15	NW	27	32	11:54
48	6/24/32	17:18	W	30	WNW	14	NW	16	W	24	27	17:04

* Month, day, year. † Height above the ground. ‡ Height above the street. § Height above the harbor.

TABLE 1—(*Continued*)

Item No.	Date*	Eastern Standard Time, in hours and minutes past midnight	Empire State Building		New York City Meteorological Observatory		*Daily News* Building		United States Weather Bureau Observatory			
									Hourly reading		Maximum	
			Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Direction of wind	Velocity, in miles per hour	Velocity, in miles per hour	Time, in hours and minutes past midnight
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Height of anemometer, in feet		....		62†		504‡		454§			
	Maximum velocities (5-min period); 3/22/36...		....	....	....	34	....	....	....	....	65	....
	Extreme velocities		....	102	....	54.4	....	....	....	....	71	....
49	6/29/32	16:55	WNW	12	NW	9	NW	7	SW	11	....	....
50	7/ 1/32	15:00	S	30	SSW	15	S	27	S	29	32	15:19
51	7/ 5/32	17:15	WNW	20	NNW	7	NW	8	NW	15	....	....
52	7/ 6/32	13:00	SW	12	WSW	9	SW	12	SW	10	....	....
53	7/ 7/32	14:40	SSE	12	S	10	S	13	S	12	....	....
54	7/11/32	13:10	NW	40	WNW	18	NW	18	NW	29	35	10:55
55	7/14/32	12:55	NNW	10	NNE	11	N	10	N	13	....	....
56	7/19/32	12:45	WNW	20	WNW	10	NW	13	NW	17	....	....
57	7/25/32	14:30	W	8	NW	6	NW	6	N	7	....	....
58	7/27/32	10:30	SSW	15	WSW	7	SW	7	S	9	....	....
59	7/29/32	12:30	WNW	20	WNW	9	W	11	NW	15	....	....
60	8/ 2/32	10:15	SSW	13	SSW	8	S	9	S	11	....	....
61	8/ 4/32	10:27	S	32	NNW	12	N	14	NW	17	....	....
62	8/ 4/32	13:50	S	38	NNW	10	NW	9	NW	15	....	....
63	8/22/32	12:00	NNW	5	NW	5	N	6	N	9	....	....
64	8/23/32	11:45	NE	10	E	12	NE	15	NE	9	....	....
65	8/24/32	13:25	S	8	SSW	9	S	11	SW	13	....	....
66	8/31/32	16:25	E	6	E	9	SE	7	SE	9	....	....
67	9/ 7/32	11:25	NNW	12	NNW	10	NE	11	N	13	....	....
68	9/ 8/32	12:45	NNW	42	NNE	24	NE	27	N	29	....	....
69	9/ 8/32	14:50	NNW	55	NNE	21	NE	26	N	34	40	14:17
70	9/ 8/32	16:10	NNW	40	NNE	21	NE	24	N	36	44	16:18
71	9/12/32	14:07	W	10	WNW	7	NW	8	SW	11	....	....
72	9/27/32	10:00	SSW	22	WSW	13	SW	17	SW	15	....	....
73	9/27/32	16:15	SSW	22	SW	12	SW	14	SW	18	....	....
74	9/28/32	9:30	SSW	42	SW	15	SW	20	SW	26	28	9:30
75	9/28/32	10:15	....	40	SW	16	SW	21	SW	26	29	10:30
76	9/28/32	14:50	WNW	40	WNW	18	NW	18	NW	32	....	....
77	10/ 3/32	13:10	S	42	SSW	14	SE	21	SE	24	....	....
78	10/ 3/32	16:30	SE	26–35	SSW	14	S	24	S	24	....	....
79	10/ 4/32	16:40	SE	35	SSE	10	SE	17	SE	18	....	....
80	10/ 5/32	9:00	SSE	52	SSE	14	S	29	SE	28	....	....
81	10/ 5/32	14:40	SSE	45	S	14	S	27	SE	27	....	....
82	10/ 6/32	9:20	SSE	65–75	S	16	S	33	SE	33	36	9:44
83	10/ 7/32	10:15	WNW	32	WNW	16	NW	15	NW	27	34	10:33
84	10/11/32	9:20	WSW	34	WSW	10	W	15	W	16	....	....
85	10/12/32	9:00	SW	35	WSW	16	W	19	SW	24	....	....
86	10/12/32	13:10	W	47	WNW	21	W	28	NW	38	50	13:39
87	10/14/32	16:50	WSW	10	NW	6	W	6	W	10	....	....
88	10/17/32	9:00	SE	22	ENE	9	E	9	E	9	....	....
89	10/18/32	11:35	NE	35	ENE	19	NE	28	NE	15	....	....
90	10/18/32	11:50	NE	42	ENE	19	NE	28	NE	15	....	....
91	10/19/32	8:50	NNE	32	NE	18	NE	24	NE	13	....	....
92	10/21/32	10:00	W	22	WNW	12	NW	16	W	18	....	....
93	10/21/32	10:20	W	28	WNW	12	NW	16	W	17	....	....
94	10/21/32	16:20	WNW	50	NW	18	NW	18	NW	32	35	16:13
95	10/22/32	9:10	NE	22	WNW	8	W	11	NW	17	....	....
96	10/25/32	13:10	E	15	ESE	10	SE	12	E	12	....	....

TABLE 1—(*Continued*)

Item No.	Date*	Eastern Standard Time, in hours and minutes past midnight	Empire State Building: Direction of wind	Empire State Building: Velocity, in miles per hour	New York City Meteorological Observatory: Direction of wind	New York City Meteorological Observatory: Velocity, in miles per hour	*Daily News* Building: Direction of wind	*Daily News* Building: Velocity, in miles per hour	United States Weather Bureau Observatory, Hourly reading: Direction of wind	United States Weather Bureau Observatory, Hourly reading: Velocity, in miles per hour	United States Weather Bureau Observatory, Maximum: Velocity, in miles per hour	United States Weather Bureau Observatory, Maximum: Time, in hours and minutes past midnight
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Height of anemometer, in feet		....		62†		504‡		454§			
	Maximum velocities (5-min period); 3/22/36...		....	....	....	34	....	....	....	....	65	....
	Extreme velocities		....	102	....	54.4	....	....	....	....	71	....
97	10/26/32	11;05	SSE	13	ESE	7	SE	6	SE	9	....	....
98	10/28/32	14:40	SSE	27–35	NW	18	NW	15	NW	29	34	14:58
99	10/29/32	8:55	SSW	4–10	W	5	W	5	WSW	7	....	....
100	11/ 1/32	9:00	SE	75	SSE	17	SE	40	SE	39	40	....
101	11/ 1/32	10:30	SE	75	SSE	18	SE	41	SE	40	....	....
102	11/ 1/32	12:00	SSE	75–95	SSE-NW	14	NW	16	NW	56	....	....
103	10/ 1/32	16:05	WNW	42	NW	17	NW	16	NW	29	....	....
104	11/ 1/32	16:50	WNW	45	NW	17	NW	16	NW	23	29	....
105	11/ 1/32	17:15	WNW	50	N	17	NW	16	NW	29	36	....
106	11/ 9/32	11:00	NE	45–57	ENE	24	NE	34	NE	12	....	....
107	11/ 9/32	12:00	ENE	45–60	ENE	27	E	36	NE	12	....	....
108	11/ 9/32	14:20	ENE	55–70	ENE	28	E	38	NE	14	....	....
109	11/ 9/32	14:45	ENE	60	ENE	28	E	38	NE	12	....	....
110	11/ 9/32	16:30	ENE	60	ENE	29	E	41	NE	13	....	....
111	11/11/32	11:00	S	25	SSW	10	S	16	S	13	....	....
112	11/15/32	9:50	SW	35	WSW	12	SW	12	SW	8	....	....
113	11/18/32	11:00	ENE	6–10	ENE	10	NE	9	ENE	7	....	....
114	11/18/32	16:00	ENE	15–16	ENE	9	E	12	ENE	6	....	....
115	11/19/32	13:10	SSE	65	SSW	22	S	37	S	44	49	13:24
116	12/ 2/32	14:00	WSW	35	WSW	13	SW	13	SW	14	....	....
117	12/ 2/32	16:00	SW	35	WSW	13	SW	16	SW	12	....	....
118	12/ 9/32	10:15	S	7	S	4	S	7	S	7	....	....
119	12/21/32	11:40	SSW	34	WSW	12	SW	14	SW	22	....	....
120	12/27/32	17:20	ENE	12	NE	11	NE	17	NE	10	....	....
121	12/28/32	9:40	SSW	12	ENE	6	SW	6	SW	9	....	....
122	12/30/32	11:45	SSW	8	E	1	SW	5	S	10	....	....
123	1/ 6/33	17:10	SW	29	WSW	12	W	15	W	11	....	....
124	1/ 7/33	12:15	WSW	50	WSW	17	W	24	W	30	35	12:06
125	1/11/33	9:10	SSW	42	SW	19	SW	24	SW	25	28	9:42
126	1/11/33	10:35	SSW	50	WSW	15	SW	20	SW	19	25	10:00
127	1/13/33	16:16	NE	15	ENE	15	E	15	NE	7	....	....
128	1/20/33	16:10	WNW	28	NW	11	NW	12	NW	18	26	16:07
129	1/25/33	9:30	ENE	58	ENE	15	NE	18	E	10	....	....
130	1/28/33	17:28	NW	47	NNW	18	NW	19	NW	32	34	17:05
131	2/ 2/33	13:45	WSW	40	WSW	14	W	17	W	25	30	12:59
132	2/10/33	10:10	W	35	W	14	W	12	W	15	....	....
133	2/15/33	10:00	NW	40	NNW	12	NW	15	NW	23	....	....
134	2/15/33	12:00	NW	46	NNW	12	NW	20	NW	31	35	11:55
135	2/17/33	16:30	WSW	18	WSW	8	SW	7	SW	9	....	....
136	2/18/33	16:30	WSW	18	NNW	8	SW	11	N	18	....	....
137	2/27/33	9:30	NW	55–65	NNW	24	NW	23	NW	41	44	9:15
138	2/27/33	10:15	NW	55–65	NNW	24	NW	25	NW	41	47	10:38
139	2/28/33	9:00	WSW	30	NNW	21	NW	19	NW	27	34	9:00
140	2/28/33	9:30	NW	50	NNW	21	NW	20	NW	27	....	....
141	2/28/33	11:00	NW	50–55	NNW	24	NW	22	NW	25	....	....
142	2/28/33	11:20	NW	45–50	NNW	24	NW	22	NW	25	33	11:47
143	3/10/33	12:30	WNW	52	NW	24	NW	18	NW	35	37	12:23

TABLE 1—(*Continued*)

Item No.	Date*	Eastern Standard Time, in hours and minutes past midnight	Empire State Building: Direction of wind	Empire State Building: Velocity, in miles per hour	New York City Meteorological Observatory: Direction of wind	New York City Meteorological Observatory: Velocity, in miles per hour	*Daily News* Building: Direction of wind	*Daily News* Building: Velocity, in miles per hour	United States Weather Bureau Observatory: Hourly reading: Direction of wind	United States Weather Bureau Observatory: Hourly reading: Velocity, in miles per hour	United States Weather Bureau Observatory: Maximum: Velocity, in miles per hour	United States Weather Bureau Observatory: Maximum: Time, in hours and minutes past midnight
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Height of anemometer, in feet		....		62†		504‡		454§			
	Maximum velocities (5-min period); 3/22/36...		....	....	....	34	....	....	....	....	65	....
	Extreme velocities		....	102	....	54.4	....	....	....	....	71	....
144	3/15/33	11:00	WNW	50	ENE-NW	11	NW	12	NW	21	34	10:45
145	3/21/33	10:45	NE	40	NNE	13	NE	23	NE	14	....	....
146	3/25/33	13:25	SSE	15	S	11	S	13	S	14	....	....
147	3/28/33	17:00	WNW	41	NNW	18	NW	17	NW	27	35	17:20
148	4/ 4/33	17:00	WNW	45	NW	19	NW	19	NW	30	30	17:04
149	4/19/33	11:20	NE	44	ENE	21	E	28	NE	12	....	....
150	2/ 3/36	14:00	....	0	WNW	4	NW	5	N	5	....	....
151	3/ 2/36	18:40	SE	35	ESE	12	SE	15	E	18	....	....
152	3/ 9/36	17:00	SW	22	SSE	8	S	9	S	7	....	....
153	3/11/36	14:45	SE	55–60	ESE	18	SE	29	SE	30	32	14:55
154	3/18/36	14:40	E	55	NE	21	E	27	E	9	....	....
155	3/22/36	10:25	NNW	90	NW	28	NW	27	NW	56	58	10:10
156	3/22/36	15:55	NW	50–60	NW	25	NW	23	NW	47	54	15:50
157	3/22/36	16:10	NW	50–60	NW	27	NW	23	NW	45	54	16:35
158	4/ 1/36	14:15	S	25	SE	9	SE	8	SE	9	....	....
159	4/ 7/36	12:25	E	20	ENE	13	NE	11	E	6	....	....
160	4/ 8/36	9:36	NNW	40–45	NW	16	NW	17	NW	36	....	....
161	4/ 8/36	9:53	....	....	NW	16	NW	17	NW	36	....	....
162	4/13/36	15:50	NNW	20	NW	11	N	11	N	10	....	....
163	4/14/36	15:45	NNW	25	SW	10	SW	13	SW	15	....	....
164	4/15/36	10:00	NW	45	SE	7	SE	4	SE	5	....	....
165	4/17/36	11:00	WNW	3	WNW	15	W	23	W	31	32	11:37
166	4/29/36	14:10	....	....	SE	11	SE	11	SE	13	....	....
167	4/29/36	15:00	....	....	SE	13	S	15	SE	18	....	....
168	4/30/36	12:05	....	....	SSE	9	S	13	SE	13	....	....
169	5/16/36	11:30	....	....	NNE	10	N	7	N	10	....	....

that same hour, as recorded by the Federal Observatory, is given in Column (11). The exact time that this maximum was taken is given in Column (12).

The maximum velocity during the storm of March 22, 1936, for a 5-min period, was 34 miles per hr in Central Park, and 65 miles per hr at the Federal Station. The extreme velocity at the Federal Station was 71 miles per hr whereas a velocity of 102 miles per hr was noted at the Empire State Building. A pressure of 20.2 lb per sq ft has been reached once in Central Park since 1919.

The Federal Observatory is in the Whitehall Building overlooking Battery Park on the south end of Manhattan Island. In the vicinity there are a number of buildings as tall or taller than the one containing the U. S. Weather

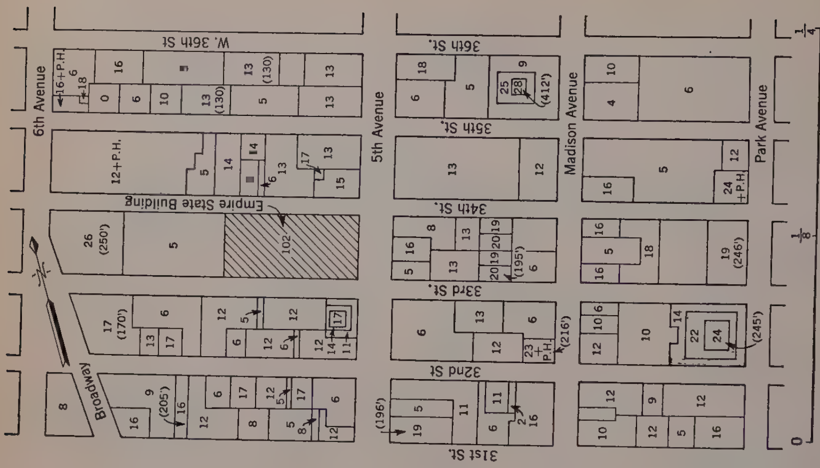


FIG. 7.—HEIGHT OF BUILDINGS IN IMMEDIATE VICINITY OF STRUCTURE; SHOWN IN TERMS OF NUMBER OF STORIES; NUMBERS IN CIRCLES ARE HEIGHTS, IN FEET

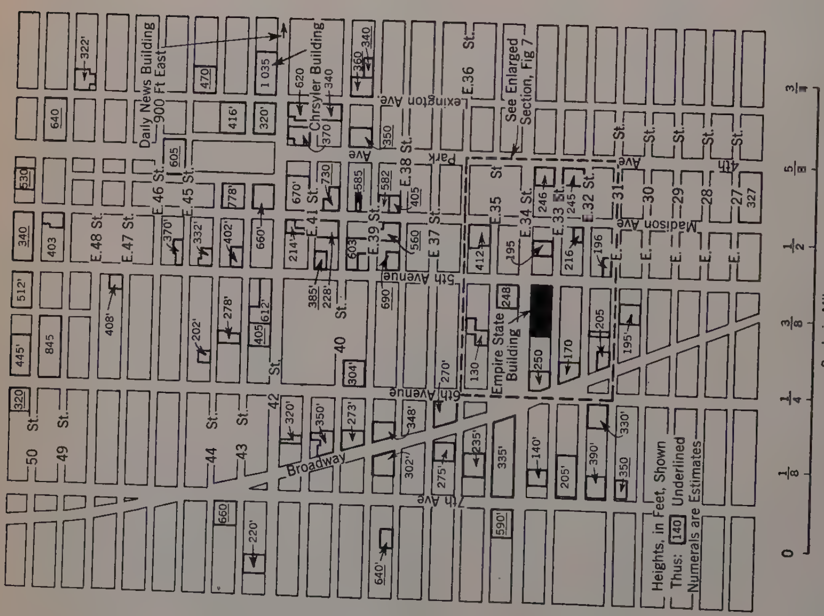


FIG. 6.—LOCATION AND HEIGHTS, IN FEET, OF BUILDINGS IN THE VICINITY OF THE STRUCTURE

Bureau Station, which to a considerable extent affect the readings of the anemometer. These tall buildings lie to the north and the northeast. To the south and west there is nothing between the instruments and the harbor. The anemometer is 454 ft above the harbor, and about 4.5 miles due south of the Empire State Building.

The News Building is on the south side of Forty-Second Street between Second and Third Avenues, and about 900 ft east of Lexington Avenue. This structure is much taller than any other building east of Lexington Avenue, except the Chrysler Tower, so that all buildings that are anywhere near the anemometer in height are shown in Fig. 6. The height of the anemometer is 504 ft above the street and 30 ft above the roof.

The New York City Meteorological Station is in Central Park, about 0.3 mile west of Fifth Avenue and 2.9 miles north of the Empire State Building. The observatory is in the approximate center of the park which has a width of about 0.5 mile and a length of a little more than 2.5 miles. The anemometer is 62 ft above the ground and is of the rotary cup type. This elevation places it well above the trees of the park but lower than the buildings on the adjoining streets.

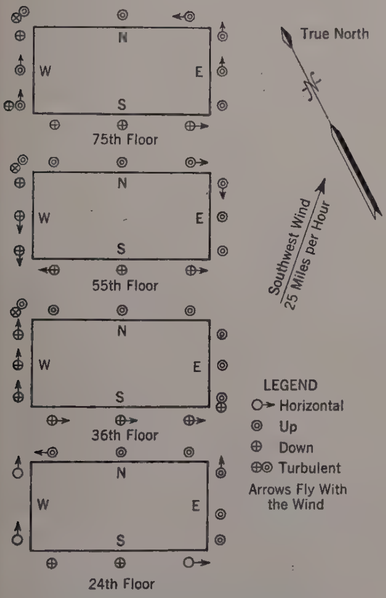


FIG. 8.—SNOW OBSERVATIONS, JANUARY 11, 1936, 2:00 TO 4:00 P.M.

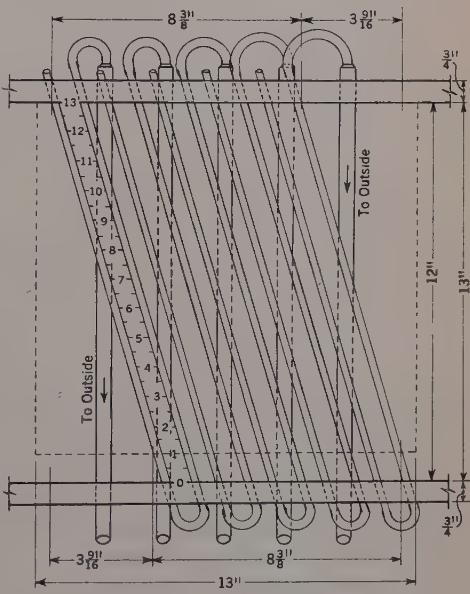


FIG. 9.—U-TUBE MANOMETERS

The direction and approximate velocity of the air currents around the Empire State Building have been observed by watching the drift of fog and of snow flakes. The results of one typical observation are given in Fig. 8 which shows that the flow of the air is greatly disturbed by the building in a manner that is far from simple. Another observation on February 8, 1936, showed definite vertical currents, up as well as down, whereas the anemometer recorded a zero

reading and the direction arrow rotated in an irregular manner over the entire range of the dial.

During a strong wind a rubber balloon was tied to a long thread and sent out from the highest point of the mast. It did not move horizontally away from the wind as was expected, but eddied about the lee side of the building and the mast. It made one vertical trip the full length of the thread (several hundred feet) and returned. It did not leave the lee side of the building until 15 min after the thread was severed. From these observations it is quite evident that the velocity of the wind acting on the surface of a building is entirely too complicated to be represented by a single reading.

Table 1 shows that recording instruments within a few miles of each other differ widely in their readings while recording a storm. This table (which justifies considerable detailed study) reveals the fact that each station has individual characteristics that must be understood, when interpreting data.

Manometers

In order to study the pressure and the pressure distribution of the wind over the surface of the building ten gages were placed on each of three floors: The thirty-sixth, the fifty-fifth, and the seventy-fifth (see Fig. 4).

The opening in the outside wall consisted of a hole, ¼ in. in diameter, placed on a spandrel of aluminum. Great care was taken in boring the hole to have the edges sharp and true. A 0.5-in. galvanized iron pipe, with specially designed air-tight couplings, led from each hole to one of two closets on this same floor. These pipes were embedded in the floor and the holes were bored during the erection of the building.

Each closet contained a rack consisting of ten glass tubes (see Fig. 9) which were fastened together in pairs at their lower ends by a short rubber tube, forming five U-tubes, which were partly filled with colored water. One end of each tube was connected to one of the galvanized pipes, previously described, by means of a rubber tube, thus subjecting it to the pressure of the outside air. The other end was open to the air pressure of the closet. Although the tubes were placed on a slant the scales were made to measure vertically; therefore, no allowance was made for this increase in length of tube. The difference in level between the two columns of water gave a measure of the difference in pressure of the air in the closet and that against the ¼-in. hole. There is some possibility that the air pressure in the two closets was not the the same during a high wind. The doors were left open during visual readings but not during the camera readings.

When the equipment was being installed it was felt that better information could be obtained by having all readings (on the manometers and on the steel) made at the same time by electrical control. In each closet a camera was focused on the five U-tubes; and each extensometer (described subsequently) had a camera attached to it. All were operated from a common button on the twenty-fourth floor. In its operation this set-up did not prove entirely satisfactory. During a strong wind storm the building not only deflects but also vibrates, and the camera may catch a reading on the steel at the maximum or minimum of the strain due to this vibration. With the manometers, on the

other hand, the fact that the wind comes in gusts might cause a reading higher or lower than the mean pressure. The average was taken in all cases when visual readings were made. It took several minutes from the time the dial was read on the first floor until the observer had made all his readings on the several floors and returned to the first floor. In a considerable number of cases the character of the storm changed very materially during this time.

The cameras were a source of annoyance because they often jammed, and, in this way, much valuable information was lost.

In computing the pressures given in Table 2, 1 in. of difference in liquid elevation was taken as 5.2 lb per sq ft on the wall surface. The manometer readings given in Table 2 show the general character of the pressure distribution during a number of characteristic storms. Where the wind changed materially during a visual observation the results were not recorded. It is worthy of note that there is a great variation in the distribution of load among the floors. Some of the storms seem to affect only the upper floors and anemometers whereas others affect some lower level. One instance was noted in which the wind registered 70 miles per hr. Below the seventieth floor it was calm whereas above that floor the wind was strong. At another time a strong wind was noted at the thirty-sixth floor and it was comparatively calm at and above the fifty-fifth floor.

The manometer tubes (see Fig. 4) recorded both positive and negative pressures. Fig. 10 gives a graphic illustration of the distribution of the wind

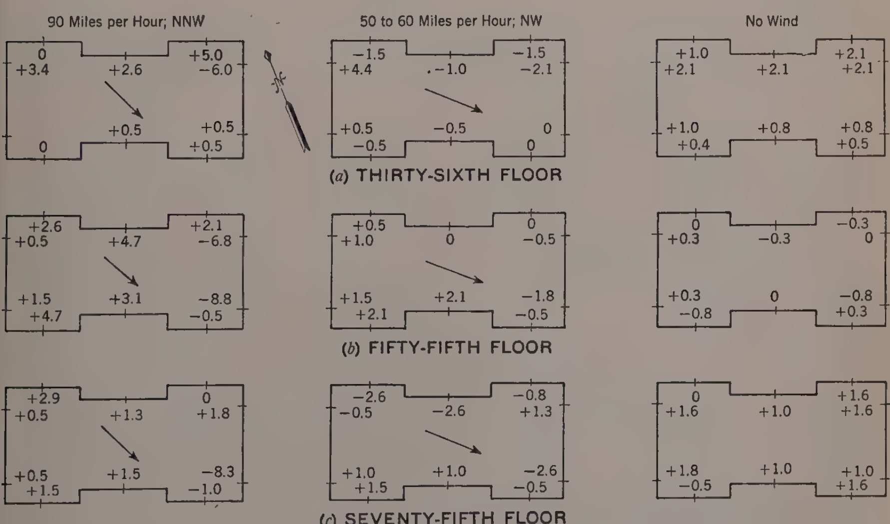


FIG. 10.—OBSERVATIONS DENOTING PRESSURE IN POUNDS PER SQUARE FOOT; ARROW INDICATES DIRECTION OF WIND

pressure over the surface of the building and impresses one with the lack of uniformity of this pressure. Three types of storms, as recorded by the anemometer, were selected.

TABLE 2.—Record of Manometer Readings (Pressure

Item No.*	Wind Characteristics		Thirty-Sixth Floor (at Observation Stations; see Fig. 4)										Fifty- (at Obser-		
	Direction	Velocity, in miles per hour	1	2	3	4	5	6	7	8	9	10	1	2	3
	SSE	68	+26	+36	+30	0	−20	−36	−19	−28	−35	+21	+50	+53	+47
	WNW	31	−12	−05	−10	−04	+30	+40	+17	+05	+10	00	−04	−09	−08
103	WNW	42	+05	+20	−07	−01	+90	+37	0	+05	+10	+10	−20	−15	−10
106	NE	50	+04	−15	−17	−03	−23	−30	+18	+28	+29	+70	−07	−26	−30
107	ENE	52	+10	−30	−40	−04	−30	−18	+26	+60	+30	+50	+16	−45	−46
109	ENE	60	0	−50	−40	−10	−40	−10	+10	+30	+40	+60	0	−30	−50
111	S	24	+20	+22	+10	0	0	−05	0	0	0	+10	+07	+10	−05
113	ENE	12	+03	+02	+05	−01	+06	+04	−07	+15	+15	+18	−08	−10	−14
116	W	30	−02	+14	+10	+05	+06	+07	0	+05	+03	+03	−03	+19	+25
117	SW	35	−04	+16	+21	+05	−03	+03	+07	+08	+06	+03	−03	+17	+23
118	SSW	7	+10	+ 7	+ 7	0	+10	+07	+09	+06	+04	+12	−10	−11	−17
119	SW	43	−15	+29	+42	+05	−03	−05	+04	+11	+13	0	−19	+16	+08
122	SSW	8	+01	+04	+04	+ 0	+03	+04	+06	+05	+03	+06	−03	−04	−09
123	WSW	35	−02	+18	+29	+07	+22	+14	+04	+05	+03	+07	−12	+09	+18
125	SSW	53	−37	+54	+45	+05	−90	−25	−08	0	+02	−20	−25	+34	+49
..	NE	13	+19	+09	+10	0	+13	+12	+ 6	+31	+27	+22	−07	−14	−16
127	ENE	16	+09	+05	+06	0	−03	+08	+19	+21	+25	+21	−09	−16	−19
128	W	35	+02	+07	+16	+01	+29	+21	−06	−14	+06	+06	−06	−05	−25
..	NE	52	−07	−75	−60	−03	−40	−20	+11	+35	+80	+40	+01	−39	−52
130	NW	50	−06	−13	−13	−03	+51	+56	−20	+36	+11	−19	−29	−26	−27
135	W	10	..	..	..	..	..	+07	+08	+03	+08	+05	−08	−06	−09
137	NW	58	..	..	..	..	..	−13	+20	+07	0	−30	−38	−47	−54
138	NW	57	..	..	..	..	..	−33	−25	+15	+39	+18	−18	−28	−42
144	WNW	52	−08	0	0	−04	+14	+19	−12	+02	+13	+02	−24	−31	−44
145	NE	32	−08	+08	+12	+02	+13	+06	+32	+35	+31	+15	−05	−05	−04
147	NW	39	−66	−09	−28	−01	+18	+31	+13	+14	+15	−07	−12	−18	−19
148	WNW	45	−03	+10	+05	−06	+50	+25	−15	−29	−15	+01	−10	−05	−45
149	NE	44	−10	−25	−60	−28	−40	−25	+14	+40	+51	+27	−09	−29	−43
150	...	15	+15	+10	+15	+08	+20	+40	+20	+40	+40	+40	−15	+05	+ 0
151	SE	35	0	0	0	0	0	..	..	..	..	..	0	+20	+20
152	SW	22	0	0	0	0	0	−20	−25	−20	−20	−50	0	0	0
153	SE	55–60	+20	0	+25	+25	+10	−15	−25	−20	+40	−130	−80	+30	+50
154	E	55	..	0	+20	+20	+10	−10	−30	−40	−50	−20	0	+10	+10
155	NNW	99	−20	0	+20	+20	+ 0	−105	+60	+35	+20	+65	+30	+60	+60
155	NNW	90	..	0	+10	+10	+10	−115	+95	+50	0	+65	+30	+90	+60
155	NNW	90	−10	+10	+10	+10	+10	..	..	..	..	..	+10	+20	+10
156	NW	50–60	0	−10	−10	0	0	−60	−15	−20	−10	+95	+15	..	+50
156	NW	50–60	+10	−10	−10	0	0	−40	−30	−20	−30	+85	+30	+40	+40
157	NW	50–60	..	0	0	0	0	−20	−15	−20	−20	−20	0	−10	+10
158	S	35	..	..	..	..	..	−20	−20	−20	−20	−40	−10	0	0
159	S	20	..	..	..	..	..	..	..	..	..	..	0	0	0
159	S	20	..	..	..	..	..	−25	−25	−30	−30	−50	0	0	0
159	S	20	..	..	..	..	..	−20	−20	−30	−30	−50	0	0	0
160	NW	40.45	..	..	..	..	..	−50	−25	−30	−35	+90	+20	+25	+20
160	NW	40.25	..	..	..	..	..	−40	−15	−25	−45	+70	+10	+20	+20
161	NNW	..	..	..	..	..	..	−70	−10	−30	−40	+70	+10	+10	+10
162	NNW	..	..	..	..	..	..	..	..	..	..	..	0	0	0
162	NNW	..	..	..	..	..	..	−35	−30	−30	−30	−10	0	0	0
163	NNW	..	..	..	..	..	..	−15	−15	−15	−10	−25	0	−20	−20
164	NW	..	..	..	..	..	..	−85	+85	+65	+10	+15	−90	−10	+30
164	NW	..	..	..	..	..	..	..	..	..	..	..	−90	−10	+30
164	NW	..	..	..	..	..	..	−70	+60	+50	+10	0	−90	−10	+30
165	WNN	..	..	..	..	..	..	−35	−10	−20	−10	−20	0	−10	0
166	WNN	..	..	..	..	..	..	−35	−10	−10	−10	−20	..	..	..

* See corresponding Item No. in Table 1.

ON SIDE OF BUILDING, IN HUNDREDTHS OF AN INCH OF HEAD OF WATER)

Fifth Floor vation Stations; see Fig. 4)							Seventy-Fifth Floor (At Observation Stations; see Fig. 4)									
4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
+08	−23	−27	−17	−30	−22	+20	−29	−44	+20	+18	−40	−24	−02	−17	−17	+30
+08	−42	+55	−10	0	−02	−03	−15	−12	−17	+03	+12	+40	−03	−12	−10	−15
00	+50	+50	00	0	0	−10	−10	−20	−20	+05	+35	+55	0	−25	−15	−05
+07	−40	+53	+32	+29	+07	−38	−07	−27	−37	+04	−37	−31	+03	+15	+20	+14
+06	−50	−38	+15	+29	+36	+56	+07	−50	−43	00	−93	−60	+10	+60	+48	+70
0	−50	−60	0	+60	+50	+30	0	−50	−55	0	−50	−50	0	+80	+50	+50
+05	−10	−10	−05	−10	−08	0	0	+07	+02	+07	−27	−12	0	−10	−20	−07
+05	−12	−13	−06	−06	−08	−05	19	−28	−28	+03	−29	−24	+02	−17	−21	−23
+22	−09	−02	−03	−07	−12	−05	02	+09	+18	+12	−08	0	+04	−06	−15	−14
+22	−16	−08	−04	−06	−08	−03	11	+15	+24	+25	−20	−07	+01	−09	−11	−13
+05	−15	−20	−08	−18	−21	−09	−35	−36	−35	0	−39	−39	−04	−27	−33	−34
+11	−35	−29	−11	−18	−29	−19	−27	+09	0	+05	−35	−40	−03	−25	−30	−33
+03	−07	−12	−06	−12	−11	−09	−17	−21	−22	+03	−24	−18	−01	−15	−22	−17
+24	+10	−03	−07	−15	−17	−08	−25	−21	−08	+04	− 2	−13	−03	−24	−29	−23
+32	−10	−40	−13	−22	−23	−25	−55	+30	−51	+20	−48	−35	0	−75	−33	−43
+04	−13	−14	−07	−09	−08	−03	−32	−44	−45	+02	−43	−44	0	−21	−27	−29
+03	−14	−15	−06	−03	−03	+03	−36	−40	−46	+01	−41	−40	0	−24	−23	−40
+06	+21	+13	−09	−15	−22	−10	−19	−30	−25	+03	+05	0	0	−20	−25	−24
+06	+46	−47	+07	+32	+41	+14	−22	−59	+33	+02	−64	−62	+08	+60	+67	−51
+05	+19	+55	+04	+11	−12	−22	−41	−51	+57	+05	−03	+44	+03	−08	−27	−39
+06	−09	−10	−19	−16	−15	−09	−25	−26	−24	+03	−24	−27	−02	−21	−27	−29
0	+36	+71	−28	−37	−30	−37	−62	−64	−72	0	+04	+06	09	−25	−50	−59
+07	+12	+45	−37	−49	−24	−26	−42	−63	+47	0	+43	+33	04	−45	−70	−41
+05	0	+21	−32	−32	−26	−22	−37	−50	−53	−04	+32	+24	11	−42	−43	−39
+04	−18	−17	+14	+24	+11	−24	−13	−26	−29	0	−27	−33	0	+03	0	−40
+03	+22	+21	+05	−05	−08	−16	−35	−39	−37	0	0	−34	−02	−05	−20	−29
+05	−47	+46	−12	−17	−23	−13	−22	−33	−83	+02	+17	+50	−02	−31	−31	−32
+02	−34	−39	0	+15	+22	+14	−29	−44	−45	0	−60	−59	−01	−02	+09	+07
+15	+05	+05	0	−05	−15	0	+20	+30	+20	−10	+35	+30	0	+20	+30	+30
−10	+10	+10	+10	+15	+25	−25	+10	+30	+30	−10	+35	..	..	..	..	..
−10	0	0	0	0	+10	0	..	..	..	..	..	..	..	..	..	..
−10	+10	+10	−20	+40	+60	−90	−70	+30	+30	0	+30	..	..	..	..	..
−20	+20	+20	−10	−30	−40	−90	0	+20	+20	−10	+20	..	..	..	..	..
−10	−150	−110	+60	+80	+20	+30	+20	+40	+40	−20	−180	..	..	..	..	..
−10	−170	−130	+40	+90	+50	+10	+10	+30	+30	−20	−160	+30	0	+25	+55	+10
−10	−45	−30	+20	+10	+10	+10	+20	+30	+20	−10	−50	..	0	+10	+15	+10
−10	−40	−50	+20	0	+20	+10	+20	+30	+20	−10	−40	+40	0	+20	+70	−90
−10	−35	−50	0	0	+10	+20	+20	+30	+20	−10	−50	+25	−15	−50	−50	−10
−10	−10	−10	0	0	0	0	0	0	0	−10	0	+05	0	0	+10	+05
−20	0	0	0	0	0	0	0	+10	+10	0	+10	..	..	..	..	..
−20	0	0	0	0	0	0	+20	+20	+10	−10	+20	..	..	..	..	..
−20	0	0	0	0	0	0	+20	+20	+20	0	+20	..	..	..	..	..
−20	0	0	0	0	0	0	+20	+20	+20	0	+20	..	..	..	..	..
−10	−35	−50	..	0	0	+10	+20	+30	+20	−10	−40	..	..	..	..	..
−20	−60	−60	+10	0	0	+10	+20	+20	+20	−10	−40	..	..	..	..	..
−20	−50	−50	+10	0	0	+10	+20	+20	+20	−20	−30	..	..	..	..	..
−10	−20	−20	−10	0	0	0	0	+10	+10	0	0	..	..	..	..	..
−10	−20	−20	−10	0	0	0	+10	+10	+10	0	+10	..	..	..	..	..
−20	−10	−10	0	0	0	0	0	−10	−10	−20	0	0	−10	0	+10	0
+10	+10	−85	+10	+20	+30	+20	+20	+20	+40	−10	−70	−80	−10	+05	+30	+10
+10	+10	−80	+10	+20	+20	+20	+20	+20	+40	0	−60	−80	−10	+05	+30	+10
+10	+10	−85	+10	+20	+20	+10	+20	+30	+40	−10	−70	−10	−10	0	+15	+10
−10	−35	−30	0	+10	+10	+10	+20	+20	+20	−10	−10	−10	..	0	+15	+10
...	...	−30	0	0	+10	+10	+10	+10	+20	−10	−20	..	..	..	..	..

Messrs. Hugh L. Dryden and G. C. Hill, of the National Bureau of Standards, subjected a model of the Empire State Building, 5 ft high, to air currents of known velocity in a wind tunnel, and recorded the pressure at a great number of points,[3] including the stations shown in Fig. 4. A comparison of the pressures on the model and those on the building shows clearly that the natural wind movements are not at all like those in a wind tunnel.

Deflection

For a study of the deflection of a tall building under wind loads a better planned structure than the Empire State Building could not be found. It is not only the tallest building in the world but it is symmetrical in plan about both planes parallel to its sides, and through its vertical axis. The stair well, or fire tower, is near this central axis (Fig. 4) and gives a clear sight from the sixth to the eighty-sixth floor.

The data reported herein are those of the movement of the eighty-sixth floor relative to the sixth. The lateral movement of the sixth floor is doubtless small and may be neglected.

In effect, the building is a huge cantilever and as such has two distinct movements when acted upon by a wind storm: It deflects from the vertical; and it vibrates with a definite period, similar to the tines of a tuning-fork when struck. A steady wind causes deflections only, whereas a gusty wind will set up vibrations with amplitudes that vary with the strength and character of the storm. Under the action of a strong wind the building vibrates about a mean position in the direction parallel to its narrow side and, with a different period, in a perpendicular direction. Apparently, these two vibratory motions are independent of each other, differing both in amplitude and period.

Two methods were used to study the motion of the upper part of the building during a storm. A specially designed vertical collimator with a right-angle eye-piece was set up on the sixth floor and sighted on a target on the ceiling of the eighty-sixth floor. From a point near this target a plumb-bob was suspended so that the bob hung near the sixth floor. The collimator proved very useful in studying the vibrations of the building both as to period and amplitude; the plumb-bob was more useful in obtaining the deflections. At the time of reading either instrument the velocity and direction of the wind at the mast head were recorded.

The Fire Tower

One of the most valuable features of the building from the standpoint of this investigation is the fire tower. Although there are several offsets in this tower there is a clear opening from the sixth floor to the eighty-sixth floor (a vertical distance of 972.5 ft) of about 54 in. in either direction, leaving sufficient room for both the plumb-bob string, and the collimator sight. In order to defeat the tendency for vertical currents of air to occur in the shaft, a diaphragm was installed at the fifty-fourth floor, holes being left for the line of sight and the plumb-bob string. Even after the installation of this diaphragm there is a strong current of air into the tower when the doors leading from the building

[3] "Wind Pressure on a Model of the Empire State Building," by Hugh L. Dryden and G. H. Hill, *Research Paper No. 525*, National Bureau of Standards.

floors are opened, particularly above the fifty-fifth floor. When the doors are closed the leakage of air under the door is quite perceptible but the volume of air admitted is small. At each floor the stairway is separated from the tower by a partition containing a door. All doors to the tower were kept closed except when in use.

As the tower is very close to the vertical central axis of the building any rotation of the building about this axis does not affect the readings. Deflections that persisted over a considerable period of time could be obtained to any desired degree of accuracy by means of the plumb-bob, whereas rapid movements could be studied by means of the vertical collimator.

The Plumb-Bob

The plumb-bob consisted of a 35-lb concrete cylinder (Fig. 11) containing considerable scrap iron. It was suspended by a three-strand sounding cable from the ceiling of the eighty-sixth floor to a convenient height above the sixth floor so that the distance from the point of suspension to the center of the bob was 969.25 ft. Due to this great length of suspending wire the bob did not follow rapid movements of the building, but maintained a position under the mean location of its point of support.

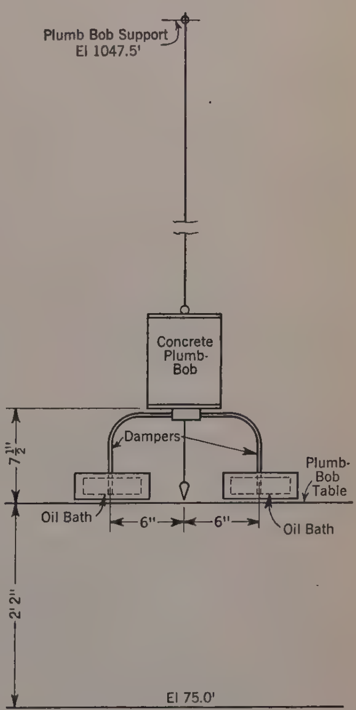


Fig. 11.—Diagram of Plumb-Bob Set-Up

The first bob designed had a brass point on the lower end but trouble was experienced by the weight bending this point when it hit the lower target (marked in tenths of an inch) on which the readings were made. In order to prevent this effect, and to allow for vertical adjustment, a very small bob was suspended from the lower end of the larger one. The point of this second bob was adjusted so that it exactly cleared the target.

Oil baths were used to damp out the swing of the bob. With the dampers the readings were easily obtained to within 0.1 in., or even closer, during a light wind. During a heavy wind the bob had a tendency to oscillate, due to the movement of the point of support and also to slight upward currents of air. As the motion in any definite direction was that of a damped harmonic it was not difficult to estimate the position of rest to within 0.2 in. and in most cases to within 0.1 in. Once or twice the position of rest was difficult to determine with an accuracy of less than 0.5 in. This occurred during very strong winds and was probably due to the shifting of the mean position of the eighty-sixth floor during the time the observations were being taken.

Some question has been raised as to the accuracy of the plumb-bob as an instrument for noting the movement of the building because of the leakage of the air into the shaft and also because of the fact that the point of support is not steady. The size of the fire tower, except at the diaphragm across the fifty-fourth floor, is more than 125 sq ft, with the result that upward currents are slight and those that do occur do not deflect the plumb-bob string in one direction more than in another. When a door was opened a serious disturbance occurred, but it subsided at once when the cause was removed. As to the effect of vibration on the action of the bob, it is to be noted that the building vibrates from 6 to 8 times per min whereas the computed period of the bob is between 34 sec and 35 sec. As the bob did not record movements of its support that are not of greater duration than the period of its swing, the building vibrations are not recorded by the bob, but their mean position is determined by it.

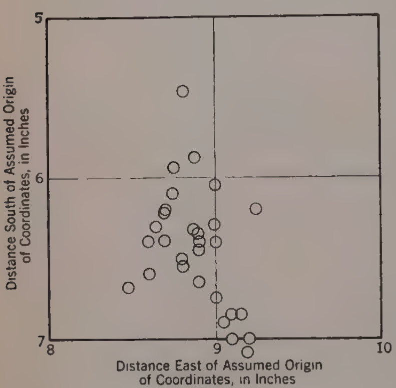


FIG. 12.—MOVEMENT OF PLUMB-BOB DURING A LIGHT WIND (0 TO 20 MILES PER HOUR)

The tidal effect on a pendulum of this length can easily be shown to be small enough to be neglected.

A large number of readings have been recorded to show the position of the plumb-bob, together with the recorded wind velocity and direction. A few representative data are given in Table 3. In plotting these readings the writer divided them into three groups for the sake of clearness: Table 3(a), for a wind velocity of less than 20 miles per hr; Table 3(b), for recorded wind velocities between 20 and 30 miles per hr; and Table 3(c), for a wind velocity greater than 30 miles per hr. Fig. 12 is an example of the form in which the data were plotted.

The first group (see Table 3(a)) shows that the building fails to return to a definite position of rest. This is to be expected as there is considerable inelastic and semi-elastic material surrounding the steel frame. It is probable that this material resists recovery, and that there is a small amount of stress left in the steel after the building is relieved of a heavy wind pressure. As a result, if the wind were to shift to the opposite direction, the building would pass through the zero point of deflection much more readily than a simple stress-strain curve would indicate. This action would, in no way, endanger the building as the phenomenon will not occur except at low stresses. Fig. 12 gives an idea of the extent of this plastic set because it can be assumed that the deflection of the building due to the wind pressure is low and so may be neglected.

The forces due to the high wind (see Table 3(c)) are great enough to overcome the plastic deformations of the storm immediately preceding the one observed and so in each case, in the third group, the displacement would be due to the wind recorded and not to the effect of a previous storm.

TABLE 3.—PLUMB-BOB OBSERVATIONS, EIGHTY-SIXTH FLOOR TO SIXTH FLOOR

(a) WIND VELOCITY LESS THAN 20 MILES PER HOUR

Item No.*	Date†	Eastern Standard Time, in hours and minutes past midnight	Wind Characteristics: Compass direction	Wind Characteristics: Velocity, in miles per hour	Distance from Assumed Origin, in Inches: East	Distance from Assumed Origin, in Inches: South
9	3/25	13:25	SSE	15	9.00	6.30
18	5/18	14:00	N	8	8.60	6.60
19	5/19	15:25	SSE	15	9.00	6.40
21	5/23	11:15	W	15	8.70	6.40
22	5/23	12:00	W	12	8.70	6.20
23	5/24	11:00	SSW	8	8.80	6.50
24	5/25	10:10	WSW	8	8.90	6.35
35	6/13	11:50	NE	15	8.60	7.20
36	6/14	15:00	NE	2	9.10	7.00
39	6/16	14:25	SSE	12	9.15	6.85
49	6/29	16:55	WNW	12	9.20	7.00
52	7/ 6	13:00	SW	12	8.90	6.65
53	7/ 7	14:40	SSE	12	9.05	6.90
55	7/14	12:55	NNW	10	9.18	7.08
57	7/25	14:30	WNW	12	9.10	6.85
58	7/27	10:30	SSW	15	8.90	6.45
60	8/ 2	10:15	SSW	13	8.45	6.65
63	8/22	12:00	NW	5	8.80	6.55
64	8/23	11:45	NE	10	8.65	6.40
65	8/24	13:25	S	8	8.75	6.10
66	8/31	16:25	E	6	8.90	6.40
67	9/ 7	11:25	NNW	12	8.65	6.30
71	9/12	14:07	W	10	8.87	5.87
87	10/14	16:51	WSW	10	8.81	5.47
96	10/25	13:10	E	15	9.00	6.05
97	10/26	11:05	SSE	13	9.25	6.20
120	12/27	17:20	ENE	12	8.75	6.95
127	1/13	16:16	NE	15	8.89	6.31
136	2/18	16:30	WSW	18	8.70	6.21
...	3/23	16:50	SW	15	8.60	6.40
165	4/17	11:00	WNW	3	8.50	7.00

(b) WIND VELOCITY FROM 20 TO 30 MILES PER HOUR

Item No.*	Date†	Eastern Standard Time, in hours and minutes past midnight	Wind Characteristics: Compass direction	Wind Characteristics: Velocity, in miles per hour	Distance from Assumed Origin, in Inches: East	Distance from Assumed Origin, in Inches: South
25	5/25	14:50	SSW	22	9.05	6.50
32	6/ 8	13:25	WNW	25	8.80	6.40
33	6/ 8	14:40	WNW	25	8.80	6.40
37	6/15	17:00	S	25	9.10	6.40
42	6/17	14:00	NNE	20	9.10	7.10
46	6/24	11:15	WNW	25	9.00	6.50
51	7/ 5	17:15	WNW	20	9.15	7.00
56	7/19	12:45	WNW	20	9.00	6.90
59	7/29	12:30	WNW	20	8.85	6.35
72	9/27	10:00	SSW	22	8.30	5.40
73	9/27	16:15	SSW	22	8.70	5.70
88	10/17	9:00	SE	22	9.20	6.10
92	10/21	10:00	W	22	8.90	5.20
93	10/21	10:20	W	28	9.20	5.65
95	10/22	9:10	NE	22	8.90	6.25
98	10/28	14:40	SSE	25	9.30	6.15
111	11/11	11:00	S	25	8.80	6.00
128	1/20	16:10	WNW	28	8.37	5.82
152	3/ 9	17:00	SW	22	8.90	6.40
158	4/ 1	14:15	S	25	8.10	6.30
159	4/ 7	12:25	E	20	8.40	7.30
162	4/13	15:50	NNW	20	8.70	7.25
163	4/14	15:45	NNW	25	8.20	6.80

(c) WIND VELOCITY MORE THAN 30 MILES PER HOUR

Item No.*	Date†	Eastern Standard Time, in hours and minutes past midnight	Wind Characteristics: Compass direction	Wind Characteristics: Velocity, in miles per hour	Distance from Assumed Origin, in Inches: East	Distance from Assumed Origin, in Inches: South
27	5/26	15:50	SSW	35	8.80	5.90
31	6/ 7	16:45	N	35	8.50	7.40
43	6/22	....	....	....	....	....
44	6/23	....	....	....	....	....
45	6/23	15:25	W	32	9.30	6.60
48	6/24	....	....	....	....	....
50	7/ 1	....	....	....	....	....
54	7/11	13:10	NW	40	9.50	7.20
61	8/ 4	10:27	S	32	8.45	5.75
62	8/ 4	13:50	S	38	8.55	5.60
68	9/ 8	12:45	NNW	42	8.30	7.52
69	9/ 8	14:50	NNW	55	8.80	8.64
70	9/ 8	16:10	NNW	40	8.60	7.48
..	9/28	9:30	SSW	42	8.80	4.78
77	10/ 3	13:10	S	42	8.72	4.45
79	10/ 4	16:40	SE	35	8.87	5.85
80	10/ 5	9:00	SSE	52	8.70	5.05
81	10/ 5	14:40	SSE	45	8.85	5.40
83	10/ 7	10:15	WNW	32	8.80	5.80
84	10/11	9:20	WNW	34	8.67	5.08
86	10/12	13:10	W	41	8.88	4.06
89	10/18	11:35	NE	35	9.07	7.51
90	10/18	11:50	NE	42	9.07	7.51
91	10/19	8:50	NNE	32	8.85	7.13
94	10/21	16:20	WNW	50	9.55	5.69
100	11/ 1	9:00	SE	75	8.20	4.75
101	11/ 1	10:30	SE	75	7.93	4.63
103	11/ 1	16:05	WNW	42	8.68	5.33
104	11/ 1	16:50	WNW	45	8.45	5.18
105	11/ 1	17:15	WNW	50	8.53	4.90
109	11/ 9	14:45	ENE	60	8.20	9.70
110	11/ 9	16:30	ENE	60	8.29	9.76
112	11/15	9:50	SW	35	8.60	5.20
115	11/19	13:10	SSE	65	8.36	3.26
124	1/ 7	12:15	WSW	50	9.39	4.23
125	1/11	9:10	SSW	42	8.36	4.23
126	1/11	10:35	SSW	50	8.57	3.82
129	1/25	9:30	ENE	58	8.72	9.21
130	1/28	17:28	NW	47	8.89	7.12
131	2/ 2	13:45	WSW	40	9.19	5.39
132	2/10	10:10	W	35	9.25	5.58
133	2/15	10:00	NW	40	9.10	7.04
134	2/15	12:15	NW	46	8.88	7.80
151	3/ 2	18:40	SE	35	8.40	6.25
153	3/11	14:45	SE	55–60	7.20	6.50
154	3/18	14:40	E	55	7.25	9.00
155	3/22	10:25	NNW	90	10.70	7.20
156	3/22	15:55	NW	50–60	9.8	7.5
157	3/22	16:10	NW	50–60	9.8	7.5
160	4/ 8	9:36	N to NW	40–45	9.25	9.25
161	4/15	10:00	NW	45	9.30	7.0

* See corresponding numbers in Table 1. † Month/day.

It is to be noted that the values in Table 3 do not indicate the maximum deflection of the building. They are only the mean position about which the building vibrates; for a maximum deflection one-half the amplitude of vibration should be added.

In order to make a further study of this plastic set in the narrow direction of the building, three sets of measurements were made to the plumb-bob string from reference points, on the several floors. As it required more than two hours for each set of readings it was necessary to make them during calm weather in order to be assured that the building did not shift during the time of the observations. These variations are plotted in the curves of Fig. 13.

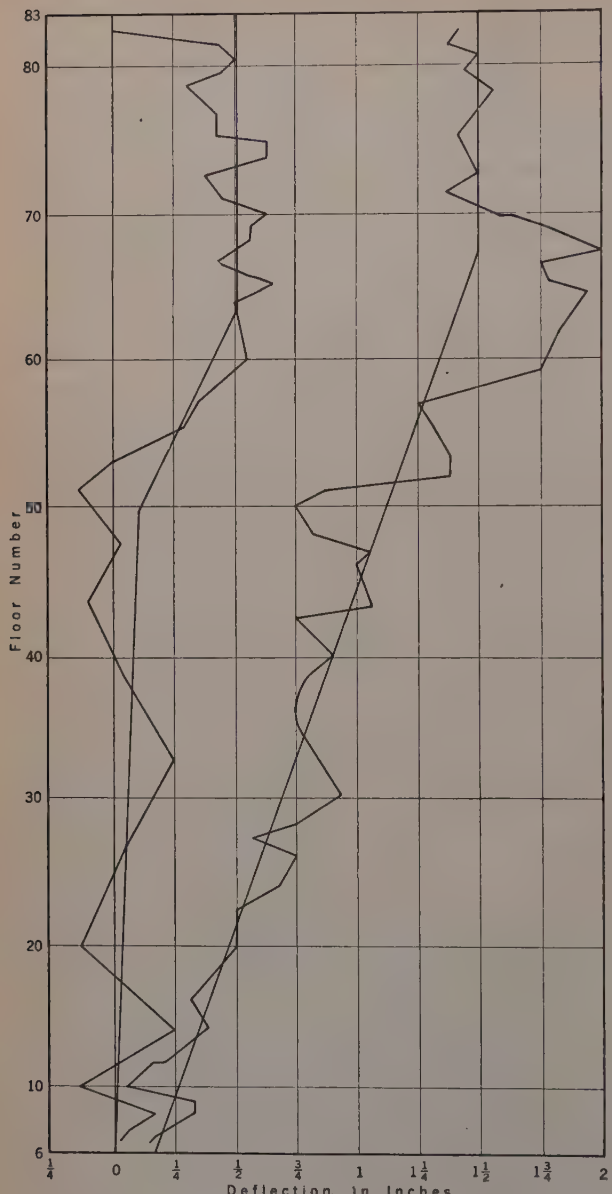


Fig. 13.—Check Measurements to Plumb-Bob String

Temperature Deflections

In order to ascertain the deflection due to the inequality of the temperature of the several sides of the building, observations were made throughout the day of August 24, 1936. The sky was clear and sunny, and the maximum temperature was 86° F at 3:00 P.M. There was little wind

blowing, the anemometer reading from 25 to 30 miles per hr, with a steady wind from the west northwest. Table 4 gives the readings of the plumb-bob showing a north deflection of 0.25 in. at noon and an east deflection of 0.15 in.

TABLE 4.—PLUMB-BOB OBSERVATIONS ON A HOT, CLEAR DAY, AUGUST 24, 1936

Description	Displacement of Plumb-Bob, in Inches, from the Origin, as Observed at the Following Times (Eastern Standard, in Hours and Minutes Past Midnight)								
	9:15	10:00	11:00	12:00	12:30	13:00	14:00	14:30	15:00
Wind velocity, in miles per hour*	30†	....	25†	28‡	....	25†	30‡	....	30§
Position of Plumb-Bob, in Inches, from the Origin:									
Southward	7.00	7.00	6.80	6.80	6.75	6.80	7.00	6.90	7.00
Eastward	8.60	8.60	8.50	8.50	8.55	8.45	8.50	8.60	8.45

* Anemometer readings. † West, northwest. ‡ Northwest. § North, northwest.

at 3:00 P.M. As the day was not cloudy, leaving the surface of the building exposed to the heat of the sun, it is inferred that the effect of temperature is only a minor factor in the results given in this paper.

The collimator was designed by the United States Coast and Geodetic Survey for the purpose of locating the reading points of triangulation survey marks above their permanent points. It consists essentially of a transit equipped with a right-angle eye-piece and a leveling bubble. The telescope has a focal length of about 8 in. One division of the leveling bubble represents 0.56 in. on the target; that is, the bubble has a sensitiveness of about 10″ of arc per division. The target can be read to 0.5 in.

As the principal value of the collimator lay in the study of the vibrations of the building it was used during high winds only. Hewitt Crosby, M. Am. Soc. C. E., reporting to the American Institute of Steel Construction,[4] stated that for some unknown reason the sixth floor tilted as much as four divisions of the dial, or approximately 40″ of arc. In light and steady winds the effect was eliminated by leveling the instrument carefully before taking a sight; but in strong or gusty winds, when readings must be made on a moving target, this was impossible. In extreme cases, the indicated error from this cause was found to be as much as 2 in. In strong winds, the motion of the target was irregular in direction, amplitude, and time.

Nevertheless, the instrument proved invaluable because it enabled one to study the movements of the top of the building qualitatively when acted upon by a high wind. Table 5 records the readings of the collimator, showing the amplitude of vibration of the building. A number of attempts were made to measure the deflection of other buildings by means of transits set up at a distance, but the writer knows of no case in which satisfactory results have been obtained.

[4] "Wind Stresses in the Structure of the Empire State Building," by Hewitt Crosby. Manuscript report to Am. Inst. of Steel Construction, New York, N. Y., June, 1933, pp. 30–33.

TABLE 5.—DEFLECTION READINGS BY COLLIMATOR

Item No.*	Date†	Eastern Standard Time, in hours and minutes past midnight	Wind Characteristics		Collimator Readings				Item No.*	Date†	Eastern Standard Time, in hours and minutes past midnight	Wind Characteristics		Collimator Readings			
			Compass direction	Velocity, in miles per hour	East-West Direction		North-South Direction					Compass direction	Velocity in miles per hour	East-West Direction		North-South Direction	
					Maximum	Minimum	Maximum	Minimum						Maximum	Minimum	Maximum	Minimum
79	10/4	16:30	SE	26	5.0	5.1	7.4	7.5	123	1/6	16:25	SW	29	6.8	7.9	6.9	8.0
101	11/1	11:00	SE	75	4.0	4.0	7.0	10.0	126	1/11	10:30	SSW	51	6.0	8.0	7.2	9.2
101	11/1	11:15	SE	8.5	4.0	4.0	8.0	12.0	....	1/13	12:00	ENE	16	6.9	7.9	4.8	6.5
102	11/1	12:00	SSE	75	4.5	5.5	6.5	9.0	127	1/13	16:00	NE	16	6.5	7.8	5.5	6.8
103	11/1	16:40	WNW	43	6.8	7.0	4.0	5.5	128	1/20	16:00	WNW	28	7.4	8.2	5.3	6.9
107	11/9	11:45	ENE	52	4.0	5.6	3.0	6.0	129	1/25	9:40	ENE	58	5.0	7.2	0	4.3
108	11/9	14:30	ENE	60	3.8	5.0	0.5	5.2	130	1/28	17:00	NW	47	6.9	8.8	3.1	5.2
114	11/18	16:00	ENE	15	4.5	5.5	3.6	5.5	....	2/17	16:00	WSW	16	6.6	7.9	6.2	6.6
117	12/2	16:00	SW	35	4.8	5.1	7.9	8.5	135	2/17	16:40	WSW	18	6.5	6.9	6.4	6.9
119	12/21	11:40	SSW	34	5.3	6.5	6.7	7.5	....	....	....	....	....	....	....	....	....

* See corresponding numbers in Table 1.
† Month/day.

The period of vibration was observed by means of the collimator on November 1, 1932 (see Fig. 14). The north-south movement is given in Fig. 15.

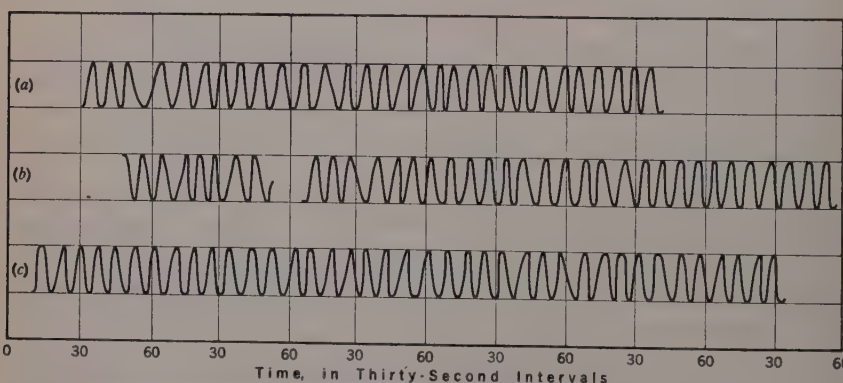


FIG. 14.—COLLIMATOR OBSERVATIONS ON THE SWING OF THE EMPIRE STATE BUILDING, NOVEMBER 1, 1932

The swing of one of the dials on the southwest corner column of the twenty-fourth floor (Fig. 4) was read on November 18, 1934. The curve thus obtained (see Fig. 15(*b*)) is the combination of two vibrations at right angles to each other, the north-south one dominating. The time of each high and low reading was taken and plotted. As the weight of the building was practically the same during these two observations the change of period from 8.38 sec to 8.14 sec is of considerable interest and is probably due to an adjustment of the several parts of the structure during severe storms.

Extensometers

Four extensometers were attached to each of four columns on the twenty-fourth floor, by fastening an 0.001-in., Ames dial 8⅞ in. below the mid-height point. One end of a rod touches the stem of the dial, the other end being

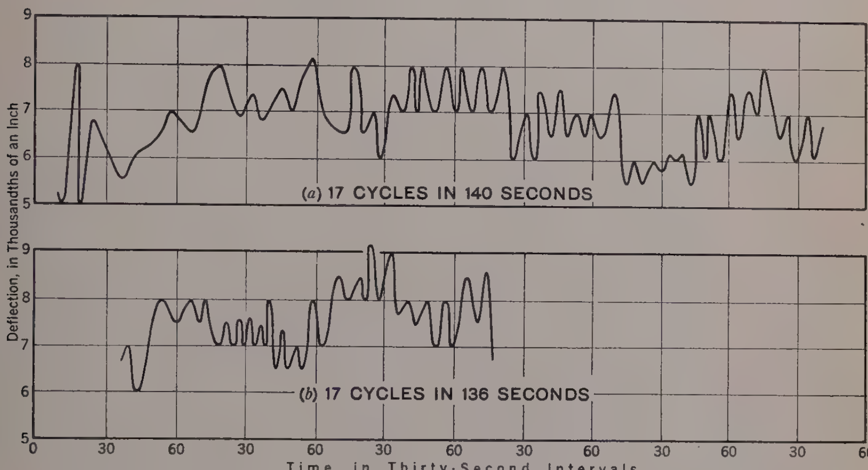


Fig. 15.—Deflection Observations of a Corner Column of the Empire State Building (Twenty-fifth Floor)

welded to the column 41⅛ in. above the midpoint of the column. In other words, the two points of attachment are 50 in. apart. They were all placed as near the outer end of the column flange (Carnegie H-beams) as possible.

With this arrangement the average of the differential readings of the dials gives the change of length of the column at its center line over a span of 50 in., thus furnishing a measure of the direct stress in the column. Although the angular change in the column can be computed it is not given herein because, unless the point of counterflexure is known, it is of little value.

The columns chosen for observation were the southwest corner column of the twenty-fifth floor and the three columns nearest to this corner. An offset in both directions places these columns in the interior on the twenty-fourth floor, thus facilitating the reading of the instruments.

Column (19) of Table 6 gives the sum of the readings of the several dials on the corner column in hundred-thousandths of an inch. Column (14) is the direct stress, in pounds per square inch, producing this strain. It is obtained by multiplying the strain by 29 000 000 and dividing by 50; or, the reading multiplied by 29 000 000 and divided by 50 × 100 000, or 5.8. In other words, the strain multiplied by the modulus of elasticity and divided by the length gives the stress, in pounds per square inch.

The stress in this same column due to the wind pressure has been computed from the manometer readings on the assumption that the building is a cantilever. The overturning moment of the wind about an elevation half-way between the twenty-fourth and the twenty-fifth floors has been computed for both a south

TABLE 6.—COMPARISON OF COMPUTED AND OBSERVED COLUMN STRESSES

Observation No.*	(a) Manometer Readings, in Inches†						(b) Stresses in Columns, in Pounds per Square Inch, for the Following Directions of the Wind Load								(c) Extensometer Readings, in Hundred Thousandths of an Inch, on the Following Corners of the Post¶				
	Elevation 36		Elevation 55		Elevation 75		Elevation 36		Elevation 55		Elevation 75								
	Manometers 2, 3, 4, 7, 8, and 9‡	Manometers 5, 6, 1, and 10‡	Manometers 2, 3, 4, 7, 8, and 9‡	Manometers 5, 6, 1, and 10‡	Manometers 2, 3, 4, 7, 8, and 9‡	Manometers 5, 6, 1, and 10‡	S	W	S	W	S	W	By computation, from manometer readings	By extensometer observations	NW	NE	SE	SW	Sum
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
2	+1.48	−1.03	+1.82	−1.20	+1.49	+0.74	+28.7	−13.4	+100.6	−44.3	+189.5	−156.0	+103.8	−148.0	+7	+25	−76	−59	−103
7	−0.51	+0.82	+0.03	+1.04	−0.01	+0.82	−9.1	+10.7	+1.6	+38.4	−1.3	+73.4	+112.9	+52.0	−2	+42	+14	−18	+36
9	−0.03	+0.62	−0.25	+1.30	+0.05	+1.05	−0.6	+8.1	−13.8	+48.0	+6.3	+94.0	+142.0	+223.0	+4	+79	+73	−2	+154
10	−1.10	−1.27	−1.16	+0.58	−0.98	−0.75	−21.4	−16.0	−63.8	+21.4	−174.4	−67.1	−171.9	−343.0	−75	−27	−66	−69	−237
11	−1.90	−1.08	−1.65	−1.60	−2.11	−2.27	−36.9	−14.1	−90.7	−59.1	−268.0	−203.0	−671.8	−693.0	−138	−114	−117	−109	−478
12	−1.80	−1.10	−1.95	−1.40	−2.35	−1.50	−35.0	−14.4	−107.4	−51.6	−299.0	−134.2	−641.6	−712.0	−146	−110	−127	−109	−492
13	+0.32	−0.35	+0.33	−0.27	+0.46	−0.32	+6.2	−4.6	+18.2	−10.0	+58.5	−28.6	+39.7	−7.2	−4	+47	−33	−15	−5
14	−0.17	−0.23	+0.01	−0.12	−0.17	−0.11	−3.3	−3.0	+0.5	−4.4	−21.6	−9.9	−41.7	−110.0	−22	+26	−54	−26	−76
15	+0.21	+0.12	+0.88	−0.03	+0.56	+0.08	+4.1	+1.6	+48.4	−1.1	+71.2	+7.2	+131.4	−63.8	−27	+34	−6	−45	−44
16	+0.21	+0.01	+0.80	−0.18	+0.83	−0.03	+4.1	+0.1	+44.0	−6.6	+105.5	−2.7	+144.4	−98.5	−27	+35	−16	−60	−68
17	−0.05	−0.05	+0.24	−0.16	−0.03	−0.09	−1.0	−0.7	+13.2	−5.9	−3.8	−8.1	−6.3	+146.2	+9	+77	+6	+9	+101
18	+0.48	+0.07	+0.93	−0.26	+0.70	−0.15	+9.3	+0.9	+51.1	−9.6	+89.1	−13.4	+127.4	+355.0	+28	+97	+66	+54	+245
19	−0.10	0	+0.19	−0.07	−0.02	−0.08	−1.9	....	+10.5	−2.6	−2.5	−7.2	−3.7	−191.3	−54	....	−31	−47	−132
20	+0.42	+0.31	+0.90	+0.27	+0.31	+0.33	+8.2	+4.0	+49.5	+10.0	+39.4	+29.6	+140.7	+116.0	+11	+55	−4	+18	+80
21	+1.10	−0.58	+1.73	0	+0.57	+0.15	+21.4	−7.6	+95.1	....	+72.4	+13.4	+194.7	+341.0	+42	+78	+64	+51	+235
23	−0.65	−0.16	−0.02	−0.17	−0.39	−0.26	−12.6	−2.1	−1.1	−6.3	−49.5	−23.3	−94.9	+187.0	+15	+57	+24	+33	+129
24	−0.54	−0.25	−0.20	−0.24	−0.38	−0.05	−10.5	−3.3	−11.0	−8.9	−48.3	−4.5	−86.5	+165.0	+14	+57	+16	+27	+114
25	+0.38	−1.42	+0.22	+0.50	−0.07	+0.48	+7.4	−15.5	+12.1	+18.5	−8.9	−143.0	+77.6	−75.3	−18	+28	−19	−41	−52
26	−2.64	−1.33	−1.65	−1.08	−1.59	−0.53	−51.3	−17.4	−90.8	−39.8	−202.0	−47.5	−448.8	−488.0	−20	−108	−112	−99	−337
28	−0.56	+1.32	−0.51	+1.25	+0.43	+1.27	−10.9	+17.2	−28.1	+46.1	+54.6	+113.8	+192.7	+59.5	−5	+47	−7	+6	+41
40	−2.18	−0.82	−1.17	−0.78	−0.98	−0.97	−42.4	−10.7	−64.4	−28.8	−124.4	−86.8	−357.5	−623.0	−115	−64	−129	−122	−430
44	−0.57	+0.05	+0.40	+0.25	+0.90	−0.15	−11.1	+0.7	+22.0	+9.2	+104.2	−13.4	+111.6	....	....	....	....	....	....
50	−1.25	−1.90	−0.40	−3.40	−0.40	−1.45	−24.3	−24.8	−22.0	−125.6	−50.9	−130.0	−377.6	−533.0	−117	−137	−67	−47	−368
52	+0.25	−1.55	+0.10	−1.15	−0.70	+0.70	−14.9	−20.2	+5.5	−42.5	−89.0	+62.8	−78.5	−226.0	−89	−115	+11	+31	−156
53	+0.60	−1.35	+0.60	−1.35	+1.55	−0.35	+11.6	−17.6	+33.0	−49.8	+197.0	−31.4	+142.8	−249.0	−97	−115	+11	+29	+72
54	+0.55	0	−0.10	−0.20	−0.20	0	+10.7	....	−5.5	−7.4	−25.4	....	−27.6	−243.0	−75	−115	−09	+31	−168
	(20 #/D′ = 384)						+74.6	+50.0	+211.0	+142.0	488.0		2 320.8§ 1 072.0‖		in hundred thousandths of an inch				

* See corresponding numbers in Table 1.
† 384 lb per sq in. is equivalent to a horizontal wind load of 20 lb per sq ft.
‡ See Fig. 4.
§ Computed stress for a horizontal load of 20 lb per sq ft on the south face.
‖ Computed stress for a horizontal load of 20 lb per sq ft on the west face.
¶ Compass directions.

pressure and a west pressure. It is assumed: (1) That the pressure is uniform between the twenty-fourth floor and an elevation half-way between the pressure openings of the thirty-sixth and the fifty-fifth floors; (2) that it is also uniform between this elevation and half-way between the fifty-fifth and the seventy-fifth floors; and (3) that between this latter elevation and the top it is uniform. These several pressures are obtained for the south face by taking the readings of Tubes 2, 3, and 4, Fig. 4, subtracting the readings of Tubes 7, 8, and 9, and dividing the resultant by 3. These summations (before dividing by 3) for the several floors are given in Table 6, Columns (1), (3), and (5); similarly, for the west pressure points. The readings of Tubes 1 and 10 are subtracted from those of Tubes 5 and 6, and the result is divided by 2 (see Columns (2), (4), and (6), Table 6).

The moment about the central elevation of the twenty-fourth floor of a 1-lb pressure, uniformly distributed on the south face, has been computed to be: (*a*) For the thirty-sixth floor, 5 411 000 ft-lb; (*b*) for the fifty-fifth floor, 15 300 000 ft-lb; and (*c*) for the seventy-fifth floor, 35 400 000 ft-lb. Similarly, for the west face these values would be: (*a*) For the thirty-sixth floor, 3 900 000 ft-lb; (*b*) for the fifty-fifth floor, 11 100 000 ft-lb; and (*c*) for the seventy-fifth floor, 26 700 000 ft-lb. To obtain the total overturning moment in either direction one would multiply the sum of the manometer readings by 5.2 to reduce them to pounds per square foot, divide by two or three, and then multiply by the proper value. The effect of the six sets of readings is additive.

Resisting this is the column steel on the twenty-fourth floor. The beam formula, $s = \frac{M\,c}{I}$, is used to obtain the stress in the steel. This assumes that the planes of the twenty-fourth and twenty-fifth floors remain planes.

The value of I about each axis has been obtained by multiplying the area of each column, in inches, by the distance to the axis, squared, in feet. The reduction to inches is indicated by the factor, 12, which cancels out of the computations. The moment of inertia, I, about the east-west axis is $31\,454\,000 \times 12 \times 12$ in.^{4}, whereas, for the north-south axis, I is $70\,970\,000 \times 12 \times 12$. The corner column is 65.25 ft and 91.25 ft from the two axes.

To ascertain the stress in the corner column corresponding to the manometer readings on the thirty-sixth floor, north and south faces, the data from Column (1), Table 6, should be multiplied by $\frac{5\,411\,000 \times 12 \times 65.25 \times 12 \times 5.2}{31\,454\,000 \times 12 \times 12 \times 3}$; or, 19.45. Similarly, for the east and west faces:

$$\frac{3\,900\,000 \times 12 \times 91.25 \times 12 \times 5.2}{70\,970\,000 \times 12 \times 12 \times 2}\text{; or, } 13.05$$

For the fifty-fifth floor the corresponding values are 0.55 and 0.3695, whereas, for the seventy-fifth floor they are 1.272 and 0.895. Columns (7) to (12) of Table 6 are obtained from Columns (1) to (6) by multiplying by these several values. Column (13) is the algebraic sum of Columns (7) to (12) and is the computed stress in the column.

A comparison of Columns (13) and (15) shows the difference between the computed and the observed stresses in this corner column. It furnishes a

rough check on the "cantilever method" of computing the stresses in the columns of buildings.

An assumed horizontal load of 20 lb per sq ft over the south face will give a computed stress of 2 330 lb per sq in. in this column, whereas the same pressure over the west face would give 1 070 lb per sq in.

PART II.—AN ESTIMATE OF THE STIFFENING EFFECT OF THE MASONRY

Introduction

Two assumptions are usually made in computing the stresses in the frames of tall buildings due to lateral forces when any of the so-called exact methods are used. One assumption is that the girders are rigidly connected to the columns and the other is that the frame is not braced by the walls, partitions, floors, and other parts of the building.

The first assumption has been given considerable study both in America[5] and in England.[6] Data have been collected and investigations are in process in some of the colleges of the United States to ascertain the percentages of error in the final results due to this assumption. It is hoped that, before long, the properties of the connections will be incorporated into the solution of the rigid frame subjected to lateral loads as a routine part of the solution.

The second assumption (neglecting the stiffening effect of the masonry) is of far greater influence but the writer has been able to find only meager reference on the subject. It is of more than academic interest, because the stresses in the frame are proportional to the deflection of the structure if the distribution of stress is the same when the masonry is present as it is in the steel frame acting alone. Therefore, the actual stress in the steel is reduced by the presence of the masonry in proportion to its influence on the stiffness. David A. Molitor,[7] M. Am. Soc. C. E., has given an estimate that the frame would be stiffened 30% to 50% if the connections were rigid. He has also estimated that the building is stiffened 300% to 400% by the materials other than the frame.

Rigidity is of considerable importance inasmuch as it affects the popularity of a building with tenants. If the structure is too limber the fact may be shown by cracks in the plaster and the movement may be noted in other ways causing lack of confidence in the safety of the structure. On the other hand, if it is too stiff the period of vibration will be short enough to be noticeable by the tenants, thus producing the same lack of confidence. In either case an unsatisfactory result is obtained. David C. Coyle,[8] M. Am. Soc. C. E., has published information on the period of vibration of a few existing structures and has collected considerable data. The problem will doubtless assume major proportions if office buildings are built higher in the future.

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[5] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 524.

[6] "Investigation on Beam and Column Connections," by C. Batho and H. C. Rowen, Second Rept., Steel Structures Research Committee, Dept. of Scientific and Industrial Research, pp. 61–137.

[7] "Structural Engineering Problems," by David A. Molitor, The Peters Co., Detroit, Mich., 1937, p. 64; also, reproduced by the Bureau of Yards and Docks, U. S. Navy Dept., 1937.

[8] "Testing the Strength of Skyscrapers," *Record and Guide*, February 7, 1931; "Sway of Tall Towers," *Architectural Record*, July, 1931, p. 109; *Civil Engineering*, May, 1938; *American Architect*, June 20, 1929; "Stiffness of Skyscrapers," *Transactions*, First Cong., International Assoc. for Bridge and Structural Engrs., Paris, May, 1932; and, *Proceedings*, Am. Soc. C. E., October, 1928, p. 2393; and August, 1932, p. 1100.

The capacity of a building to resist loads is not dependent upon the steel frame alone but upon the combination of steel and masonry, acting as a unit. As a large percentage of the material in the structure does not obey Hooke's law the stresses incurred at low loads may be of an entirely different character from those that would occur if the building were subjected to a dangerous wind pressure. It is doubtless true that, due to the greater stiffness and high elastic limit of the steel composing the frame, the stiffening effect of the masonry becomes less valuable as the loads increase. It is customary to ignore this source of strength, and probably wisely so, when designing a building; but it may account for the fact that few, if any, steel buildings have been destroyed by wind or earthquake.

An estimate of this stiffening effect should be of value, and, therefore, a study of some of the data taken from the Empire State Building investigation has been used to find the stiffening of this one building in the given range of working loads. The accuracy of the result is open to some question as it is based on only one set of computations and on data from only one building. A second set of computations has not been made because of the time and labor that is required to make the necessary model. However, this value gives a general idea of the ratio between the rigidity of a building without this bracing and one with it. For a building of different construction the ratio can be estimated with greater accuracy as a result of this study. The entire problem is one that does not allow for accurate work and great refinement; nor does it require it.

In evaluating the rigidity of the building, as actually constructed, advantage is taken of the fact that the period of vibration is a simple function of the rigidity. The building vibrates in the same manner as does the tine of a tuning fork; it is an elastic cantilever secured at its base and vibrated by the gusts of wind (not a steady force). The structure is composed of an elastic steel frame and of parts that are more or less plastic. This plasticity has a dampening effect on the vibrations but does not alter the period.

In order to form an estimate of the period of vibration, similar in all respects to the one observed but supported by the steel frame alone, a model was constructed and observed. It was constructed so that the principles of similitude could be applied, and the period of vibration computed, for a steel frame of the size of the building, and weighted to represent the dead load of the building.

From the period of vibration of the model and that of the building one can compute the relative stiffness of the model and that of the finished building. The one has rigid (soldered) connections and no masonry stiffening, whereas the other has riveted connections and a great mass of masonry surrounding the steel. The model represents the steel skeleton, proportionally weighted. It is assumed that the effect of the difference between the model and the building (masonry and connections) will be the same on the several floors of the structure.

The period of vibration does not depend upon the severity and the character of the horizontal loads but rather upon the several properties of the building that can be obtained from the engineers' plans, such as the weight, weight distribution, linear dimensions, and stiffness of steel members. Figs. 16 and 17 are views of the model used.

If two structures of the general nature of the one under consideration are built similar to each other, except for the scales of weight, length, modulus of elasticity, and moment of inertia of the sections, it can easily be shown that their periods when subjected to flexural vibration will vary directly as the weight and the cube of the length, and inversely as the modulus of elasticity and the moment of inertia of the members.

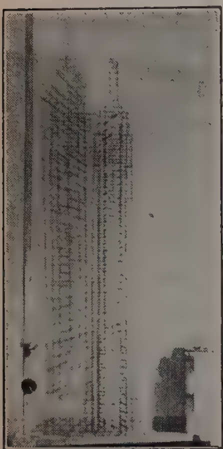

FIG. 16.—MODEL OF STRUCTURE

FIG. 17.—TESTING EQUIPMENT FOR MODEL

The period of vibration, expressed as a time dimension, T, or the ratio of homologous times, is expressed by the following equation,

$$T = \frac{W\,L^3}{E\,I} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which W = the weight of the building or the model—or the ratio of homologous weights; L = the linear dimensions—or the ratio of homologous dimensions; E = modulus of elasticity—or the ratio of homologous elastic moduli; and I = the moment of inertia—or the ratio of homologous moments of inertia. Substituting known values in Equation (1),

$$T = \sqrt{\frac{50\,530\,000 \times 144^3}{2.07 \times 43\,000 \times 10^5}} = 130$$

The observed period of the model is forty-four vibrations in 6 sec; whence $\frac{6 \times 130}{44}$, or 17.8 sec, is the computed period of an unstiffened weighted frame. The building has an observed vibration period of about 8.25 sec (see Fig. 15). The ratio of these periods is then 8.25 to 17.8 or 1 to 2.16, which makes the stiffness ratio 2.16^2 to 1, or about 4.65 to 1. It can be concluded, therefore, that the structure is very close to four and one-half times as stiff as it would be if the steel frame alone held it in position.

Fig. 18(*a*) is the deformation curve of the model due to a pull at the eighty-sixth floor of sufficient magnitude to produce a unit deflection at that floor. The weight used was 5 lb, producing a deflection of 0.244 in. or 0.0488 in. per lb. Assuming similitude between the model and the building the force of the wind against the side of the structure can be estimated by the use of this curve and the deflection of the building. In computing the wind pressure its distribution

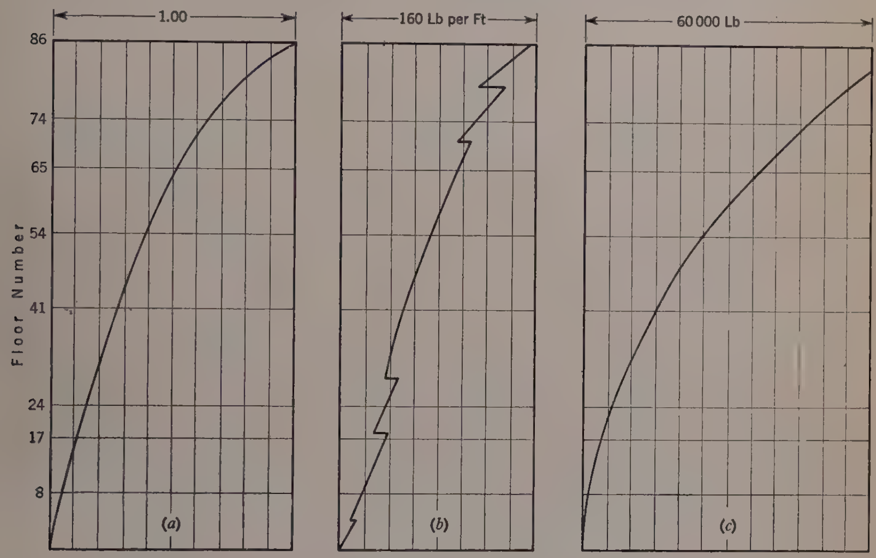

FIG. 18

must be assumed because a wind at an upper floor has more overturning effect than one lower down.

The deflection due to a load at the end of a cantilever is given by the formula,

$$\Delta = K \frac{P L^3}{E I} \quad \text{(2)}$$

in which K is an abstract constant depending upon the manner in which $E I$ varies along the length of the cantilever (the same in the model as in the prototype); that is, using ratios of similitude:

$$\Delta = \frac{P L^3}{E I} = P \frac{L^3}{E I} \quad \text{(3)}$$

in which Δ = the deflection; and P = a concentrated load. By Equation (1), $\frac{L^3}{E I} = \frac{T^2}{W}$; whence,

$$\Delta = \frac{P T^2}{W} \quad \text{(4a)}$$

and,

$$P = \frac{\Delta W}{T^2} \quad \text{(4b)}$$

In Fig. 18, if the ordinates of Fig. 18(*a*) are multiplied by the width of the building, the curve in Fig. 18(*b*) is obtained. In Fig. 18(*c*) is the integral of the curve of Fig. 18(*b*) obtained by plotting as ordinates the area below each point in Fig. 18(*b*). The total area of Fig. 18(*b*) is 64 600. Assuming a uniform pressure throughout the building of x lb per sq ft, and observing the deflection, the pressure is found to be as follows: From the plumb-bob readings in Table 3 the maximum south reading is found to be 9.76 in. and the minimum is 3.82 in. Assuming the difference is twice the deflection: $\Delta = 2.97$ in. The ratio of this deflection to that produced by a 1-lb load on the model is $\frac{2.97}{0.0488}$, or 60.86; the ratio of the times of vibration, T, is $\frac{8.25}{0.1385}$, or 59.6. The weight ratio has been computed as 50 530 000. Whence the horizontal pull at the eighty-sixth floor necessary to produce a deflection of 2.97 in. is $P = 60.86 \times 50\,530\,000 \times 59.6^2 = 865\,700$ lb. From the curves of Fig. 18 the same deflection would be produced by a uniform load over the south face of: $\frac{865\,700}{64\,600} = 13.4$ lb per sq ft, which includes the negative, as well as the positive, pressure.

It is to be noted that the foregoing data are based on the greatest recorded deflections. That storms producing greater deflections have occurred is doubtless true, so this value must be accepted with caution.

Conclusion 9, in Part III, is based partly on the values given herein and those of Column (14), Table 6. It is hoped discussion will bring out valuable information on this point.

Description of the Model

The steel frame of the Empire State Building lends itself readily to model analysis. It is unusually symmetrical and, with very few exceptions, the girders and columns can be reproduced as a rectangular grill with the columns extending from the foundation to the roof. The model was designed to have elastic properties similar to the frame when deflections in the narrow direction of the building (that is, parallel to Fifth Avenue) are being studied. It consisted of three grills fastened together in such a way that they were forced to vibrate as a unit. An examination of the plans shows that the frame at the foundation is composed of twenty-two rows of columns. The first eight rows consist of ten columns, the next six of nine columns and the last eight of ten columns. The twenty-two grills thus formed are forced to act as a unit by the concrete floors and other connections between them, and to vibrate with the same period. It was considered proper, therefore, to combine these grills into three, as indicated in Figs. 3, 16, and 17. In each grill the columns and girders were designed so they had a moment of inertia proportional to the sum of all eight (or six) corresponding columns or girders. At each floor the girders are secured to the columns by shallow wind-brace connections which, in this paper, have been considered as rigid. In addition, at certain joints, knee-braces extend from the intersection of a column and a girder to about the third-point on the girder in the floor above. These braces are represented in the model in their

proper geometric relation by stiff brass wire (21-gage) which is a minimum allowable size for compression as a column.

In one or two cases, as at the Fifth Avenue entrance, there is a variation within the group of eight grills, which condition prevails only on the six lower floors. Another place where the building required more than three grills was between the seventy-second and the eighty-first floors, because the knee-braces did not lend themselves to a single pattern. In this case the model's grill is separated into two parts over this section of the structure; and so at these places the model contained five grills.

The distances from center to center of columns, and those between floors on the model, were laid out to a scale of 1 in. equals 12 ft, or a scale of 1 to 144. The grills were constructed of cold drawn brass 1/16 in. thick, which thickness was maintained for both girders and columns except in the case of some of the heavy columns in the lower part of the building where ⅛-in. material was used. The moments of inertia computed were divided by an arbitrary constant which was of such a value that the members of the model would be of a convenient size. These depths were calculated and the brass milled to the nearest thousandth of an inch. Each column was of one continuous piece, varying in depth to correspond with the computed value of its moment of inertia, whereas at each floor the girders were represented by a continuous member whose depth was changed at the points where it intersected the columns.

Each of the grills was formed by laying the girders on the columns and soldering them with soft solder. It was considered unwise to use hard solder for fear of drawing the temper in the brass.

The three grills were fastened together at approximately every eighth floor by means of horizontal trusses having no vertical stability but considerable lateral rigidity. Between the third, and also the eighth, sets of columns vertical trusses were placed between the two grills so that the model would have rigidity perpendicular to the direction of vibration. These trusses each consisted of a piece of cold drawn brass wire (21-gage) bent to form the webbing of a Warren truss whose chords were the columns. These trusses had no stiffening effect in the direction of vibration but acted as spacers between the grills, holding them about an inch apart. The posts were set in sockets in a steel frame, 4½ by 13/16 by 24 in., and doweled securely so that the rigidity of the base was as stable as the rock on which the building itself rests.

Representative specimens of the brass strips were tested, after milling to a depth of approximately that of an average beam, in order to find the value of E, the modulus of elasticity. This was done again after the strip was treated to considerable heat, which was applied in the same manner as was done in making the soldered joints. No change in modulus was noted.

Some trouble was experienced with the soldered joints flowing when subjected to a long continued stress. In order to determine whether these joints were strong enough to carry the stresses during the period of vibration (about 1/7 sec), the model shown in Fig. 19 was constructed. The joint at Point A was first made perfectly rigid by clamping the two pieces of brass together with a C-clamp which exerted enough pressure to prevent slippage. The lever was then vibrated and the period noted by the mark of a pen on a drum revolving

42.5 times per min. The clamp was removed and the joint soldered in the same manner as the joints in the model. The lever was again vibrated, and if the joint had not held satisfactorily, part of the motion would have gone through this joint into the section beyond and thus would have slowed down

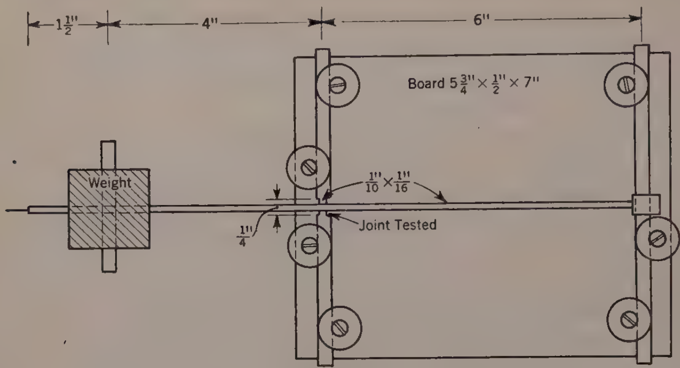

FIG. 19

the period. The period of vibration used was approximately the same as that of the model, the weight being adjusted to produce it. Five readings over a fixed period of time showed, for the soldered joint: 27.5, 27.6, 27.6, 27.7, and 27.8 half vibrations, or a mean of 27.64. For the clamped joint, on the other hand, the readings were: 27.5, 27.8, 27.6, 27.8, and 27.6, with a mean of 27.66. The difference between the two means is low enough to warrant the conclusion that the soldered connection is satisfactory when the stress is low and of short duration.

In making the model it was impracticable to control both the area and the moment of inertia of the several members as the constant width of 1/16 in. had to be maintained, that being the thickness of the brass ribbon used. The areas, therefore, are not proportional to those of the building. In the case of the girders the direct stress is low and so the lack of proportionality is not objectionable. With the columns, however, the direct stress and the areas do enter into the deflections of both building and model and thus introduce a small error into the results. The fact that the areas of the braces in the model are too large for similitude has little influence on the results.

The weight of each floor was obtained from the column schedule of the steel plans. The total weight of the building above each floor was obtained by adding the stresses in all columns on that floor. The increment for each floor was obtained from these data. The partition loads and live loads were subtracted where the floors were not occupied. In the case of the occupied floors an estimate was made of the added weight and the model vibrated under conditions both allowing for this added weight and omitting it. The difference was not noticeable.

These computed floor loads were divided by an arbitrary constant and the weights of the floors in the model were increased to the computed value by the addition of pieces of steel.

In computing the weight of the model the weight of the brass girders and columns was first computed and distributed at each floor. The weights of the knee-braces and the several stiffening trusses were obtained by observing the length of the wire used in making them. The model was then weighed and the difference between the computed and actual weight was taken as the weight of the solder. This was distributed among the several floors in proportion to the number of joints on that floor. To each floor was then added enough weight in the form of steel rods to bring its weight up to the proper ratio. In a few cases this added weight was computed to be negative, in which case one-half the value was subtracted from the weight to be added to the floor below and the other half from that to be added to the floor above. The weights were secured in position by means of rubber bands whose weight was also considered.

The model, as thus designed and constructed, was similar to the building in the following particulars:

(*a*) Its geometrical proportions (both vertically and in a direction parallel to its narrow face) were to a scale of 1 to 144;

(*b*) Its weight distribution was the same as that of the building, and its weight, or mass ratio, was 1 to 50 530 000;

(*c*) The ratio of modulus of elasticity of the materials used in the model to that of the steel frame of the building was 14 to 29; and,

(*d*) The ratio of the moments of inertia (and, therefore, the ratios of the stiffness of the various members of the model, to the corresponding members of the building) was 1 to $43\,000 \times 10^5$. This ratio was chosen because it produced sizes of members in the model with which it was practical to work.

The model was not similar to the building in the following particulars: (1) In the dimensions perpendicular to the narrow face of the building; (2) in the method of fastening girders to columns; (3) in the fact that it is not braced by masonry surrounding the frame—floors, walls, partitions, and other building materials; and, (4) in the cross-sectional areas of the several members.

Fig. 17 shows the method of obtaining the vibration characteristics of the model. A 5-lb weight was used to produce the horizontal pull. The connection was broken suddenly by means of a trigger trip.

The horizontal movement was recorded by means of a thread attached to the model at one end and held taut by a rubber band at the other. A lever, holding a pen, was attached to the thread in such a manner that the pen traced the movements of the model on a revolving disk. A clock indicated the seconds on the disk. The lever exaggerated the movement of the model in the ratio of 478 to 240. Fig. 20 gives the curve produced with: (*a*) The load and recorder attached to the eighty-sixth floor; (*b*) the load on the eighty-sixth floor and the recorder on the sixty-fifth floor; and (*c*) the load on the sixty-fifth floor and the recorder on the eighty-sixth floor. The recorder was placed on the eighth, seventeenth, twenty-ninth, forty-fourth, fifty-fourth, sixty-fifth, seventy-fourth, and eighty-sixth floors. For each position the load was applied and suddenly released at all the floors named. Thus, a series of sixty-four curves similar to those of Fig. 20 were produced. The period of vibration was the same in all the curves.

The curve of Fig. 18(*a*) was plotted from measurements taken from this series of curves. The position before and after releasing the 5-lb weight was used as the deflection due to the pull.

Theory

The derivation of the following formulas is given so that one may note the approximations involved in Equation (20) and thus estimate the errors if the result of neglecting them is open to question.

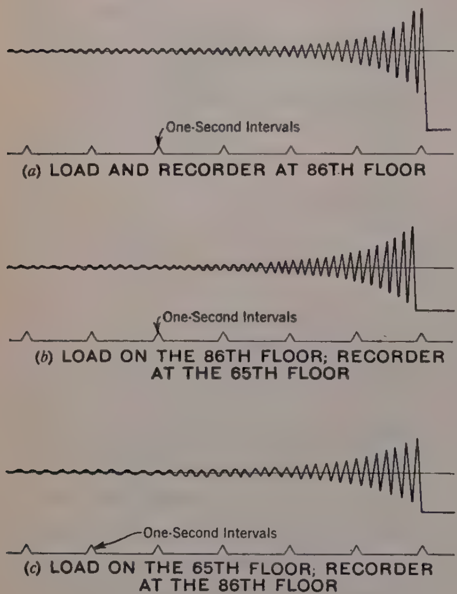

Fig. 20.—Vibration Curves of Model

Let A B in Fig. 21 represent a cantilever secured at Point A and vibrating about its neutral position, in a plane parallel to the paper, with a simple harmonic motion. This cantilever may be made up of a complicated set of springs, as girders and columns; or it may be a simple spring. A mass, $m\,dx$, is fixed at Point P, the symbol, m, being the mass per linear unit; it may be a function of x. When the particle, $m\,dx$, is in a position of maximum deflection the velocity is zero, and all energy is potential. This energy is stored up in the stressed fibers of the cantilever. If the cantilever is a building frame this energy is the internal energy of the strained columns and girders. When the particle, $m\,dx$, is in the neutral position all the energy is transferred to kinetic energy. If the period of vibration is T and the amplitude of vibration is y the velocity, v, at the central position is given by the equation,

$$v = \frac{2\,\pi\,y}{T} \quad \text{.......................... (5)}$$

The kinetic energy, W_k, of a moving mass is $\frac{m\,v^2}{2}$, and the energy in the mass is,

$$W_k = \left(\frac{m\,dx}{2}\right)\left(\frac{4\,\pi^2\,y^2}{T^2}\right) = \frac{2\,\pi^2}{T^2}\,m\,y^2\,dx \quad \text{.............. (6)}$$

in which both m and y are functions of x. In some cases it may be possible to express m as a mathematical function of x, but in any specific example the distribution of the mass over the length of the structure can be found. At the point of maximum deflection the acceleration of the mass, $m\,dx$, is equal to,

$$a = -\,\frac{4\,\pi^2\,y}{T^2} \quad \text{.......................... (7)}$$

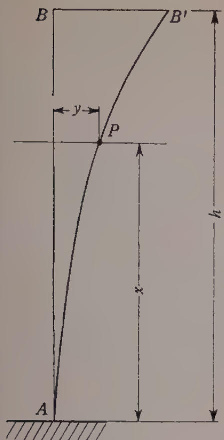

FIG. 21

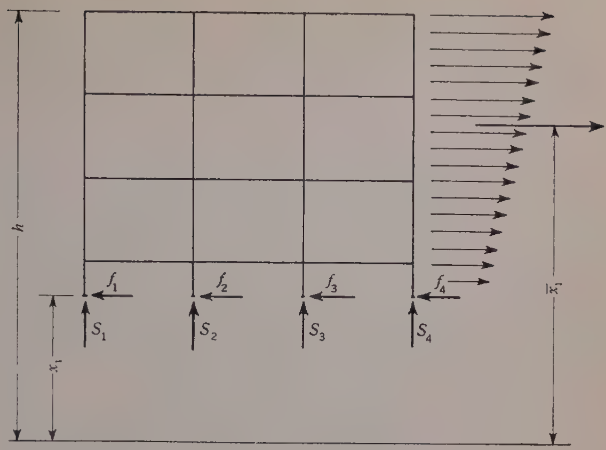

FIG. 22

and, since $f = m\,a$, the force exerted by, or on, Mass m, due to its acceleration, is,

$$df = -\frac{4\,\pi^2}{T^2}\,m\,y\,dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

This is the force that produces the stress throughout the cantilever and determines the value of y and, consequently, the shape of the curve of maximum amplitude. In addition to this energy of horizontal motion the point, P, will fall or rise a slight distance, b, producing a change in potential energy of $b\,g\,m\,dx$, which gives:

$$W_t = \frac{2\,\pi^2}{T^2}\int_0^h m\,y^2\,dx + g\int_0^h b\,m\,dx \ldots\ldots\ldots\ldots (9)$$

as the total energy which changes from kinetic to potential as the particle moves from the central position to the position of maximum amplitude. In Equation (9), g = the acceleration due to gravity.

The second term in the problem considered in this paper is of minor significance and value, although in some cases, as a pendulum, it might be a dominating factor. Let Fig. 22 be that part of the frame forming the cantilever of Fig. 21, above which is a plane half-way between the nth and the $n + 1$ floor, or a distance, x, from the base. At maximum amplitude the structure is acted upon by the forces, F, shown in the diagram, in which,

$$F = \frac{4\,\pi^2}{T^2}\int_{x_1}^h m\,y\,dx \ldots\ldots\ldots\ldots\ldots\ldots (10a)$$

and the moment above Point C is,

$$F\,(\bar{x} - x_1) = \frac{4\,\pi^2}{T^2}\int_{x_1}^h m\,y\,x\,dx - F\,x_1 \ldots\ldots\ldots\ldots (10b)$$

Each column resists a fraction of the shear force, F, and a thrust, S, as well as a

moment, M, which is not shown. The values of the forces (such as S) are assumed to be proportional to the moment about Point C as given by Equation (10*b*). The values of the column shears have a definite but unknown distribution among the columns and as their sum total is equal to F they are proportional to this value and may be represented by the fractional force, $\frac{p\,F}{q}$, in which p and q are abstract, dimensionless numbers. The values of S are probably approximately proportional to their distance from Point C. Let e be the distance from Point C to any given column and $\frac{c}{d}$ (a ratio of dimensionless abstract numbers) be the fraction that corrects for any lack of uniformity in the stress distribution. The value of Moment M is equal to the shear times the distance to the point of counterflexure or r, whence,

$$\Sigma\, S\, e = \frac{4\,\pi^2}{T^2}\int_{x_1}^{h} m\, x\, y\, dx - F\, x_1 \qquad (11a)$$

and,

$$S = \frac{1}{e}\frac{c}{d}\left[\frac{4\,\pi^2}{T^2}\int_{x_1}^{h} m\, x\, y\, dx - F\, x_1\right] \qquad (11b)$$

The energy in a column under an axial load, however, is equal to one-half the stress times the strain, or,

$$W_a = \frac{S}{2}\frac{S\,l}{A\,E} = \frac{S^2 l}{2\,A\,E} \qquad (12a)$$

or,

$$W_a = \frac{16\,\pi^4}{2\,T^4}\frac{1}{e^2}\left(\frac{c}{d}\right)^2\frac{L}{A\,E}\left[\int_{x_1}^{h} m\, x\, y\, dx - x_1\int_{x_1}^{h} m\, y\, dx\right] \qquad (12b)$$

The shear on the post is,

$$V = \frac{4\,\pi^2}{T^2}\frac{p}{q}\int_{x_1}^{h} m\, y\, dx \qquad (13a)$$

and the moment in the column at any point is,

$$M_c = \frac{4\,\pi^2}{T^2}\frac{p}{q}\, r\int_{x_1}^{h} m\, y\, dx = \frac{j\,r}{T^2}\int_{x_1}^{h} m\, y\, dx \qquad (13b)$$

if $4\,\pi^2\,\frac{p}{q} = j$, a substitution constant. The moment at the end of a girder is proportional to the moment in the column adjacent to that end, plus the difference in the thrusts, S, on this column above and below the joint, times the distance from the joint to the point of counterflexure in the girder:

$$M_b = M_c\frac{u}{v} + (S_n - S_{n+1})\, 0 \qquad (14a)$$

but, since $S_n - S_{n+1}$ is proportional to F:

$$M_B = \frac{u}{v}\times\frac{4\,\pi^2}{T^2}\times\frac{p}{q}\, r\int_{x_1}^{h} m\, y\, dx + \frac{i}{n}\, 0\,\frac{4\,\pi^2}{T^2}\int_{x_1}^{h} m\, y\, dx$$

$$= \frac{K\,r}{T^2}\left[\int_{x_1}^{h} m\, y\, dx\right] \qquad (14b)$$

in which $K = \frac{u\,p}{v\,q} + \frac{i\,o}{n\,r}$ is an abstract number; but the energy in the beam or column due to bending is equal to, $W = \int_0^l \frac{M^2\,dx}{2\,E\,I} = \frac{1}{2\,E\,I}\int_0^l M^2\,dx$, in which l is the length of the member. Substituting the values:

$$M_c = j\,r \int_{x_1}^{h} m\,y\,dx \ldots\ldots\ldots\ldots (15a)$$

and,

$$M_b = \frac{K\,r}{T^2} \int_{x_1}^{h} m\,y\,dx \ldots\ldots\ldots\ldots (15b)$$

the energy in the column is expressed by:

$$W_c = \frac{j^2}{2\,E\,I\,T^4}\left[\int_{x_1}^{h} m\,y\,dx\right]\left[\int_0^l r^2\,dx\right] \ldots\ldots\ldots (16a)$$

and, the energy in the beam, by:

$$W_b = \frac{K^2}{2\,E\,I\,T^4}\left[\int_{x_1}^{h} m\,y\,dx\right]\left[\int_c^l r^2\,dx\right] \ldots\ldots\ldots (16b)$$

the second integral in each case being taken from the point of counterflexure to either end of the beam or column. Collecting these terms and equating the energy of motion given by Equation (9) against that of the stored energy of the deflected structure:

$$\frac{2\,\pi^2}{T^2}\int_0^h m\,y\,dx + g\int_0^h b\,m\,dx$$
$$= \frac{16\,\pi^4}{2\,T^4}\sum \frac{1}{e^2}\left(\frac{c}{d}\right)^2 \frac{L}{A\,E}\left[\int_{x_1}^{h} m\,x\,y\,dx - x_1\int_{x_1}^{h} m\,y\,dx\right]^2$$
$$+ \frac{1}{2\,T^4}\sum \frac{j^2}{E\,I}\left[\int_{x_1}^{h} m\,y\,dx\right]^2\left[\int_c^l r^2\,dx\right]$$
$$+ \frac{1}{2\,T^4}\sum \frac{K^2}{E\,I}\left[\int_{x_1}^{h} m\,y\,dx\right]^2\left[\int_0^l r^2\,dx\right]_x \ldots\ldots\ldots (17)$$

Using capital letters for the dimensionless terms, which are, therefore, independent of the character of the model and its relationship to its prototype, Equation (17) becomes (after multiplying by T^4):

$$T^2\,A\int_0^h m\,y^2\,dx + T^4\,B\,g\int_0^h m\,b\,dx = C\sum \frac{l}{e^2\,A\,E}\left[\int_{x_1}^{h} m\,x\,y\,dx\right.$$
$$\left. - x_1\int_{x_1}^{h} m\,y\,dx\right]^2 + D\sum \frac{r^3}{E\,I}\left[\int_{x_1}^{h} m\,y\,dx\right]^2$$
$$+ F\sum \frac{r^3}{E\,I}\left[\int_{x_1}^{h} m\,y\,dx\right]^2 \ldots\ldots\ldots (18)$$

in which A, B, C, D, and F are abstract numbers in the terms representing the energy: (a) Of motion of the mass of the structure; (b) of position of the mass of the structure; (c) in the columns due to direct thrust; (d) in the columns due to bending; and (f) in the beams due to bending. Neglecting the second term as small, and the third term as being probably small:

$$T^2 = \frac{D \sum \frac{r^3}{E\,I} \left[\int_{x_1}^{h} m\,y\,dx \right]^2 + F \sum \frac{r^3}{E\,I} \left[\int_{x_1}^{h} m\,y\,dx \right]^2}{A \int_0^h m\,y^2\,dx} \quad \ldots\ldots (19)$$

Writing the dimensions of these terms for similitude:

$$T^2 = \frac{\frac{L^3}{E\,I} \left[\frac{M\,L^2}{L} \right]^2}{\frac{M\,L^3}{L}} = \frac{M\,L^3}{E\,I} \quad \ldots\ldots (20)$$

that is, the square of the period of vibration of the frame varies directly as the mass and the cube of the linear dimension and inversely as the modulus of the material and of the moments of inertia of the members. As the mass ratio is the same as the weight ratio the formula is the same as Equation (1).

PART III.—CONCLUSIONS

The following conclusions have been suggested by the data recorded in this paper:

(1) The distribution of the wind pressure on a tall building is very complicated and irregular, the air currents having been broken up by the surrounding structures and by the building itself.

(2) The action of a building under horizontal loads is plastic as well as elastic, with the result that strains and deflections are not proportional to the forces that produce them.

(3) The manometer readings do not give a true picture of the forces acting on the building, but do give the pressure at definite isolated points. This is because the wind comes in gusts that affect only a small surface. Frictional resistance has some influence.

(4) The deflection of the building due to the sun shining on one side of it (and expanding that side) is practically negligible.

(5) The horizontal deflection of this building during a wind storm is notable because of its popular interest and the small amount of data on this subject.

(6) Theoretically, there is danger of designing a building too stiff as well as not sufficiently rigid.

(7) The cantilever and portal methods of design have neither been corroborated or refuted by these data.

(8) The masonry increases the rigidity of the building about 350% more than that of the unsupported frame.

(9) The data herein given have not shown that 20 lb per sq ft is an improper value for the assumed lateral load on a building of the type of the one investigated when in a similar environment. This leaves the masonry as an added, but not perfectly reliable, factor of safety.

Acknowledgments

The design and installation of the instruments herein described were made by the engineering staff of the American Institute of Steel Construction. The firm of Shreve, Lamb and Harmon, architects of the building, co-operated in the installation of the instruments and the gathering of the data. Many data and many suggestions were furnished and made by the late H. G. Balcom, M. Am. Soc. C. E., a member of the Institute's Committee on Research and Engineer of the design and construction of the building. F. H. Frankland, M. Am. Soc. C. E., member of the Committee and Chief Engineer of the Institute, supervised the design and installation of the manometers, extensometers, and cameras, and the collection of the first set of data (including that for the area shown in Fig. 7). Mr. Hewitt Crosby obtained some of the collimator data and assisted in obtaining some of the other readings. The second group of readings (the vicinity map of Fig. 6), the deflection curves, and Weather Bureau reports were obtained by the writer assisted by Walter J. Gray, Jun. Am. Soc. C. E., and other students from the College of the City of New York, in New York City. The model was made by the writer in the laboratories of the City College. The curves and readings from the model were made by Mr. Arthur Gatterdam. The balloon experiment was made, and many helpful suggestions were given, during the first period of the research, by Mr. Coyle.

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PAPERS

TRANSPORTATION OF SAND AND GRAVEL IN A FOUR-INCH PIPE

By G. W. Howard,[1] Jun. Am. Soc. C. E.

Synopsis

Tests on 4-in. pipe, transporting water mixed with sand and small gravel, are described in this paper. Although a description of the flow characteristics of sand and water mixtures and gravel and water mixtures in a 4-in. pipe constitutes the main part of the paper, an attempt is made to generalize the findings in regard to other sizes of pipe in so far as it is believed to be applicable. A discussion is given of existing general formulas for the transportation of materials in pipe lines, and it is attempted to prove that the type of material to be transported and the experience of the designer should have more bearing on the solution of a problem than any solid-transportation formula now extant.

No new formulas are presented. An insufficient number of investigations have been made to justify the use even of the formulas now in existence. The attempt is made, however, to consolidate all known facts concerning the transportation of sand and water mixtures in order to show that, at present, only general characteristics can be determined for this phenomenon.

Introduction

One of the earliest discussions on the transportation of sand mixtures in pipes is given in a paper entitled "Works for the Purification of the Water Supply of Washington, D. C.,"[2] by the late Allen Hazen and E. D. Hardy, Members, Am. Soc. C. E. Only a brief treatment is presented in the paper, however, because the topic of sand transportation in filter washing is incidental to the primary subject. The pipes for which friction curves are given had diameters of 3 and 4 in. Types of transportation and other flow characteristics are not considered, as friction losses were the only features that were of interest. A discussion of the paper of Messrs. Hazen and Hardy by Nora Stanton Blatch,

Note.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted **November 15, 1938.**

[1] Jun. Engr., U. S. Waterways Experiment Station, Vicksburg, Miss.

[2] *Transactions*, Am. Soc. C. E., Vol. LVII (1906), p. 307.

Jun. Am. Soc. C. E., provides more definite data than any other available reference on the subject.[3] Using the results from tests on 1-in. pipe as a basis for her discussion, Miss Blatch compared the data of Hazen and Hardy with additional data from the Mississippi River Commission on the dredges *Alpha*, *Beta*, and *Gamma*. Due to the fact that the test line was equipped with a transparent section, Miss Blatch covered, in detail, the types of movement that occur at different velocities. An analysis of all data led to a general formula for head loss in pipes ranging in diameter from 1 in. to 32 in., and for all variations of solid concentration. Her findings are generally used as references in textbooks on hydraulics.[4]

The data available from pipe-line dredges are so few that it might be considered surprising until the difficulties in making accurate field measurements are considered. Unstable pumping conditions with subsequent variations in pipe-line velocities and solid concentrations make data of this nature extremely difficult to correlate. Due to the fact that the load in the pipe line of a dredge varies so frequently, it is almost impossible to obtain consistent pressure-gage readings; therefore, an average value of velocity and solid concentration is all that can be hoped for in tests on large pipe lines. Discussions of tests on pipe-line dredges have been presented more frequently in recent years,[5] but no sets of curves, such as were presented by Miss Blatch, have been included in any of these papers.

Although friction losses for pipe lines of hydraulic dredges have been determined,[6] few instances have occurred in which these lines were loaded with solids. Since actual design work for this plant must consider the material that is being moved, the losses found for water alone have definite limitations.

The purpose of this paper is to present the findings from tests on 4-in. pipe carrying varying concentrations of sand and gravel. These findings are believed to be of general interest to those engaged in work dealing with the transportation of solids through closed conduits, and the results presented herein are given with the view of advancing the limited knowledge in this field.

Program of Experiments

Two separate series of tests were performed, each on a different material. The investigation of the sand and water mixtures was more complete than that of the gravel mixtures. This was due to the fact that the specific purpose of the gravel experiments, to which the data presented herein are only incidental, had been limited in scope, because of findings from the sand tests. During each series the following conditions were varied within certain limits: (*a*) Pipe-line velocities; and (*b*) solid concentrations.

[3] *Transactions*, Am. Soc. C. E., Vol. LVII (1906), p. 403.

[4] "Hydraulics," by Ernest W. Schoder and Francis M. Dawson, Members, Am. Soc. C. E., McGraw-Hill Book Co., 1934, p. 259; also "Hydraulics and Its Applications," by A. H. Gibson, D. Van Nostrand Pub. Co., 1930, p. 210.

[5] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 625; "Hydraulic Dredging and Methods of Placing the Fill," *Engineering News-Record*, August, 1935; and "Hydraulic Fill at Quabbin Dike," *Engineering News-Record*, June 18, 1936.

[6] "Friction in Dredge Pipes," by James H. Polhemus, M. Am. Soc. C. E., and John R. DuPriest, *Transactions*, A. S. M. E., Vol. 49–50, 1927–28, HYD–50–7; also, "Velocity Tests in Hydraulic Dredge Pipes," *Engineering News-Record*, January 6, 1921.

Description of the Apparatus

General.—The apparatus used to maintain a constant solid concentration consisted of a sump, a pump, and a variable head tank that discharged through the 4-in. test line back into the sump. This caused a uniform circulation of any mixture and provided a means for obtaining constant pressure and an even distribution of solids over the cross-sectional area of the pipe. The arrangement of this apparatus is shown diagrammatically in Fig. 1.

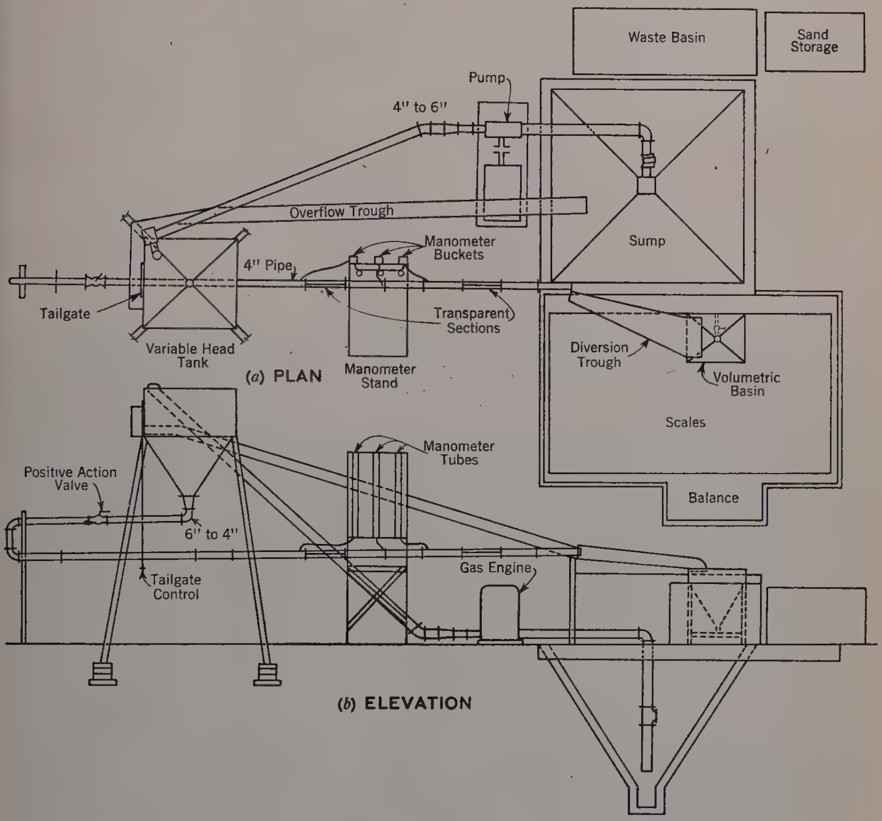

Fig. 1.—Apparatus Used on Tests for Sand and Gravel

Test Line.—Starting from the variable head tank the 4-in. line extended 8 ft horizontally and then curved downward and reversed its direction through two 90° ells. The upper 8-ft section was equipped with a positive-action cock to give control over the lower velocities. After reversing direction, the 4-in. line extended a distance of $86\frac{1}{4}$ diameters and discharged into the sump. The lower $43\frac{1}{3}$ diameters of the line were used as the test section. Two 2-ft sections of transparent pyralin pipe were installed in the test line and three open-end manometers were connected to the test section. Due to the fact that considerable material was usually carried on the bottom of the pipe, it was not con-

sidered practical to use a manometer ring or four manometer taps. Two openings, at 45° with the horizontal plane, were used at each connection. The up-stream connection was $43\frac{1}{3}$ diameters from the end of the line, and the lengths of the other connections were varied for different tests; the readings of these two gages were used as a check on the up-stream manometer.

Solid Concentration Determination.—The device used to determine the percentage of solids of the mixture consisted of a volumetric basin supported on scales. A diversion trough, which was moved under the end of the pipe when samples of the mixture were taken, was attached to the volumetric basin.

Concentrations of material over the cross-sectional area of the pipe were determined by inserting a tube, $\frac{7}{16}$ in. in diameter and bent to an angle of 90°, into the end of the pipe. To insure comparable locations for all samples, the pipe was held in place by a plate that could be raised or lowered between two hinges in order to give vertical control. Horizontal control was secured by a wing nut which held the tube firmly in place. Three ranges across the pipe were used for observations: One at the center of the pipe, and the other two at one-half the distance to the edge of the pipe on either side of the center range. This apparatus is shown in Fig. 2.

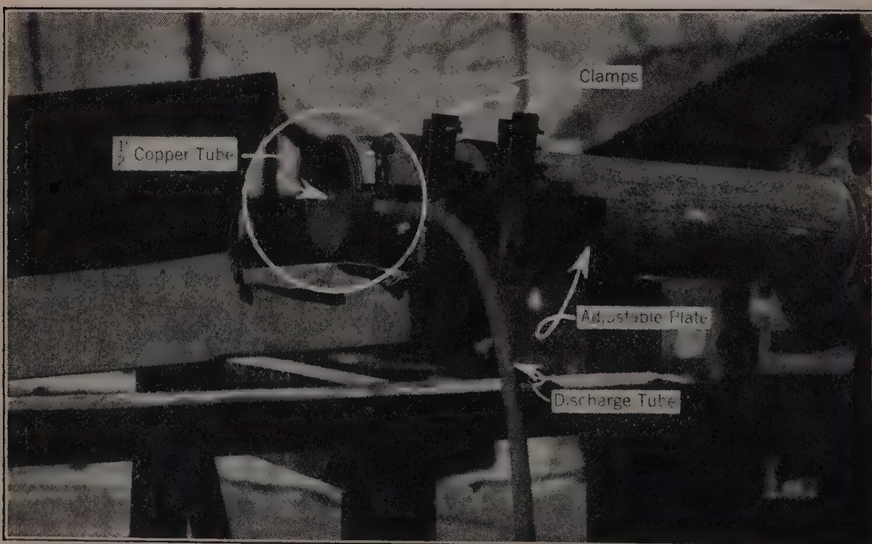

FIG. 2.—APPARATUS FOR DETERMINING THE CONCENTRATION OF SOLID MATTER

PROCEDURE OF EXPERIMENTATION

The material tested is known at the United States Waterways Experiment Station as Pearl River sand, an analysis of which is shown on Fig. 3. The gravel in Fig. 3, known locally as "pea gravel," had the general specification that no particle should be larger than $\frac{1}{4}$ in.

Sand Tests.—The velocity was first set at about 5 ft per sec and a solid concentration introduced into the circulating system that caused about 10 lb

per sec to be discharged at this velocity. (The percentage of solids used for these investigations was apparent; that is, included water in the voids. This resulted in a specific gravity for wet sand of 2.08 and wet gravel of 1.94, instead

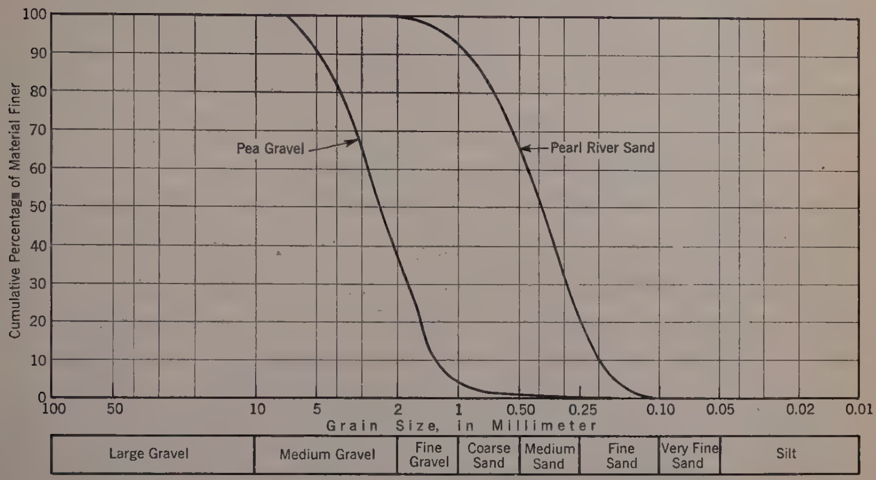

FIG. 3.—MECHANICAL ANALYSIS OF PEARL RIVER SAND AND PEA GRAVEL, TESTED IN A FOUR-INCH PIPE

of the 2.65 customarily used.) When the condition had been obtained and checked, a complete set of manometer readings was taken and the velocity increased to about 6 ft per sec. Another set of manometer readings was taken at this increased velocity. In this manner the data for the complete velocity range obtainable with the apparatus were obtained for a sand discharge of 10 lb per sec. The equilibrium set up in the system was such that, when the velocity was increased, although the percentage of solids decreased, the same weight of sand was discharged. Clear water was used in the manometers and this pressure was later converted to feet of mixture of the solid concentration.

After obtaining the data for the 10-lb per sec discharge, the velocity was reduced and material added until the discharge was 15 lb per sec. The foregoing testing procedure was followed, for the new sand discharge, throughout the entire range of the carrying capacity of the pipe, with the sand discharge increased in 5-lb increments.

The distribution of solids over the cross-sectional area of the pipe was determined by inserting the bent tube sampler and taking observations at different places in the section. Besides obtaining data for the percentage of solids passing a certain point, information was also obtained relative to the velocity in the pipe at that location.

Gravel Tests.—The primary difference between these tests and the sand tests lay in the fact that the testing range was considerably decreased. Velocities of from about 5.5 to 8.5 ft per sec constituted the entire range covered in the investigation. Since an equilibrium similar to that obtained during the sand tests occurred during this investigation, it was possible to observe the entire velocity range for any given weight of material discharged without

changing the quantity of gravel in the circulating system. Gravel discharges of from 10 to 30 lb per sec were used and the same test routine was followed as that used for the investigation of sand. Considerably more difficulty was experienced in obtaining check readings for these tests on gravel than for those with sand. The distribution of solids over the cross-sectional area could not be obtained on account of the fact that the size of the gravel was such that it would plug the sampling device. Although the particles would enter the tube, the curved outlet caused blocking.

Method of Transporting the Material

Sand.—Three distinct types of transportation were found for sand and water mixtures in pipes. The first of these was at low velocities, where practically all material was on the bottom of the pipe. A layer of sand was built up and the material rolled over this bed in much the same manner as in an open flume. The second type of movement was one in which the sand layer that was built up at low velocities would move forward slightly and then halt, while other material would still move freely over this bed. This spasmodic movement can be termed most correctly, "jerking," because the material along the bottom moved freely one instant and was quiet the next. The third type was one in which the material was moving over the entire cross-sectional area, although the greater percentage was concentrated in the lower part of the pipe. These three types of movement cause head-loss determinations for sand and water mixtures naturally to be quite complex, because of the indeterminate line of differentiation. For a 4-in. pipe transporting the sand that was used in the studies at the U. S. Waterways Experiment Station, the type of movement in which the material is on the bottom of the pipe will generally

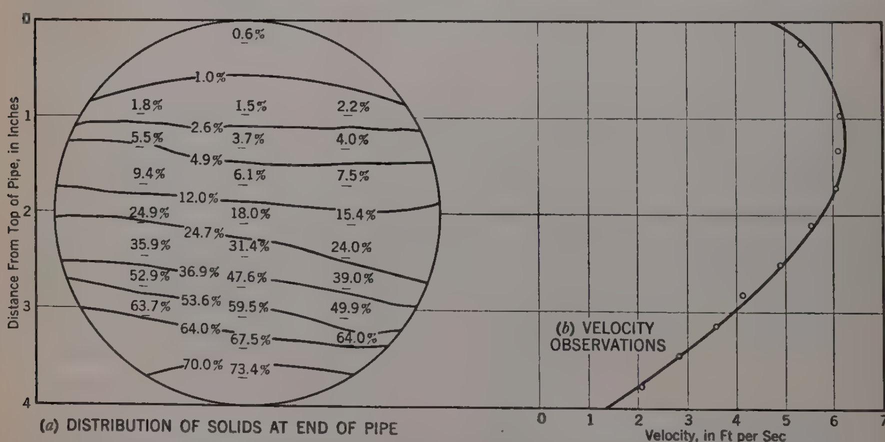

FIG. 4.—DISTRIBUTION OF SOLIDS AND VELOCITY OBSERVATIONS IN THE CROSS-SECTION OF A PLAIN FOUR-INCH PIPE

occur for velocities of less than 6.0 ft per sec; the type for material that jerks along the bottom, for velocities of from 6.0 to 7.5 ft per sec; and the type for material that is moving at velocities greater than 7.5 ft per sec. Although the

foregoing values will vary somewhat with solid concentrations, it is possible to use them for general purposes.

The manner in which the material is concentrated over the cross-sectional area is shown in Fig. 4, together with the variation of vertical velocities. These velocity observations are considered as weights rather than true values because it was not possible to obtain a satisfactory calibration of the sampling device, due to the variation of the solid concentrations, coefficient of friction, and mean grain size of material transported at various locations in the pipe. General data relating to these observations are:

Solid matter discharged, in pounds per second	14.6
Percentage of solids	14.4
Velocity of flow in pipe, in feet per second	8.88

Although it can be seen that the greater concentration of material is on the bottom of the pipe, this does not mean that the greater quantity of material is carried in this area. Fig. 5 presents a curve showing the distribution of

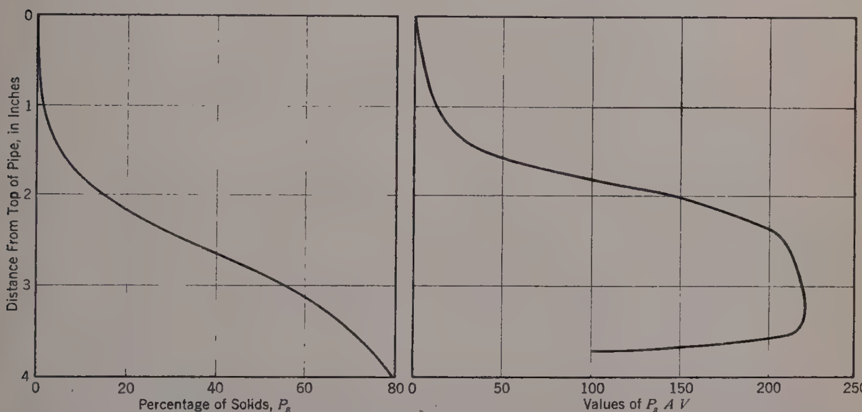

Fig. 5.—Curves of Solid Concentration and Quantity of Material Discharged

solids in a vertical section and a curve showing the volume of material discharged from this vertical section. The volume curve was obtained by measuring the areas between contours shown on Fig. 4 (by planimeter) and multiplying these areas by a value obtained from the velocity curve.

Gravel.—Types of movement similar to those observed for sand were found for gravel, with the exception that the first type (that in which the material rolled over a motionless layer) covered such a small range that it should scarcely be considered in a class by itself. The stage at which the material "jerked" along the bottom extended over a much greater velocity range than had been observed for sand, but, due to the very gradual change from this range to the one in which all particles were in motion, it was not possible to determine a definite upper transition velocity. The lower transition velocity was determined to be about 5.0 ft per sec. At velocities less than this value, for the solid concentrations that were investigated, the pipe became blocked frequently.

This was caused by the lower gravel particles becoming motionless on the bottom and the other particles piling rapidly on top of each other instead of rolling along over a motionless layer as did the sand particles.

HEAD LOSS VARIATIONS

Curves showing changes in head loss for sand and gravel mixtures in the 4-in. pipe are presented in Fig. 6. These curves show tendencies that are to be expected from the aforementioned types of movement. At velocities where material is settled or greatly concentrated on the bottom, there is a greater head loss per foot of pipe than for slightly higher velocities, because of the pressures created by the constriction of the cross-sectional area. The minimum head loss was obtained during the velocity range in which the jerking movement occurs, since the constriction was being removed. Curves of the losses for sand (Fig. 6(*a*)) show this condition more clearly than do those for gravel

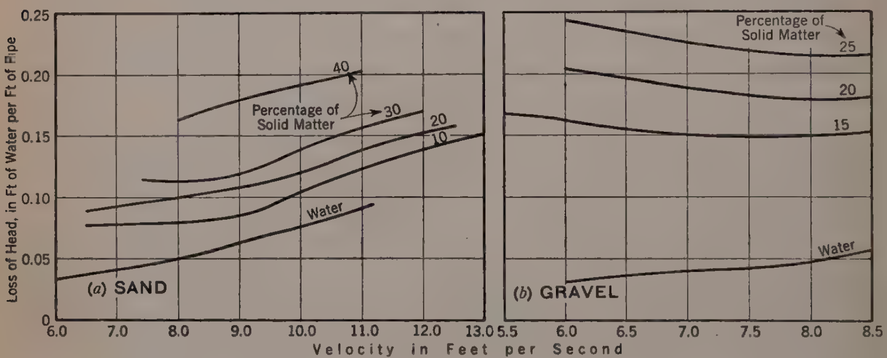

FIG. 6.—HEAD LOSS CURVES FOR VARIOUS SAND AND GRAVEL MIXTURES

(Fig. 6(*b*)), because of the small velocity range investigated for gravel and the fact that gravel was found to travel along the bottom in a high concentration at all velocities. The curves for sand show points of minimum loss and the gradual increase in head loss at high velocities. The gravel curves, however, show only the great losses found at low velocities and the gradual decrease in head loss for a corresponding increase in velocity.

When applied to water, exponential pipe formulas are more accurate than the Darcy formula,

$$h_f = \frac{f\,L\,V^2}{2\,g\,D} \qquad (1)$$

which fact more than offsets the extra computation required. Changes in head loss for pipes carrying sand or gravel ,however, can best be shown by the formula,

$$f = \frac{2\,g\,D\,h_f}{L\,V^2} \qquad (2)$$

and the results from an investigation of f-values will be presented before consideration is given to any exponential formulas. (In Equations (1) and (2): h_f = loss of head due to friction; f = a friction factor; L = length of pipe,

in feet; V = average velocity, in feet per second; g = acceleration due to gravity; and D = mean inside diameter of pipe, in feet.)

The Common Formula for Head Loss.—Values of f for various concentrations of material are presented in Table 1. (It should be noted that these values

TABLE 1.—VARIATIONS IN THE FRICTION FACTOR, f,* FOR CHANGES IN SOLIDS CONCENTRATION IN A FOUR-INCH PIPE

Velocity, V, in feet per second	(a) VALUES OF f FOR THE FOLLOWING PERCENTAGES OF SAND IN THE MIXTURE							(b) VALUES OF f FOR THE FOLLOWING PERCENTAGES OF GRAVEL		
	10	15	20	25	30	35	40	15	20	25
5.5	....	....	....	....	....	....	....	0.089	....	....
6.0	....	....	....	....	....	....	....	0.084	0.102	0.118
6.5	0.035	....	....	....	....	....	....	0.069	0.084	0.096
7.0	0.030	0.031	0.033	....	....	....	....	0.058	0.070	0.082
7.5	0.027	0.028	0.030	....	0.034	....	....	0.050	0.060	0.068
8.0	0.024	0.026	0.027	0.029	0.030	....	0.038	0.044	0.051	0.058
8.5	0.022	0.024	0.026	0.026	0.027	0.029	0.036	0.040	0.045	0.051
9.0	0.019	0.023	0.024	0.025	0.025	0.029	0.033	....	....	....
9.5	0.020	0.022	0.023	0.023	0.024	0.028	0.031	....	....	....
10.0	0.020	0.021	0.021	0.022	0.023	0.026	0.029	....	....	....
10.5	0.020	0.020	0.020	0.021	0.022	0.024	0.027	....	....	....
11.0	0.020	0.020	0.020	0.021	0.021	0.023	0.025	....	....	....
11.5	0.019	0.019	0.020	0.020	0.020	0.022	....	....	....	....
12.0	0.019	0.019	0.019	0.019	0.019	....	....	....	....	....
12.5	0.018	0.018	0.017	....	....	....	....	....	....	....
13.0	0.017	....	....	....	....	....	....	....	....	....

* All values of f are computed from a head that is in feet of mixture rather than in feet of water.

are determined from a head that is expressed in feet of mixture rather than in feet of water.) Inspection of Table 1(*a*) shows decrease in f-values for increases in velocity as well as increases in f-values for practically all increases in solid concentration. Observations through the transparent section of the pipe indicated that these variations in f could be expected, because of changes in transportation characteristics that were observed.

TABLE 2.—VARIATIONS IN FRICTION FACTOR, f,* FOR CHANGES IN SOLIDS CONCENTRATION IN PIPES OF VARIOUS SIZES

Velocity, V, in feet per second	ONE-INCH PIPE			THREE-INCH PIPE			FOUR-INCH PIPE		
	Values of f for the Following Percentages of Solid Matter:								
	10	20	30	10	20	30	10	20	30
4.0	0.060	0.084	0.098	0.070	0.081	0.098	0.076	0.100	0.123
5.0	0.046	0.054	0.066	0.048	0.059	0.069	0.055	0.071	0.085
6.0	0.038	0.041	0.050	0.039	0.047	0.053	0.044	0.055	0.064
7.0	0.036	0.037	0.041	0.034	0.039	0.043	0.037	0.045	....
8.0	0.034	0.034	0.038	0.030	0.034	....	0.033	0.038	....
9.0	....	....	....	0.028	0.030	....	0.029	0.034	....

* All values of f are computed from a head that is in feet of mixture rather than in feet of water.

Investigation of the variation of f-values when gravel was transported shows characteristics similar to those obtained for sand. These values are shown in Table 1(*b*). The changes in values of f can be seen to follow the same trends that were found for sand.

Investigation of the 1-in. pipe by Miss Blatch and the 3-in. and 4-in. pipe of Hazen and Hardy shows tendencies similar to those obtained from the 4-in. pipe at the U. S. Waterways Experiment Station. Computations of f-values, made for the 1-in., 3-in., and 4-in. pipes are presented in Table 2. The weight of wet sand in these tests was assumed equal to 129.5 lb per cu ft.

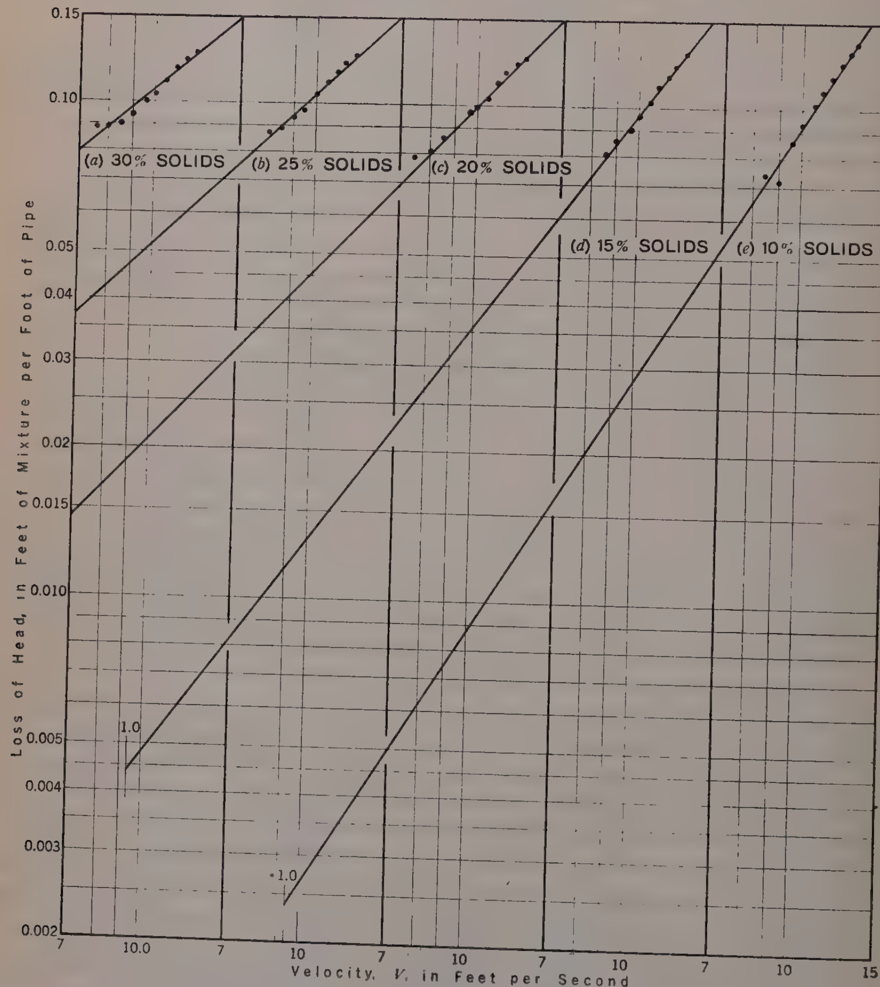

FIG. 7.—LOGARITHMIC PLOT OF HEAD LOSSES FOR VARIOUS SAND CONCENTRATIONS

The only data available for f-values in clear water referred to the 1-in. pipe, and these values were found to range from 0.037 at 3.0 ft per sec to 0.035 at 11.0 ft per sec. Values of f for the 4-in. pipe at the U. S. Waterways Experiment Station varied from 0.0206 at 5.4 ft per sec to 0.0164 at 11.2 ft per sec.

Results from the 4-in. pipe for the Hazen and Hardy investigations cannot be compared with data obtained at the U. S. Waterways Experiment Station,

because of the differences in pipe and sand. The test pipe described herein has two 2-ft pyralin sections and one 2-ft sheet metal section in the test line. No sand analysis curves were presented in the Hazen and Hardy paper.

These results show the differences that can be expected in head losses due to the type of material that is being transported. It can be seen from a review of these data that a very fine sand should be transported with much less head loss than a large sand or small gravel. Changes in the gradation of the materials are the primary causes of variations, and it is due to these changes that it is not possible to obtain a formula for head loss that is applicable to any but specific gradations of material.

Exponential Formulas.—Attempts were made, without success, to determine a satisfactory approach to an exponential formula for pipes carrying sand. Due to the fact that head losses were believed to vary with the three types of sand movement, the attempts at developing exponential formulas were confined to the velocity range in which all particles were in motion.

Logarithmic plots of varying solid concentrations were made and the results of these plots are shown in Fig. 7. From these curves it was found that the slope of the lines varied with changes in solid concentration, the formulas of the type,

$$h_f = m\,V^x \qquad (3)$$

being, for 30% solids:

$$h_f = 0.0160\,V^{0.830} \qquad (4a)$$

for 25% solids:

$$h_f = 0.0118\,V^{0.950} \qquad (4b)$$

for 20% solids:

$$h_f = 0.0089\,V^{1.050} \qquad (4c)$$

for 15% solids:

$$h_f = 0.0043\,V^{1.360} \qquad (4d)$$

and, for 10% solids:

$$h_f = 0.0024\,V^{1.590} \qquad (4e)$$

The next step was to plot values of the intercept of these curves against the slope of the line as shown on Fig. 8. The equations for these curves (of the type of Equation (3)) being:

For Curve (1),

$$m = 0.0044\,P_s^{1.740} \qquad (5a)$$

and for Curve (2),

$$x = \frac{6.4}{P_s^{0.595}} \qquad (5b)$$

(In Equations (5*a*) and (5*b*): m = the intercept; P_s = the solid concentration; and x = the exponent for the general equation.) Although this action provided a satisfactory set of curves which were applicable to all solid concentrations, it was realized that this formula would yield data that were satisfactory for only one size of pipe transporting only one particular class of sand. A family of curves might yield data applicable to general cases, but the foregoing data considered only one of the types of movement. Since data were lacking for all

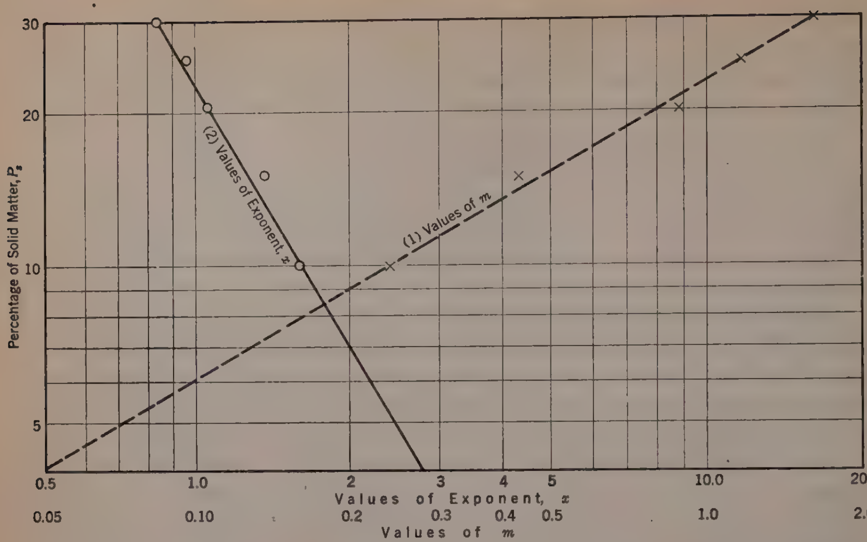

FIG. 8.—LOGARITHMIC PLOT OF EXPONENT AND m-VALUES FOR VARIOUS SAND CONCENTRATIONS

except one set of curves, and it was believed that this method would result in more complications relative to transition velocities for the types of movement, efforts to obtain an exponential formula were abandoned.

THE ECONOMICAL VELOCITY FOR SAND TRANSPORTATION

The velocity at which any given volume of sand per hour can be transported through a given length of pipe with the least expenditure of power per unit volume of sand transported, has been called by Miss Blatch[7] the "economical velocity." Curves taken from diagrams in her discussion can be found in several references on sand transportation, and it is the information contained in these curves rather than the definition itself that will be treated herein.

In order to determine an economical velocity for transporting material in any given size of pipe, the following factors should be considered: (1) The length of line to be used; (2) the energy required to produce the desired velocity; (3) the type of material to be transported; and (4) the solid concentration that will cause a minimum loss of head at the velocity that is desired. The resultant economical velocity, therefore, is a delicate balance among these factors.

Of the aforementioned considerations, it is believed that the type of material to be transported, Factor (3) is most pertinent. A comparison of Figs. 6(a) and 6(b) shows clearly the great effect on head loss that will result from variations in gradation of the material. These differences in grain sizes have a decided effect on all transportation characteristics and the results obtained from one class of material cannot be expected to hold for another. The fact

[7] *Transactions*, Am. Soc. C. E., Vol. LVII (1906), p. 406.

that the data from the U. S. Waterways Experiment Station fail to check those of Hazen and Hardy is attributed directly to this cause. Therefore, it is suggested that exceeding care be taken in the use of curves giving "economical velocities" or "velocities at which all material is in motion," as these curves fail to consider the changes in the physical composition of the material to be carried. Too great emphasis cannot be placed on this point, because a blind acceptance of values from curves of unknown origin will lead to unusual and unexpected results.

Although no method for determining the "economical velocity" or "the velocity at which all material is in motion" is presented in this paper, it is desired to emphasize the fact that for each type of material there will be variations in characteristics that will definitely affect the economical velocity. It is recommended that for all exact design purposes a test line be used to determine the characteristics of the material, rather than a curve or formula, because any data that can be obtained relative to the characteristics of a particular material in a pipe will be of far more value than all theoretical approaches to the solution.

Transferability of Experimental Results

Results from the investigations on 4-in. pipe cannot be applied, by means of any model law, to larger pipe used in dredging. General variations of type of movement, head losses, and method of transportation, discussed previously, occur in a similar manner in larger pipe. Because of the constant variations in the material pumped by dredges, the results from 4-in. pipe can be considered only qualitatively, when applied to the larger pipe lines. Velocity determinations in regard to the ranges of the different classes of movement are of primary importance in attempting to apply results from smaller pipe to the larger sizes. Observations through the transparent section of the test line showed that a velocity range of from about 5.0 ft per sec to 8.5 ft per sec in the 4-in. pipe would cover all types of movement that occur in the discharge lines of dredges. It can also be expected that, depending on the length of the discharge line of the dredge, the material in the line will move in accordance with one of the three types of movement previously described. For long discharge lines, where the velocity drops to about 14 or 15 ft per sec in a 32-in. line, or to 9 or 10 ft per sec in a 20-in. line, the material will settle on the bottom of the pipe, provided sand is being pumped. As the length of the line is decreased and the velocity is correspondingly increased, the types of movement change until the material is being literally blown through the line, passing through exactly the same stages of transportation as those found in the 4-in. pipe.

The determination of an economical velocity for the discharge lines of dredges cannot be made directly from characteristics of the transported material, however, because it is usually true that power developed by the pumping machinery and the fuel consumption have more bearing on this velocity than head losses in the line. This fact makes the economical velocity for each dredge an individual problem which cannot be solved until a sufficient

period of operation has elapsed to furnish data relative to output and fuel consumption for various pump speeds. Although it can be found that the minimum head loss for the discharge line would occur at a certain velocity, it must also be considered that a greater quantity of material might be moved, at a higher velocity and with a slightly greater fuel consumption, but with no real increase in operating overhead. It can be seen, therefore, that, besides the theoretical conditions for the economical velocity in the pipe line, the consideration of operating costs and depreciation must necessarily play a greater part in determining velocities for use in large pipe lines.

Conclusions

In attempting to generalize the findings of this investigation, the conclusions are confined to a range in which as many statements as possible can be confirmed by data from previous investigations. This is done, whenever possible, to prevent the forming of conclusions from only one set of data.

The investigations described herein, with information available from other studies conducted on somewhat similar lines, justify the following conclusions:

(1) Sand is transported in pipes by rolling along the bottom at low velocities, "jerking" along the bottom at medium velocities, and by having all particles in motion at velocities greater than the range in which "jerking" occurs;

(2) The largest quantity of material is transported in the lower third of a pipe rather than along the bottom;

(3) For pipes carrying sand, values of f in Equation (1) will decrease with an increase in velocity;

(4) Values of f will increase with an increase in solid concentration, for any velocity;

(5) A general formula for use in determining head loss in pipes carrying sand is definitely dangerous for use by any person who has not had considerable experience with the use of the formula;

(6) The economical velocity for transporting solids depends upon the character of the material to be carried, and each class of material will probably have a different economical velocity for the same size of pipe;

(7) A pipe line, transporting material of a large grain size at low velocities, will become blocked much more frequently than pipes carrying material of a small grain size, because the larger particles tend to become locked, obstructing other particles, and do not form a smooth bed over which the material can move; and,

(8) The transfer of results from a small pipe line to a line of greater diameter must be qualitative and not governed by any law of corresponding velocities.

Acknowledgments

The investigations described herein were conducted at the U. S. Waterways Experiment Station, at Vicksburg, Miss., in connection with an investigation dealing with methods of increasing the capacity of pipe lines transporting

solids. They were made under the direction of F. H. Falkner, Lieutenant, Corps of Engineers, U. S. Army, Jun. Am. Soc. C. E. The writer wishes to thank the present Director of the Station, Paul W. Thompson, Lieutenant, Corps of Engineers, U. S. Army, Jun. Am. Soc. C. E., for his co-operation in making this paper possible and for suggestions in connection with its preparation. He also wishes to thank Joseph B. Tiffany, Jr., Jun. Am. Soc. C. E., and V. G. Kaufman, for reviewing the contents prior to final assembly.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

PRINCIPLES APPLYING TO HIGHWAY ROAD-BEDS

By Ira B. Mullis,[1] Esq.

Synopsis

The purpose of this paper is to present, for criticism and discussion, a process of reasoning and development in road-bed design and construction rather than to set forth definite rules and data; to apply the laws of mechanics of materials to the solution of road-bed problems; and to stimulate research and thought in the rational design of road-beds. The mechanical properties of typical earthy formations are discussed with reference to their suitability for road-bed use. Construction methods and experience are presented and discussed.

Essential Structural Requirements

With respect to the design, construction, and use of any structure, certain physical properties are required. These properties and their inter-relationships constitute that branch of science known as Mechanics of Materials. Other branches of science are frequently required to aid in the explanation or interpretation of certain physical facts; but with respect to structures of all kinds, the mechanics of materials furnishes a direct means of approach.

The first essential to the design of all structures, including road-beds, is the determination of the specific structural requirements, such as type and magnitude of load to be supported by the several members of the structure. A satisfactory road-bed must possess the following requirements:

(*a*) Compressive strength in excess of that of the maximum wheel load to be used;

(*b*) Effective pore space so limited in volume that it cannot receive sufficient water to produce plasticity;

(*c*) Water content so restricted in quantity as to cause the road-bed to remain an elastic solid; and

(*d*) Surface protection against the destructive forces of traffic, weather, and erosion.

Note.—Written comments are invited for imediate publication; to ensure publication the last disussion should be submitted by **November 15, 1938.**

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To satisfy the foregoing requirements, both designer and builder must possess adequate knowledge of the structural requirements of road-beds and must be familiar with the mechanical properties of the available materials to be utilized.

Gases

Every one knows that matter exists in three states: Gases, liquids, and solids. Gases belong to that form of matter which is elastic and tends to expand indefinitely. The gas that affects road-beds most is air. Its chief characteristic, as it relates to road-beds, is that of expansibility. If a pint of liquid is poured into a quart bottle, it does not expand and fill the bottle. In contrast, an inflated pneumatic tire is punctured, and the escaping air expands and occupies considerably more room than it had before.

During the period of road-bed construction the engineer is concerned with the elimination of air from earthy materials. This removal is essential before earth particles can unite. After a road-bed has been completed the presence of a limited quantity of air in its pores restricts the quantity of water that these pores may receive. In case of freezing, the air is either compressed or removed, and room is thereby furnished for the expansion of the freezing water within the pores without disrupting the solid mass. Thus, it will be seen that a limited air content in the fine pores of a road-bed is an element of safety. For this reason the better grades of sand-clay gravel roads do not fail under frost action.

Liquids

A liquid is a body of matter, such as water, which offers resistance to any change in size, but none to change of form. The only liquid to be considered herein is water which, when pure, has a specific gravity or density of 1.00 gram per cu cm at 4° C. Within the scope of this paper this value is satisfactory for all atmospheric temperatures above the freezing point. Pure ice, at the freezing point, has a density of 0.9175. With the density of water considered as unity, therefore, it will be seen that 1 unit-volume of pure water, upon freezing, becomes 1.08+ units, an expansion of 8 per cent. However, under field conditions, this expansion (including entrapped air) is generally considered to be about 10 per cent. When freezing water is rigidly confined, its force is enormously destructive. The only known method of avoiding this destruction to road-beds is that of restricting the quantity of water that may enter them.

The forces exerted by water are both gravitational and molecular. The gravitational force is too well known to require more than mention. The molecular forces, because of their finite scope and the conditions under which they act, frequently evade interpretation. They include adhesion, cohesion, and chemical affinity. Discussion of how these forces act is beyond the scope of this paper, but knowledge of the conditions under which they apply is essential.

As a constituent of road-beds, water is essential at all times. Its importance as a means of removing air during the period of construction has already been stressed; but the removal of air is not enough. Particles must be cemented together, if all their potential strength is to be utilized. On the other hand,

the excessive use of water is to be avoided; otherwise, a weak and excessively porous or fissured structure is obtained. "Spare the water and increase the weight of the roller" is generally a safe maxim when compacting earth, but the presence of excessive volumes of air in the compacted mass should not be tolerated.

Solids

A solid body is one which offers resistance to any force that tends to change either its form or its size. The ultimate particles of a solid are considered as being bound together by some kind of intermolecular force which causes them to resist forces tending to separate them.

The theory of solids includes the concept of continuity of matter, but, in a strict sense, this is never true. Even atoms contain much open space. Although steel is an excellent example of an elastic solid, it possesses a crystalline structure. When crystal size in metals or non-metals is reduced and the crystals are more closely arranged, strength is increased. Moreover, it has been shown that certain metal surfaces can be ground and polished to planes so perfectly fitted as to bring them within range of molecular forces sufficient to offer considerable resistance to their separation.

Bodies of earth within the definition of that of a solid are to be found occurring, both naturally and artificially, everywhere. It is this state of aggregation of earth particles that is most desirable for road-beds.

To solidify masses of earth, the constituent particles and molecules must be united. This process involves the removal of air, the development of cementing properties, and close molecular arrangement of all constituent particles. In this process, water is essential both as an aid in the removal of air and also in the preparation of the clay molecules for the uniting process. Another essential is adequate pressure. With the aid of water and pressure, the molecules are brought into that state of aggregation in which intermolecular force-functioning becomes possible. The mass may now be considered as having been "welded" into a solid body.[2] Upon completion of the welding process it may be considered as a malleable solid possessing considerable compressive strength.

Of course, small masses of clay particles may also be united into strong bodies by the simple process of thorough wetting and mixing, followed by controlled drying. This method may be termed "aqueous fusion" and is somewhat analogous to the fusion of metal scraps into a solid body. Under certain conditions, such as that of constructing a sand-clay mat, the aqueous fusion process may be used advantageously in highway work; but its use should be restricted to granular or thin masses where adequate drying is possible during the process of construction. This method should not be attempted in heavy clays where the development of shrinkage fissures will certainly cause trouble, unless they are solidly filled during the period of construction.

With respect to the matter in which solids resist forces, hardness, brittleness, elasticity, and plasticity may be mentioned as being important mechanical properties applying to road-beds.

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[2] "A Treatise on Metamorphism," by C. R. Van Hise, *Monograph 47*, U. S. Geological Survey, 1904.

Hardness, in the broadest sense, is that property of any material that causes it to resist deformation or abrasion by external forces. This characteristic depends, first, upon the nature of the chemical substances it contains; and, second, upon the state of molecular aggregation. The quantity of water present in rocks and minerals is an important factor of hardness. Hydrous minerals are invariably softer than those of a similar character except that they contain less water; for example, quartz is harder than opal. The only chemical difference between quartz and opal is the presence of water in the opal in varying percentages ranging from about 2 to 13.[3] The softness of wet brick, wet stone, and other non-ferrous materials in contrast to the same materials in the dry state are too well known to require discussion. It may be mentioned, however, that clays are hardest when in the densest and dryest condition; and, of course, they soften and also weaken in proportion to increases of porosity and water content.

Brittleness is characteristic of materials having little or no plasticity. Such materials have a very low elastic limit and this is poorly defined. Failure of this class of materials under stress, therefore, is sudden and generally without warning. Dry clays are characteristically brittle and such materials are not well adapted to road use. Moreover, dry particles are easily sheared, form dust, and, therefore, produce traffic hazards.

Elasticity is that property of solids which causes them to recover or tend to recover change of form or size upon removal of the deforming force. Within the elastic limit, stress and strain are proportional; but when the pressure exceeds that of the elastic limit of a given material, plastic flow has begun.

One of the essentials of any traveled way, whether it be path or road, is that of elasticity. The road must be able to resist wheel loads adequately and also to recover after their passage. The elastic limit of the road-bed must be adequate for all loads under expected conditions of weather.

Inasmuch as bone-dry clays are characteristically brittle and wet ones plastic, obviously there must be a state of moisture at which a given clay may be caused to attain its maximum elasticity. The methods for developing elasticity will be discussed later.

Plasticity may be described as follows: A body is said to be plastic when it has the ability to change its shape, even to a marked extent, without rupture. A perfectly plastic material is one on which an applied load, however small, produces a permanent deformation. Plasticity in the wide sense of the word, according to the New Century Dictionary, means the possession of a structure weak enough to yield to an influence, but strong enough not to yield all at once. The word is defined by Alfred B. Searle[4] as "that property of a material by means of which it may be deformed or changed in shape and yet retain that shape when the deforming force is removed." Heinrich Ries[5] defines plasticity as "the property which many bodies possess of changing form under pressure, without rupturing, which form they retain when the pressure ceases, it being

[3] "Textbook on Mineralogy," by E. S. Dana, Fourth Edition, 1932.

[4] "The Chemistry and Physics of Clay and Other Ceramic Materials," by Alfred B. Searle, Second Edition, p. 274.

[5] "Clays, Their Occurrence, Properties and Uses," by Heinrich Ries, Second Edition, p. 119.

understood that the amount of pressure required and the degree of deformation possible will vary with the material." The late Mansfield Merriman,[6] M. Am. Soc. C. E., defined a material as plastic when it has no elasticity, so that the smallest forces cause permanent deformation.

Work done by Bennett H. Levenson and verified by Charles Terzaghi,[7] M. Am. Soc. C. E., showed that pressures as small as 5.6 lb per sq in. were sufficient to produce plastic deformation in all wet bodies of earth tested (including both fragile and tough clays). His experiments also showed that plastic deformation occurred at equal moisture contents in duplicate bodies of earth regardless of the magnitude of the loads applied. The loads applied were 5.6, 12.0, 16.0, and 32.0 lb per sq in., respectively. In all cases, the bodies were free to deform in one or more directions.

Yield Point Due to Water Content.—The moisture content at which earth and other wet ceramic bodies change from the elastic to the plastic state is known as the yield point due to water content. By this definition it will be seen that earth containing water, equivalent to that of its yield point, will behave as a plastic body. All unconfined plastic bodies not only deform when loaded, but continue to do so without any increase in the applied load. This yield point of plastic clay bodies under load may be said to be analogous to the yield point of a loaded mass of red-hot steel which is also plastic. Both metallurgists[8] and structural engineers[9] recognize the effect of applied heat in the reduction of the strength of metals. Gustav Tammann states, "With increasing temperature the tensile strength of metals decreases at first slowly, then rapidly so that at a red heat most metals have only a very small tensile strength." The law of "plastic yield" governs in both cases. Moreover, it is evident to the physicist that in the processes of drying wet clay and of cooling hot steel, both first lose their plastic properties and then pass into states of elasticity. It must also be remembered that beyond certain limits of dryness of the clay or of the coldness of the steel the elastic properties disappear and the bodies are then said to be brittle. The effect of water content on the several states of coherent earth is illustrated in Fig. 1. By substituting the words, "Temperature increasing," instead of the words, "Water content increasing," on the X-axis of the diagram, the effect of temperature on the strength of metals may be illustrated.

By definition of Ries,[5] the yield point of plastic clay is equivalent to Albert Atterberg's "lower plastic limit," [10] which is restricted by the point of rupture or rolling-out limit. However, Atterberg's method of determination is made in the reverse order, that is, by a process of moisture decrease instead of that of moisture increase. A comparison of the essential rupture feature in the Atterberg test with other definitions of plasticity reveals the harmony of the definitions of all investigators of plasticity. Furthermore, the yield point due to water content of fine sands and silts was found by Levenson[7] to approximate

[6] "Mechanics of Materials," by Mansfield Merriman, Tenth Edition, p. 44.

[7] "Simplified Soil Tests," by Charles Terzaghi, *Public Roads*, Vol. 7, No. 8, pp. 153–162.

[8] "A Textbook of Metallography," by Gustav Tammann, Third Edition, 1925, p. 129.

[9] "Textbook of the Materials of Engineering," by H. F. Moore, Fifth Edition, pp. 36–45.

[10] International Rept. on Pedalogy (1911); also, "A Study of the Atterberg Plasticity Method," by Charles S. Kinnison, National Bureau of Standards, *Technical Paper No. 46*.

75% of Atterberg's "lower liquid limit" value. Moreover, in many types of plastic earth the relationship between these limits is such that 75% of the

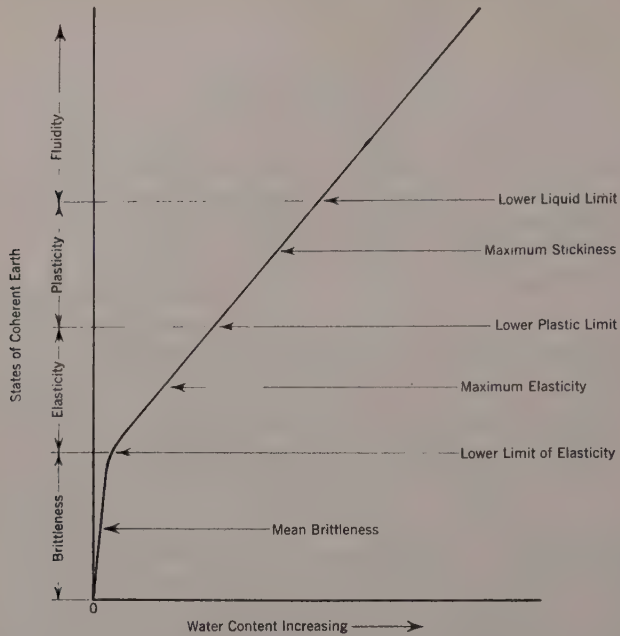

FIG. 1.—ILLUSTRATION OF EFFECT OF WATER CONTENT ON THE SEVERAL MECHANICAL PROPERTIES OF EARTHS

lower liquid limit value frequently approximates the lower plastic limit value, but important exceptions to this relationship are to be found.

GRANULAR MASSES

Material in a granular state is exemplified by sand and gravel deposits, which are characterized by their limited compressibility and the ease with which they receive and lose water. Owing to the surface tension of water, moist sand under traffic is more stable than dry sand.

The ease with which pores or voids in masses of course sand and gravel receive the discharge water is too well known to require discussion. In contrast to this, the pores in masses of very fine sand and silt are noted for retaining much of the water so readily received. Assuming a usual bulk density of sand and gravel to be 1.70 and that of silt to be 1.36 and both having an absolute density of 2.65, the former will have a pore space of 35.8% and the latter, 48.7 per cent. To fill the voids in the mass of sand and gravel with water would require a quantity equivalent to 21.0% of their dry weight; however, probably less than 5% of it would be retained. To fill the voids in the mass of silt would require a quantity of water equivalent to 35.8% of its dry weight and most of this water would be retained against gravitational drainage. The water retained in the mass of sand or gravel would be an asset in aiding traffic

over it, but the water retained in the mass of silt would convert it into a quagmire.

Loose, Amorphous, Plate-Like, Fibrous, and Cellular Matter

Loose material, regardless of other attendant structural properties, should have no place in a road-bed. Pore space is always large in loose materials and when the individual openings are so small that the force of surface tension is greater than that of gravity, water is removed only through evaporation.

Dry Clay.—The brittle properties of consolidated masses of dry clay have already been discussed under the heading, "Solids." Loose masses of dry clay are composed of particles or chunks containing much air. In this state, the mass can be neither compacted nor welded into a solid. Dry masses of earth utilized in the formation of road-beds should be expected to contain many air-filled cavities which are likely to contribute to future saturation and settlement. Dry earth should be utilized in road-bed construction only for blending or conditioning wet earth so as to obtain little variation of moisture throughout the structure.

Although loose deposits of moist clay or well-graded mixtures of sand and clay have possibilities for being converted into road-beds of considerable strength and durability, even these should have no place in a road-bed until they have been well compacted. Masses of earth particles which do not conform to the definition of a solid as given by the physicist cannot function as a part of a satisfactory road-bed.

Wet Clay.—Whether it is in a plastic or fluid condition, wet clay is generally in a state of expansion; it has very little supporting capacity and, therefore, should not be utilized. When it must be placed in road-beds, it should be used sparingly and distributed so that excess water will become absorbed by contiguous dryer layers of earth during the period of road-bed construction.

Road-beds constructed of wet earth shrink upon drying and, therefore, lead to the development of shrinkage fissures. These fissures become water-filled during rainy seasons and thereby contribute in large measure to the never ending cycle of excessive moisture changes and their attendant instability.

Plate-Like Particles.—Plate-like fragments or particles belong to one of three general classes: Brittle, elastic, or flexible matter. Of these, brittle, plate-like matter, such as spalls of rock, may best be utilized only when sandwiched between layers of moist, compact earth so that bond is provided for these incoherent fragments.

Cellular Matter.—Another type of brittle matter is that of diatomaceous or radiolarian earth. Both these types consist of shells of siliceous matter mixed with greater or less quantities of earth. These shells or frustules have various shapes, but all are more or less cylindrical, cup-like, cellular, or rod-like in form and have many openings which give the particles a kind of lace-like appearance. The pore space within, and also between, these minute shells forming the mass is enormous, and all pores are so small as to resist gravitational drainage when filled with water. Due to the fragility of the particles and also to the enormity of the pore space, neither diatomaceous nor radiolarian earth is satisfactory for use in road-beds.

Volcanic ash deposit contains enormous pore space also. Its utilization in the formation of road-beds, therefore, should be restricted to the minimum. In no case should it be placed in the upper part of the road-bed nor on road-bed slopes where it may be subjected to erosion.

Micaceous Earth.—The only minerals that possess marked elasticity so that they spring back when deformed, are the micas. These mica flakes when associated with common earth tend to produce a fluffy mass. Even the presence of small quantities of mica flakes increases mass porosity. Although the individual flakes possess considerable elasticity, the mass cannot be considered an elastic solid. Due to lack of stiffness and other structural weaknesses, bodies of micaceous earth may be utilized only when better material is not available. However, if this material is utilized, the upper 2 ft or more of the road-bed must be formed of select earth if maintenance troubles are to be averted.

Peat.—Spongy cellulose matter such as peat, humus, leaves, and straw has an absolute density of about 1.5. The absolute density values obtained for peat deposits from five localities along the Atlantic Coast and one from the State of Washington as determined by Messrs. I. C. Feustel and H. G. Byers[11] ranged from 1.105 to 2.161, the mean value being 1.533. Abnormally high density values are obviously indicative of mineral contamination to a considerable extent. In further substantiation of the absolute density of cellulose matter being about 1.5, the research personnel of the United States Forest Products Laboratory states that the absolute density of the actual wood substance of all structural timbers is about 1.54.[12]

The bulk density values of dry peat as given by Messrs. E. K. Soper and C. C. Osbon[13] are as follows: Turfy peat, 0.11 to 0.26; fibrous peat, 0.24 to 0.67; earth peat, 0.41 to 0.90; and, pitchy peat, 0.62 to 1.03. The bulk density values of dry peat from localities already mentioned, as determined by Messrs. Feustel and Byers,[11] ranged from 0.06 to 1.21, the mean value being 0.53. The effect of the presence of considerable quantities of mineral matter is again apparent in its influence upon density increase. A tabulation of thirty-three commercial woods tested in the U. S. Forest Products Laboratory shows bulk density values, when corrected to a 0% moisture content, ranging from 0.29 to 0.65, with a mean value of 0.44. It becomes apparent, therefore, that the only essential difference between the structural properties of timber and those of peat consists of difference in porosity and bond between fibers in the respective materials.

Due to its enormous pore space and also to its marked affinity for holding water,[11] [12] deposits of peat are to be avoided as road locations when possible.

Measurements of the Properties of Road-Bed Materials

The design of road-beds, like that of other structures, must be based necessarily on the mechanical properties of the materials to be utilized. The usual

[11] "Physical and Chemical Characteristics of Certain American Peat Profiles," by I. C. Feustel and H. G. Byers, *Technical Bulletin No. 214*, U. S. Dept. of Agriculture.

[12] "Wood Handbook," Forest Products Laboratory, U. S. Dept. of Agriculture.

[13] "The Occurrence and Uses of Peat in the United States," by E. K. Soper and C. C. Osbon, *Bulletin No. 728*, U. S. Geological Survey.

location survey must be supplemented by a materials and profile survey which includes soundings, collection of cores and samples of all strata, and, finally, the measurements of the mechanical properties of the specimens and samples collected. For obvious reasons no designing should be started until all design data, including materials tests, are complete.

Density

The term, density, is used in two senses: (1) Absolute density, for expressing the weight per unit volume of constituent particles of a solid or mass, exclusive of voids; and (2) bulk density for expressing the weight per unit volume of mass, including voids. In both cases, the numerical value of density is expressed in grams per cubic centimeter.

Bulk density is frequently expressed in the English system as weight in pounds per cubic foot, but the metric system is to be preferred for these measurements since absolute density is always expressed in that system. Furthermore, when bulk density is expressed in the English system, the two terms, absolute density and bulk density, must be expressed in the same system before pore space can be computed. Moreover, the metric system is better adapted to scientific work since it does not involve the awkward volumes and weights of the English system.

Absolute Density.—The term, "absolute density," is frequently called absolute or true specific gravity and in geology, "mineral specific gravity." In all cases it signifies the ratio of weight of a given volume of dry matter, unaffected by porosity, to the weight of an equal volume of distilled water. Absolute density may be determined by any one of several well-known methods, all of which require precise weight and absolute volume measurement of the matter being investigated.

Bulk Density.—The term, "bulk density," signifies the unit weight, in grams per cubic centimeter, of dry matter being investigated, including its contained pore or void space. It is synonymous to the term, "bulk specific gravity," and also to the term, "rock specific gravity," frequently used by geologists. Bulk density measurements may be performed by any one of several well-known methods, but the use of kerosene in the overflow volumeter (shown in Fig. 2) is recommended for volumetric measurements in the laboratory. This volumeter was designed by Mr. L. L. Marsh of the Kansas State Highway Commission. It is of the siphon type, dripless in performance, and capable of giving precise results on specimens of considerable size. The balance

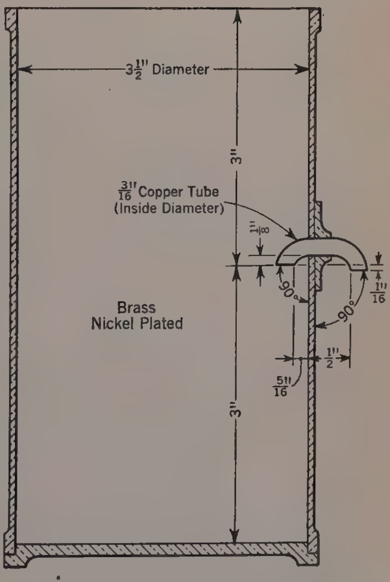

Fig. 2.—Overflow Volumeter

should be accurate to 0.1 gram for laboratory work and to 0.5 gram for field work.

Volumetric measurements of earth in cut and also in fill are readily made by the use of a 4-in. post-hole auger for obtaining the sample, and its volume may be obtained by filling the hole with common S.A.E. 30 oil or dry, screened sand, of known density, from the spout of a pot containing a known weight of the material utilized. A table for converting the weight of these materials into metric volumes is a recommended convenience, based on the formula,

$$\text{Bulk density} = \frac{\text{Bulk volume of mass, in cubic centimeters}}{\text{Dry weight, in grams}} \qquad \ldots\ldots(1)$$

Subdivision of Bulk Density.—Inasmuch as bulk density measurements are utilized in three distinct states or conditions of compactness, it becomes necessary to restrict its meaning to suit its precise application to a given problem. The term, bulk density, therefore, has been subdivided into three brief expressions: "Cut density," "fill density," and "standard density."

The term, "cut density," will be applied to all bulk density measurements made on specimens of undisturbed rock or earth taken from "cut," "borrow," or other sources from which "fill" material is to be obtained.

Fill density will be the term applied to all bulk density measurements made on specimens taken from "fill," after the fill material has been placed in its final position or in a given state of compactness. Further restrictions of this term can be made as desired by adding a limiting adjective, such as "rolled," "tamped," etc.

Standard density of earth utilized in road work may be defined as that state of compactness in which all pores have been reduced to the minimum without crushing any of the constituent particles, and all water has then been removed. Samples for this determination obviously must be representative of the material sampled. Deposits containing rock fragments too large for this determination are given special consideration through other measurements. For obvious reasons, the size of the sample taken must be proportional to the size of the largest rock particles in it.

The sample is prepared for test by vigorous boiling. This serves three purposes: It breaks up the clod structure without crushing the rock fragments; it reduces the labor involved to the minimum; and, it removes all air from the pores and thereby permits the wet mass to dry and shrink to a reproducible numerical value of density. Boiling should be continued until the water content is reduced to the minimum boiling limit. When the boiling process has been discontinued, more water may be removed by pouring the wet mass into a hot, porous, plaster-of-Paris mold similar to those used in the pottery industry. Finally, the sample is dried in an oven. The dried and cooled specimen is then weighed, soaked in kerosene, and its volume measured in the kerosene-filled overflow volumeter. This method of testing has been developed by the Iowa State Highway Commission to the extent that close check results are obtained and many tests can be completed by two operators within 24 hr of their receipt in the laboratory. Moreover, the usual laborious work of

drying and pulverizing the material prior to testing is entirely eliminated by this improved technique.

Application of Density Measurements.—Density measurements are utilized as follows:

(1) The ratio of shrinkage or swell which earth and rock masses undergo when transferred from "cut" to "fill" may be determined in advance by means of density values obtained from representative specimens of materials in place and also from the density value designated for the proposed road-bed;

(2) A means is furnished for computing pore space in bodies of earth in any state of compactness or wetness; and,

(3) Density measurements furnish a simple means of expressing the quality of earth compaction in numerical terms.

Standard density is utilized as follows: (*a*) As a distinctive and important property essential in the classification of earths proposed for structural use; (*b*) as a reproducible density, essential for specimens subjected to the hardness test; and (*c*) as a means for comparing the mechanical properties of thoroughly "welded" clays utilized in road-beds, with those utilized in the manufacture of clay products the mechanical properties of which have been measured. Experience has shown that the elastic limit and the yield point due to water content are to be preferred as standards of compaction instead of any standard based on density alone.

Porosity

All matter is more or less porous. The large pores in deposits of gravel cannot escape detection. The pores in deposits of silt are not so obvious, but water readily finds its way into them. Even iron is known to be porous and, therefore, allows water under enormous pressure to be forced through it.

The invariable weakness of highly porous masses made up of minute cells which collect and retain large quantities of water has already been discussed. Therefore, attention will be directed to the discovery and measurements of pore space for the purpose of controlling it.

The measurement of pore space is sometimes attempted by the water-absorption method. When the pores are large and open, the water-absorption method may be saisfactory; but where pores are more or less closed and tortuous, the entrance of water is restricted. Hence, in the one case, the volume of water absorbed may be equivalent to that of the total pore space; but, in the other, only a small percentage of the total pore space may be filled.

The most precise method of determining pore space is by computation. The essential measurements required for this method are: (*a*) Numerical value of absolute density of the constituent particles, ρ_a; and (*b*) numerical value of bulk density of the mass, ρ_b. Then, from the well-known relationship between the two types of density, the volumetric percentage of space occupied by pores, P, is expressed by the following equation:

$$P = \left(1 - \frac{\rho_b}{\rho_a}\right) 100 \qquad \ldots\ldots(2)$$

Obviously, Equation (2) may be applied either to cut, fill, or standard porosities by the substitution of actual numerical values for ρ_b and ρ_a. The volumetric percentage of solid matter, V_s, in a given road-bed is expressed by the following equation:

$$V_s = \frac{\rho_b}{\rho_a} 100 \quad \text{.......................... (3)}$$

The percentage porosity of a given deposit having been determined by formula, then that part of the space actually occupied by water may be computed from the determination of percentage of moisture by weight made on a

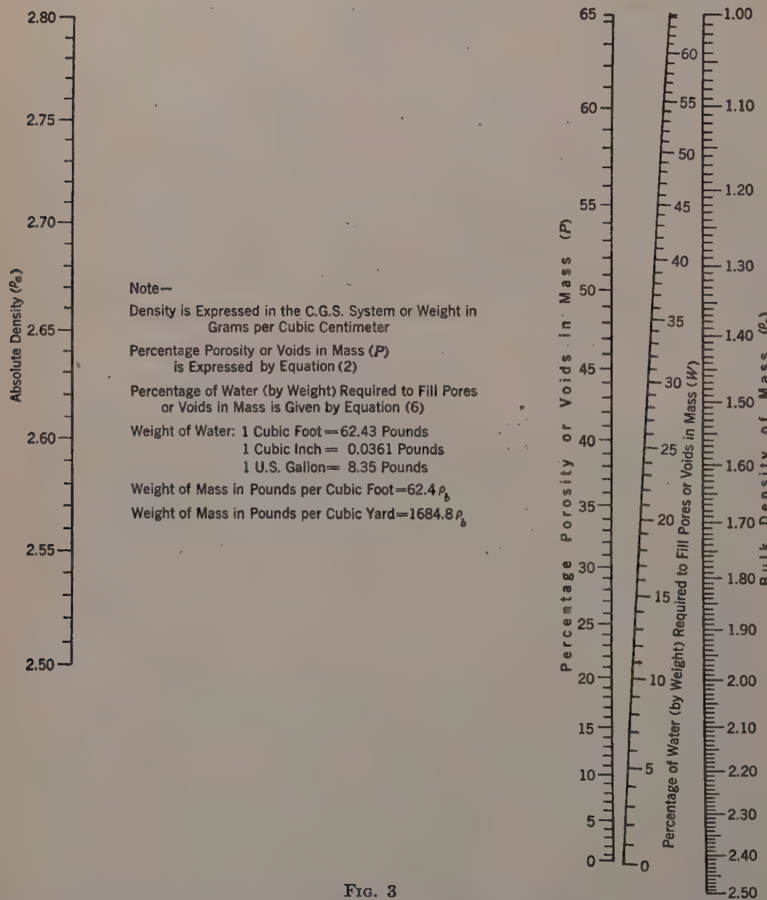

FIG. 3

representative sample of that deposit. Since this value is based on the dry weight of earth, the volumetric percentage is,

$$V_w = P_a \rho_b \quad \text{.......................... (4)}$$

and the percentage of space remaining must be occupied by air and is,

$$V_a = 100 - (V_s + V_w) \quad \text{.......................... (5)}$$

The percentage of weight by water (P_w) required to fill the pores and voids completely in a mass of a given state of bulk density is equal to the percentage of voids in that mass divided by its bulk density; or,

$$P_w = \frac{P}{\rho_b} \qquad (6)$$

To facilitate the computation of pore space in earth masses and also to determine the quantity of water required to fill the pore space in these masses, the nomograph shown in Fig. 3 has been devised. This nomograph, which is based on Equations (2) and (6), can be used to demonstrate the inter-relationship of density, pore space, and water capacity of earth masses. This is a major concept of this paper.

Theoretical water capacity is the quantity of water necessary to fill the actual pore space in a mass of matter of known porosity. It may be expressed on a basis of either total dry weight or total dry volume. In this paper it is expressed as a percentage of dry weight.

TABLE 1.—Characteristics of Certain Formations Encountered in Road Building

Item No.	Material	Density: Absolute	Density: Bulk	Porosity	Percentage of water, by weight, required to fill the pores	Characteristic bearing capacity
	(1)	(2)	(3)	(4)	(5)	(6)
	(a) Material in Its Natural State					
1	Average peat bed	1.50	0.20	86.7	433.5	Low
2			0.60	60.0	100.0	
3	Mica flake bed	2.88	0.40	86.1	215.2	
4	Diatomite bed	2.15	0.45	79.1	175.7	
5			0.96	55.4	57.7	
6	Volcanic ash bed	2.60	0.92	64.6	70.2	
	(b) Material in "Standard" Condition of Compactness					
7	Rock flour, or silt bed	2.65	1.20	54.7	45.6	Medium to low
8			1.50	43.4	28.9	
9	China clay, or kaolin bed	2.62	1.30	50.4	38.8	
10	Common sand bed	2.65	1.60	39.7	24.8	High*
11			1.80	32.1	17.7	
12	Common heavy clay bed	2.65	1.70	35.9	21.1	High
13			2.10	20.8	9.9	
14	Sand-clay road surface	2.65	2.16	18.5	8.5	
15			2.16	18.5	8.5	
16			2.22	16.3	7.3	
17			2.26	14.8	6.5	

* When moist.

Actual water capacity is the total quantity of water contained in a mass of earth that has ceased to absorb available water. Trial determinations for verifying the possibility of inducing further absorption may be made by means of shallow water-filled wells cut in the mass to be tested. Moisture determinations around wells, at short increments of distance from their centers,

should be started after the rate of subsidence of water in the wells has been materially reduced or after a suitable interval of time of percolation from the wells. From these data and from the pore space previously determined, the percentage of pore saturation may be determined both for given distances from free water in the wells and also after given intervals of time of percolation.

Table 1 is shown for the purpose of familiarizing the reader with important inter-relationships of density, porosity, and water capacity of certain formations encountered in road building. (In the case of Items Nos. 14 to 17, Table 1, the determinations were made during the first "spring thaw." The actual mean percentage of water held was 2.4, the maximum was 2.7, and the minimum, 2.2.)

Brittleness, Elasticity, and Plasticity

The effects of stress are deformation and rupture. The mechanical properties of earths, like metals, may be classified, in a general way, in three groups: (*a*) Resistance to deformation and rupture; (*b*) capacity for deformation without rupture; and (*c*) work of deformation or rupture.

In Group (*a*), which may be designated by the general term, "hardness," are included the elastic limit, modulus of elasticity, compressive strength, transverse strength, shearing strength, endurance limit under repeated stresses, Brinell hardness, etc.

Group (*b*) may be covered by the term, "plasticity," and includes the relative compression without rupture in the compression test, the deflection at failure in the transverse test as well as the general properties denoted by the term, "malleability."

The work of deformation and rupture is the product of the resistance to deformation by the amount of deformation, and can be computed from the complete data of any test in which both load and deformation are observed continuously or at close intervals. It is represented by the area under the stress-strain curve.

The method used in the measurement of brittleness, elasticity, and plasticity is that usually followed to determine stress-strain relationships. Stress-strain tests are performed either on specimens in the laboratory or on the road-bed during construction. However, a sufficient number of both laboratory and field tests should be made for correlation after which field tests made by means of the roller should be adequate.

Tests on molded and dried specimens of known density can best be made in the laboratory. It is known that dried specimens possess an ultimate strength, show little elasticity, and fail suddenly under load. It is also known that very little moisture is required to develop elastic properties in earth in a well-compacted state, but exact information is lacking. Crusts of sand-clay roads carrying heavy traffic have been examined by the writer in which the moisture content was less than 4 per cent. The elastic properties of the crusts were not measured, but their condition under the traffic carried would lead one to infer considerable stiffness. Research on the quantity of water needed to change a brittle earth into an elastic one is needed.

Although the exact quantity of water necessary to develop the maximum elastic properties of compact bodies of earth is not known, it is known that an excess quantity not only weakens the elastic properties, but continues to do so in proportion to the increase in water content, until the yield point due to water content is reached. This is the point at which the material changes from the elastic to the plastic state as previously discussed.

Several investigators, including C. C. Williams,[14] M. Am. Soc. C. E., the late John H. Griffith,[15] M. Am. Soc. C. E., and others, have applied the principles of elasticity to the solution of problems in the mechanics of earth solids.

The application of the stress-strain method to the study of the elastic and plastic properties of bodies of a given earth is shown in Fig. 4. Earth utilized in these tests had been uniformly tamped in a cylindrical container of adequate diameter and depth for loading a 1-sq in. circular bearing-block. To prevent undue restraint against plastic flow, the distance between the bearing-block and the confining walls of the cylindrical container was spaced several inches. The loading was applied at a constant rate on all bodies tested.

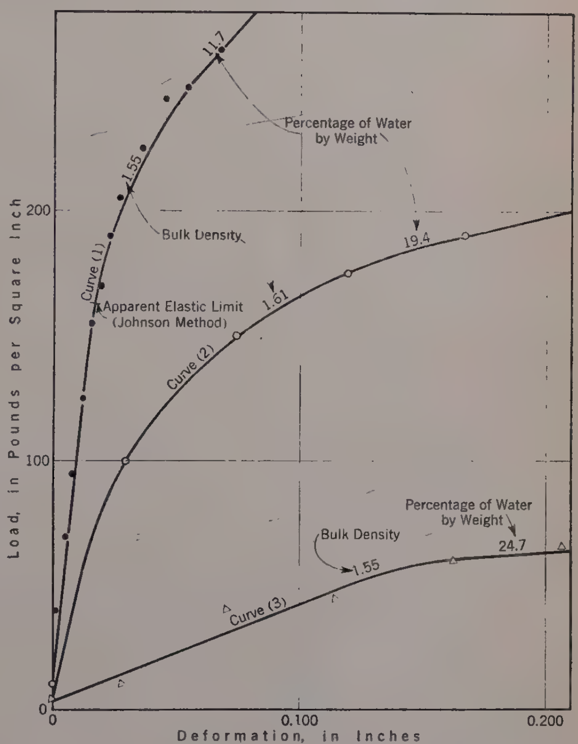

FIG. 4.—STRESS-STRAIN CURVES PLOTTED FROM TESTS ON A MASS OF EARTH COMPACTED TO INDICATED DENSITIES, AND WITH GIVEN MOISTURE CONTENTS

Curve (1) in Fig. 4 was compacted to a density of 1.55, with a water content of 11.7 per cent. Its apparent elastic limit, as computed by the Johnson method,[16] is 164 lb per sq in. It will be noted from the several curves shown in the diagram that the stiffness of the bodies tested decreased with increasing water content. This is to be expected. Although Curve (3) is the densest body of earth tested, its water content causes

[14] "Foundations," by C. C. Williams, in "Civil Engineering Handbook," by L. C. Urquhart, M. Am. Soc. C. E. and others, pp. 632–659.

[15] "Physical Properties of Earths," by John H. Griffith, *Bulletin 101* (Iowa State College Eng. Experiment Station).

[16] "Johnson's Materials of Construction," by M. O. Withey and James Aston, Fifth Edition, 1935, pp. 9–10; also "Standard Specifications for Highway Materials and Methods of Sampling and Testing," adopted by the Am. Assoc. of State Highway Officials, 1935, pp. 291–303.

it to show less stiffness than any, except the one having 24.7% of water. It will be noted that the stiffest and also the most plastic bodies tested had equal densities of 1.55; but the stiffest one contained only 11.7% and the most plastic one contained 24.7 per cent. With a water content of the latter amount, no elasticity whatever is indicated. This is to be expected since the water content in this case is 24.7% and that of the yield point due to water content is 20.1. A comparison of the water content represented by Curve (3) with that at the yield point due to water content, shows a difference of only 0.7 per cent. It will also be seen that this curve indicates little, if any, elastic properties. It does, however, indicate plastic flow under a limited load.

Rapid stress-strain tests may be made by means of a sheepsfoot roller resting on the spot to be tested. Deformation readings may be made on the feet at both ends of the roller at frequent intervals of time by means of a wye-level and rod. From the data thus obtained stress-strain data may be computed. Elastic limits in excess of the weight of the roller may be determined by constant increases in the weight of the roller, applied by means of bags of earth of known weight.

The term, "ultimate strength," is applied to the crushing strength of molded and dried specimens only. The maximum utilizable strength of a moist compact body of earth will be considered as being equivalent to its compressive strength at the elastic limit.

The term, "elastic limit," may be applied appropriately either to laboratory specimens or to full-sized field tests. In either case it is the unit stress below which the material would fully recover its form upon removal of the load. It may also be defined as the limit of proportionality between stress and strain.

For use in this paper, the yield point due to pressure will be regarded as being coincident with that of the elastic limit. In its 1921 report the Special Committee of the Society on the Bearing Value of Soils recognized the analogy of the yield point due to pressure in soils to that of the yield point of steel.[14]

The "yield point due to water content" is defined as the moisture content at which clay or clay-like bodies change from the elastic to the plastic state, so that the smallest forces cause permanent deformation. The moisture content at this point, as previously stated, is equivalent to Atterberg's lower plastic limit in clays and equivalent to 75% of his lower liquid limit in silts and other fine friable materials.

In regard to the inter-relationship of mechanical properties, both science and experience reveal that the magnitude of energy contained in a body is the measure of its capability to resist forces. Although one may not know the exact magnitude of a given form of energy possessed by a certain body, well-known laws aid in estimating the capacity of these bodies to resist certain types and magnitudes of forces. For example, the mechanical energy stored in hard or dense materials is known to be greater than that of soft or less dense ones. As a further example it may be stated that the hardness of steel, as determined by the Brinell method, has a direct relation to the tensile strength, and is equal to the product of a coefficient, C, into the hardness number.[17]

[17] Mechanical Engineers' Handbook, by William Kent, Ninth Edition, Rev., 1916, p. 365.

Moreover, if a given material shows considerable tensile strength, one may reasonably expect it to be able to offer considerable resistance to shear, compression, etc.

Other conditions being equal, dense materials, including clays, clay products, concrete, stone, wood, and metals are stronger and harder than their more porous respective equivalents. With respect to the strength of earths, Williams states:[14] "The supporting capacity of sands, gravels, clays and other earths, within the range of each soil type, varies approximately with the weight per cubic foot." He states furthermore that a committee of the Society "from a study of the influence of colloidal content of earths upon their behavior under pressure, found the strength of loam and clay specimens increased with the percentage of colloids contained."

Gumbo clays, noted for their high liquid limit (water-fusing point), possess the highest tensile strength of any known clay. In contrast, kaolins have low liquid limits and also are among the weakest of clays. Moreover, the strongest metals generally have a high fusing point, a good example of which is tungsten wire, the strongest metal known, which has a fusing point of 6 152° F. In contrast to the strength of tungsten wire, lead is one of the weakest of metals, and it also has one of the lowest fusing points of metals.

The relationship between density and elasticity of earth masses is quite marked. Loose masses (low density) under stress manifest no elastic properties, but solidified masses (high density) show elastic properties as do other solids under stress. Increasing the density of a given body is equivalent to increasing its elasticity.

Considerable experimental data on the inter-relationship of the several manifested forms of energy in given bodies have been published by Griffith.[18] He shows certain relationships between the strength of bricks and rocks and their absorption and hardness. In each of the *Bulletins* mentioned he discusses the structural organization of matter and shows the similarity of laws governing different forms of energy, including elasticity, plasticity, hardness, density, etc. In support of his experiments and hypotheses, he cites the findings of others among whom are included A. E. H. Love[19] and A. Nadai.[20]

Detailed studies based upon experimental evidence of the relationshid between stress and strain in masses of earth of certain density and moisture content should reveal a wealth of information to the highway engineer. In like manner, the effect of admixtures of sand and clay on the development of structural properties of road-beds could also be determined.

Design of Road-Beds

Obviously, the design of road-beds should be based on the essential structural requirements outlined and discussed in detail in this paper. Information obtained from large transportation companies, tire manufacturers, and pro-

[18] "Dynamics of Earth and Other Macroscopic Matter," by John H. Griffith, *Bulletin No. 117*, Iowa Eng. Experiment Station, 1934; also, "Physical Properties of Typical American Rocks," *Bulletin No. 131*, Iowa Eng. Experiment Station, 1937.

[19] "Treatise on the Mathematical Theory of Elasticity," by A. E. H. Love, Second Edition, p. 1.

[20] "The Phenomena of Slip in Plastic Materials," by A. Nadai, *Proceedings*, A.S.T.M. (1931), Pt. 2, pp. 11–26; also "Plasticity," by A. Nadai, McGraw-Hill Book Co., New York, 1931.

ducers of heavy equipment show that the pressures in balloon tires now rarely exceed 75 lb per sq in., with a tendency toward much lower pressures. Inflation pressures of tires do not now exceed 100 lb per sq in. Therefore, it seems safe to assume a maximum wheel pressure on the road to be 100 lb per sq in. Due to the limited use of solid rubber tires, these tires are not to be considered.

Factor of Safety Against Pressure.—To avoid the possibility of over-stressing the road-bed, the application of a factor of safety seems essential. Following the usual custom, it would seem that the minimum factor of safety recommended should be not less than 2. Based upon the maximum tire pressure of 100 lb per sq in. and a factor of safety of 2, the road-bed should be designed for pressures not less than 200 lb per sq in.

Elastic Limit Required.—To meet the load requirement, obviously the material of which the road-bed is to be made must be capable of being formed into a solid having an elastic limit not less than that required for the given load and safety factor against pressure, or not less than 200 lb per sq in.

Stiffness Required.—Obviously, the stiffness of a road-bed should be developed adequately to resist the load for which it is designed; but before any numerical value of stiffness can be set up as a standard, considerable field and laboratory data must be obtained. Moreover, until a satisfactory standard of stiffness can be established the designing engineer should make every effort possible to develop much needed data on this feature of road-bed design.

Development of Hardness, Elasticity, and Stiffness.—The properties of hardness, elasticity, and stiffness may be developed in wet coherent material in either of two ways. A reduction of water content will develop these properties to some extent, provided the pore space is not too voluminous; but since the capacity for holding water is predicated on the available pore space, these properties of strength may best be developed by increasing the density of the road-bed under controlled moisture conditions.

If the road-bed is too wet to be compacted, its density may be increased by the well-known method of adding dry sand, gravel, or other hard rock fragments and pounding the added material into the wet spongy mass. By this process, densities of 2.20 or more may be developed in a layer of desired thickness. Obviously, the greater the thickness and density of the layer thus built, the greater will be its resistance to mechanical forces. Sand, fine gravel, or fine crushed stone is to be preferred since fine pores offer greater resistance to the passage of wet clay than coarse ones. Aggregates graded in sizes from that of silt to that of gravel are essential in the rapid choking of the pores. Moreover, the aggregate must be cemented into a solid mass by means of clay or other adhesives.

Where a thin layer is sufficient, the work may be accomplished by the use of a tamping roller applied to thin layers of aggregate. Where rigid and dense layers of considerable thickness are to be placed across swamps, the use of a heavy rammer of the pile-driver type is essential for constructing the lower part of the densely constructed stratum.

Factor of Safety Against Water Absorption.—This factor of safety must be determined by actual test prior to the work of designing. From actual compaction tests, determine a density which gives an actual water capacity, less than the quantity of water required to produce the yield point due to contained water. Then, from these data,

$$S_W = \frac{s_{yw}}{P_a} \quad \text{.......................... (7)}$$

in which S_W = the factor of safety against water absorption; s_{yw} = the yield point stress due to water content; and P_a = the actual water capacity of pores, expressed in terms of percentage by weight. The true criterion for determining what density is essential for a road-bed is the relationship between actual water capacity and the yield point due to contained water. This relationship seems to be expressed best in the form of the "factor of safety against water absorption." It is this relationship which controls the water softening of the road-bed. That compacted bodies of earth offer a greater resistance to water softening than bodies of loose earth was probably first discovered by prehistoric Man as he walked along a path loosened here and there by some burrowing animal just before a rain; but since the writer is not aware of an existing method for determining the critical point of softening and then guarding against its occurrence, he has been forced to apply the method now termed, "factor of safety against water absorption."

Many bituminous mat-road surfaces which had failed due to the apparent weakness of the road-bed have been examined. Of these failures, no case is recalled in which the bulk density of the road-bed was in excess of 1.50. Based on this value of bulk density and on an absolute density of 2.65, the pore space is found to be 43.4% and the theoretical water capacity, therefore, is 28.9 per cent. A pore space of this magnitude will probably have an actual water capacity almost as great as its theoretical capacity. Since the yield point due to water absorption in many clays is less than 28.9%, the reason for the occurrence of many failures in road-beds having a density less than 1.50 becomes evident.

As a basis for further study and research, the writer offers the following limiting values for use in designing road-beds:

(*a*) Do not allow the pore space in the road-bed at all depths greater than 2 ft below subgrade, to exceed 40% of the volume;

(*b*) Reduce the pore space in the upper 2 ft of the road-bed so that its theoretical water capacity shall be not greater than 70% of the lower plastic limit of the earth used in forming the road-bed.

Materials Balance Factor.—As previously stated, this factor is based upon the ratio of density of mass in a fill to that in a cut. The results to be expected from the use of this factor depend entirely upon the precision of the original weighted density values obtained and also upon the precision with which the compacting specifications are executed.

Road-Bed Protection.—Experience teaches that thoroughly compacted deposits of earth do not change in volume unless acted upon by outside forces.

Bench-marks set below the frost line in wet earth or even on tree roots have been found to remain stable unless they come within range of landslides or destructive earthquakes. Even deposits of clay subjected to enormous pressures through long ages do not bulge when the pressures are removed by the excavation of the overlying deposits. Moreover, there is no such thing as a "natural density" for any given material, regardless of its environment. For example, a given type of clay should be expected to have densities proportional to its compacting environment—that is, they should be low in arable fields, higher in old pastures, and still higher in well-trodden paths, or in barnyards.

The agency most destructive to compacted road-beds is weather. It matters not how well a road-bed may be compacted, if it is left exposed to weather, the exposed part should be expected to become less dense. Therefore, the greater the number and magnitude of weather changes, the greater the change to be expected in the original density.

To prevent the weather from producing marked changes in road-beds, they must be shielded against its influence. Ground-water must be intercepted and surface water must be diverted from the entire road-bed, including the shoulders. A pavement or bituminous surface alone is not sufficient protection against the weather. Pavement cracks generally develop and joints are seldom water-tight. For this reason provision should be made to prevent excessive moisture changes near the subgrade and in the shoulders. Some form of effective stabilization seems to be the only method of preventing road-bed softening and plastic flow in the region of the subgrade which leads to pavement sag, rupture, and faulting.

Construction Methods and Equipment

Organized Personnel.—Before any grading work is to be started, the resident engineer should organize the inspection service and make certain that everything is ready for construction operations. All data that have been collected must be compiled on the plans and shown as part of the road-bed design. The moisture conditions of the material to be excavated must be ascertained immediately before construction work is to be started.

Systematic Inspection.—The system of inspection must be planned so that an inspector is constantly on duty while road-bed construction is in progress. The minimum number of tests allowable should be specified. Results of tests should be recorded by station and elevation.

Testing Equipment.—Field-testing equipment should be portable in type and located conveniently near the center of construction activity.

Control of Materials and Methods of Construction.—The essential qualifications for the proper execution of this work are: (*a*) Thorough knowledge of the inherent structural properties of the road-bed materials to be found; (*b*) methods by which these materials may best be utilized in the construction of the road-bed; and (*c*) essential diplomacy and firmness of purpose in dealing with the contractor.

The inspectors should be competent to advise with the contractor in all details of the work. They should aid in planning the work so that construction

operations may be made to fit the weather with the least delay, but under no circumstance should they allow weather conditions to lower the quality of the work. Basin-like areas must not be permitted—either in excavation or in embankment—at any time. These areas may collect water and hinder the work. All embankments should be completely compacted and finished with a smooth surface before closing the work at any time.

Conclusions

Five pertinent conclusions are supported by the foregoing text:

(1) The structural properties of earth masses are enhanced by increasing their density;

(2) The resistance offered by earth masses to the penetration of water increases with increasing density;

(3) The porosity of earth masses can generally be decreased to a certain value by means of a roller; but the porosity of granular masses such as gravel, sand, and silt, can be reduced best by the use of suitable admixtures followed by heavy rolling;

(4) A satisfactory standard of compaction is that based on a suitable factor of safety against water absorption; and

(5) Surface protection against the ravages of weather on clay-like matter is essential, regardless of the degree of compaction.

Acknowledgment

The principles set forth in this paper are from various sources. Most of them are well known, especially in other fields of work, and all are believed to be in accord with fact. Working hypotheses were first written and then tested by a series of observations and researches, most of which were done on the field, covering hundreds of miles of roads of the several types of materials already described. Conclusions were not formed until these hypotheses had been either abandoned or verified by competent evidence.

The work was done as a part of the writer's official duty as a Supervising Engineer in the United States Bureau of Public Roads on Federal Aid work. Clifford Shoemaker, M. Am. Soc. C. E., District Engineer, had long recognized the need for more stable road-beds and, in 1931, directed the writer to give special attention to this feature of the work. For this opportunity and for many helpful criticisms the writer acknowledges his grateful appreciation. Acknowledgment is also made to the Engineers of the several State Highway Departments in the District for their splendid co-operation in the collection of data used in the development of this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THE DESIGN OF ROCK-FILL DAMS

Discussion

BY MESSRS. L. R. EAST, AND FRANCISCO GOMEZ-PEREZ AND MIGUEL JINICH

L. R. EAST,[42] M. AM. SOC. C. E. (by letter).[42a]—Owing to his association with an inquiry into the causes of the partial failure of a large rock-fill dam at Eildon Reservoir, Victoria, Australia, in 1929, the writer has found this paper of particular interest. This dam comprised a rock-fill embankment extending for a distance of 2 525 ft across a comparatively wide valley to a mass concrete spillway of the side-spillway type providing for overflow along a weir section, 682 ft long.

In the river section abutting the concrete spillway, the embankment is approximately 140 ft high, and the rock-fill is founded on bed-rock which is composed of fairly hard metamorphic rock with sandstones and slates dipping almost vertically and with strike diagonally across the line of the dam. Across the river flats, the rock-fill was placed directly upon the natural surface, with from 22 to 24 ft of clay and gravel wash and about 2 ft of sand overlying the bed-rock. On this side of the valley the rock available for fill was shales and mudstones of a less durable character, with bands of slate.

For water-tightness a central vertical core-wall of reinforced concrete was provided, with a clay wall on the up-stream side to assist in staunching any possible leakage through the concrete. This wall varied in thickness from 6 ft at bed-rock (into which it was keyed to a depth of 5 ft) to 2 ft at crest level. The clay wall, of very stiff puddle trafficked with horses and drays during construction, averaged about 30 ft in thickness at the natural surface.

The rock-fill was quarried on both ends of the dam, loaded by hand on trucks (the stone being broken to "one-man" size) and side-tipped into position. The

NOTE.—The paper by J. D. Galloway, M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1937, by Messrs. Cecil E. Pearce, and H. B. Muckleston; January, 1938, by Harold K. Fox, M. Am. Soc. C. E.; February, 1938, by Messrs. Charles H. Paul, and A. Floris; March, 1938, by Messrs. Howard F. Peckworth, Oren Reed, Walter L. Huber, Samuel B. Morris, and L. F. Harza; April, 1938, by Messrs. Paul Baumann, O. W. Peterson, and George W. Howson; May, 1938, by Messrs. John E. Field, John H. Wilson, Frederick H. Fowler, I. C. Steele and Walter Dreyer, and F. Knapp; and June, 1938, by Messrs. Ralph J. Reed, F. J. Sanger, and C. S. Jarvis.

[42] Chairman, State Rivers and Water Supply Comm., Melbourne, Victoria, Australia.

[42a] Received by the Secretary April 12, 1938.

stone from the river quarry was comparatively hard and durable, but that from the other quarry was soft, easily crushed, and weathered rapidly.

Owing to financial difficulties, construction was slow, and the embankment was not brought to its designed height until practically twelve years after the

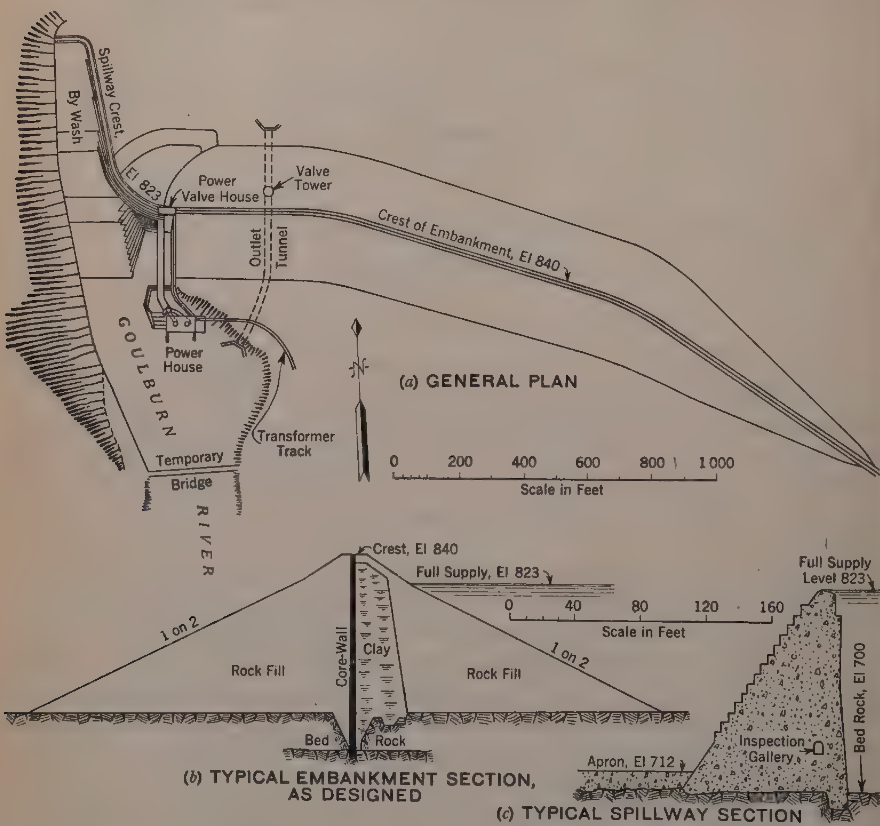

FIG. 26.—TYPICAL CROSS-SECTIONS, EILDON RESERVOIR

commencement of the fill. At no time, however, did construction actually cease. There was time, therefore, for considerable settlement and consolidation during this period, particularly as water was stored to as high a level as possible during construction.

Fig. 26 shows the plan and typical cross-sections of embankment and spillway as designed, the basic data being as follows:

Description	Quantity
Height of bank above river bed, in feet.	140
Height of spillway above river bed, in feet.	123
Length of core-wall, in feet.	2 300
Length of spillway crest, in feet.	682

Description		Quantity
Storage capacity, in acre-feet		306 400
Area submerged, in acres		8 000
Length of waterway, in miles:		
Delatile River	11.5	
Goulburn River	10.5	
Total		22
Length of water frontage, in miles		70

Fig. 27 shows a typical cross-section of the embankment as completed in 1927 when the reservoir was filled to its full capacity of 306 000 acre-ft for the first

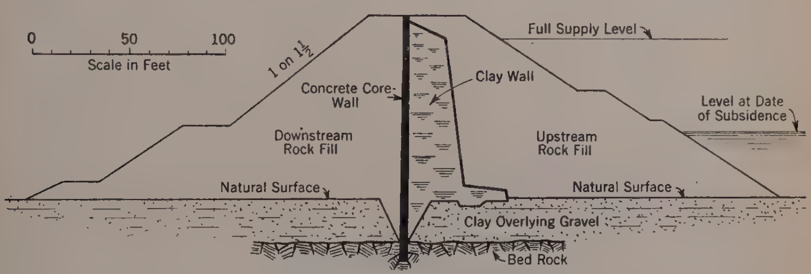

FIG. 27.—TYPICAL CROSS-SECTION OF EMBANKMENT, EILDON RESERVOIR, AS COMPLETED IN 1927

time in August of that year. It was not drawn down very far the next year and in the spring of 1928 it was again filled to capacity, the water flowing over the spillway to a depth of 4 ft 2 in. in October. The irrigation season then re-

FIG. 28.—VIEW OF EXPOSED CORE-WALL, JUNE 16, 1929

sulted in a heavy demand on the storage and in four months, to April 1929, the reservoir was drawn down to the extent of 50 ft.

At that stage, without warning, the rock-fill on the up-stream side of the core-wall subsided suddenly and the concrete core-wall itself was exposed over a length of about 1 200 ft to a maximum depth of 26 ft (see Fig. 28). The greatest subsidence occurred against the core-wall itself which was deeply scored or scratched by the rock-fill sliding against it. The exposed top of the core-wall showed deflections of as much as 4 ft 8 in. in lengths of 400 ft and 800 ft originally constructed as straight lines.

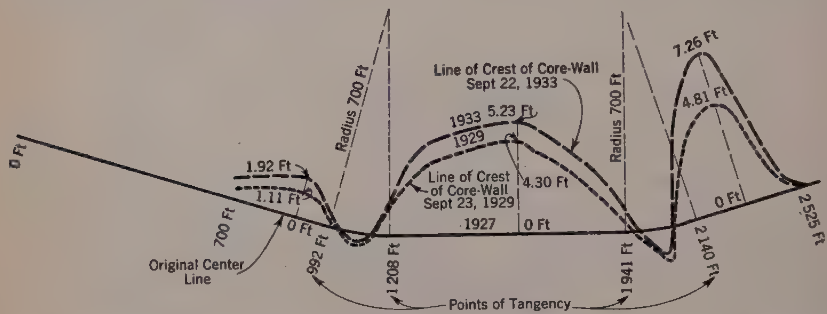

FIG. 29.—DIAGRAM SHOWING LATERAL DEFLECTION OF CORE-WALL, EILDON RESERVOIR

Fig. 29 shows the line of the core-wall as constructed, its position in September, 1929, after the subsidence, and its position in 1933, since when movement has been very slight—the movement during the 5 yr since 1933 being less than 2 in. at the point of maximum deflection. Fig. 30 shows a typical cross-

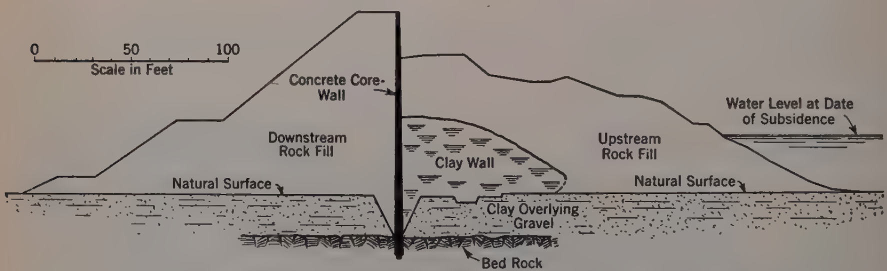

FIG. 30.—TYPICAL CROSS-SECTION OF EMBANKMENT, EILDON RESERVOIR, AFTER SUBSIDENCE, APRIL, 1929

section of the embankment after the subsidence, the position of the clay wall having been determined by borings which enabled it to be located within a few inches. Samples showed the clay to be in excellent condition and to have the consistency of very stiff putty into which a pencil could be forced by hand only with difficulty. The moisture content was about 23 per cent.

There appeared to have been little or no movement of the foundation clays and gravels upon which the rock-fill had been built. The exposed core-wall showed several vertical cracks, notably at the junction with the massive wing-

wall at the river end, and later investigation showed it to have fractured horizontally for some distance along its junction with the rock foundation. There was, however, practically no leakage through the horizontal crack and very little through the vertical cracks which were fairly easily staunched.

The Eildon Weir Inquiry Board was appointed by the Governor in Council on May 24, 1929, to report on the failure. After careful investigation the Board concluded that the subsidence had arisen directly as a result of the action of the clay wall; that the clay had acted as a stiff fluid and pushed the rock-fill out into the water; and, at the same time, had exerted great pressure down stream causing the concrete core-wall to deflect. Although there was little leakage through the concrete core-wall, the clay underlying the down-stream rock-fill was very wet, and as the rock-fill was much lighter than an earth bank (being only 90 lb per cu ft) there was considerable apprehension as to the risk of the entire structure failing by sliding of the down-stream part of the bank. The remedies suggested by the Board were:[43]

"(*a*) To restore the subsidence of rock-fill on the up-stream side by deposition of more loose rocky material to provide for greater weight and stability.

"(*b*) To provide drainage under the base of the down-stream side of the embankment, including a tunnel to carry off such water as may pass through the core-wall, and to add a large quantity of rock-fill to this part of the embankment for the purpose of increasing its supporting power.

"(*c*) To repair all cracks of importance in the concrete core-wall and to provide, where necessary, staunching material on its up-stream side in place of the clay formerly in position for that purpose."

The Board also made recommendations in regard to outlet works and spillway, but the writer proposes to confine his discussion to the rock-fill dam. Fig. 31 shows the additional rock-fill proposed by the Board.

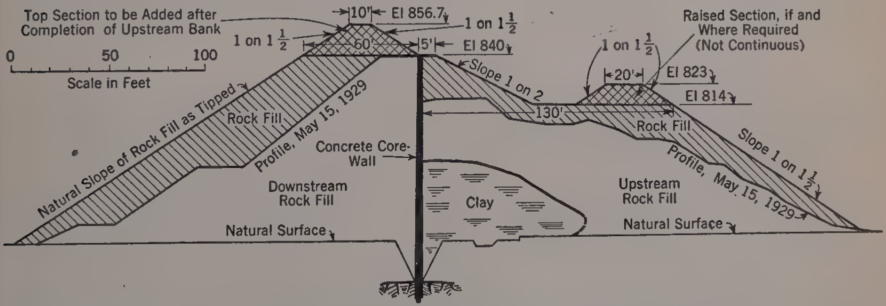

FIG. 31.—TYPICAL CROSS-SECTION SHOWING ADDITIONAL ROCK-FILL PROPOSED BY BOARD, EILDON RESERVOIR

All the remedial measures proposed by the Board have been completed at a cost of £A380 000 (Australian pounds = \$1 500 000), approximately £A206 000 (\$825 000) was for the placement of additional rock-fill, and £A36 000 (\$144 000) for the drainage of the foundation under the down-stream

[43] Rept. of the Eildon Weir Inquiry Board, 1929, p. 15, Govt. Printer, Melbourne, Victoria, Australia. (This report may be examined at Engineering Societies Library, 33 West 39th Street, New York, N. Y.)

section of the embankment. The remainder of the expenditure was on outlet works and spillway.

Regular observations have been made to detect bank movements or deflections of the core-wall, as well as of outflow from the drainage system; and there has been little movement since the completion of the remedial works in 1933. Fig. 32 shows the dam after the completion of the remedial works.

FIG. 32.—VIEW OF EILDON RESERVOIR AFTER REMEDIAL WORKS HAD BEEN COMPLETED, FEBRUARY, 1936

It might be mentioned that the total leakage through the entire structure (in which there is an area of 280 000 sq ft of core-wall) does not exceed 50 gal per min, three-fourths of which comes from one definitely located small fracture at the junction of the core-wall and the outlet tunnel.

The writer is in complete agreement with the principles of design indicated by the author, departure from which was undoubtedly responsible for the failure at Eildon. The dam was not founded on rock, much of the material of the fill was unsuitable in that it was likely to crush and disintegrate, and the up-stream half of the embankment was made a liability instead of an asset by placing the impervious element in the center of the dam. Experience at Eildon shows that the use of a thick blanket of puddled clay, loaded with rock to protect, and confine it, might have serious consequences unless steps are taken to provide for the enormous pressures exerted by clay in the mass.

A rock-filled dam very similar in design to that at Eildon was constructed at Melton, Victoria, on the Werribee River, in 1916, and has given very little trouble. The embankment of this dam, however, was constructed of harder and

more durable stone (basalt from the spillway excavation) and was founded on a rock foundation in a comparatively narrow gorge, so that the resistance against sliding of the up-stream part of the rock-fill embankment would be much greater than in the case at Eildon Reservoir.

A comprehensive description of the failure at Eildon and of the remedial measures to the embankment, spillway, and outlet works, has been published by Mr. R. G. Knight.[44]

FRANCISCO GOMEZ-PEREZ,[45] ASSOC. M. AM. SOC. C. E., AND MIGUEL JINICH,[46] ESQ. (by letter).[46a]—The very complete paper presented by Mr. Galloway on the design of rock-fill dams is based on the historical development of this type of structure. Limiting the scope of the historical summary he presents details of the construction of various dams of the rock-fill type that have been built during the last half century, mainly in the western part of the United States.

Any type of structure must necessarily be adapted to the conditions prevailing in the region where it is to be built and the rock-fill type of dam is no exception to the rule. Conditions in Mexico make low-cost labor available and, at the same time, make equipment that is generally imported a high-cost item of the construction project. These two conditions combined have influenced the development of the rock-fill type of dam in Mexico. In general, the trend has been toward using smaller rocks than would have been the case if mechanical equipment could have been used to a greater extent.

As a result of the use of smaller rocks that necessarily have a larger percentage of voids, other parts of the structure have also been affected and, therefore, the size of the panels that constitute the up-stream slab has been greatly reduced in comparison with the size generally accepted in the United States. Furthermore, and considering the possibility of seismic disturbances, a laminated type of slab has been included in one of the Mexican rock-fill dams completed in 1933. Other elements of current practice in Mexico are presented in this discussion.

General Considerations.—In sites that are far away from roads and railroads, for which the transportation of manufactured products, such as cement, steel, and timber, is very expensive—as well as those sites where such manufactured products are relatively expensive, but where labor is relatively cheap—the most economic type of dam is generally that in which the engineer can use to maximum advantage the materials found on the site. Such is the case of earth and rock-fill dams, and the combination of both. This explains why the rock-fill type has been very advantageous for many sites in Mexico. The Mexican National Commission of Irrigation has made comparative studies of various types of dams for several sites and has adopted the rock-fill type; and it has

[44] "The Subsidence of a Rockfill Dam and the Remedial Measures Employed at Eildon Reservoir, Australia," by R. G. Knight, *Journal*, Inst. C. E., March, 1938.

[45] Technical and Administrative Asst. to Chf. of Designing Dept., Mexican National Comm. of Irrig., City of Mexico, Mexico.

[46] Chf. Designer, Designing Dept., Mexican National Comm. of Irrig., City of Mexico, Mexico.

[46a] Received by the Secretary May 23, 1938.

been built successfully both from the economic and from the technical point of view.

The following rock-fill dams have been completed in Mexico: Charcas Dam, 49 ft high, in the State of Guanajuato in 1933; Taxhimay, 137 ft high, and Madero, 153 ft high, both in the State of Hidalgo (the former in 1934 and the latter in 1938); San Ildefonso Dam, 189 ft high, in the State of Querétaro, designed in 1937 (under construction in 1938); and the Tepuxtepec Dam, 126.5 ft high, in the State of Michoacán, in 1934.

Foundation Requirements.—There is no doubt that sound, durable rock is the ideal material for the foundation of a rock-fill dam, as for any other types; but, in certain cases, it is necessary to use this type at sites where the rock may not be so sound. In the latter case, the site should fulfill the following requisites:

(1) Even when saturated, the material on which the dam is to be founded must be capable of resisting all the load transmitted to it both by the fill and by the hydrostatic pressure when the dam is filled, without any appreciable settlement. The saturated condition must be considered even if it is assumed that all precautions must be taken to prevent filtration, by means of cut-off walls and an up-stream slab.

(2) The material must resist chemical solution and all tendencies to become eroded.

The Charcas Dam had been designed as a gravity structure, but for economic reasons (and mainly due to the character of the foundation rock) it was decided to change the design to a rock fill. The rock was a rhyolite in an advanced state of disintegration that did not furnish the necessary support for a gravity dam; but it complied with the aforementioned requirements for a rock-fill dam.

The Taxhimay Dam (shown in Fig. 33) was a small rock-fill structure that had been built in 1918. When it was taken over by the Commission, in 1927, to be used as a reservoir for one of its irrigation districts the engineers decided to enlarge its capacity by increasing the height of the structure, and it became necessary to study the problem relative to including the old structure within the section of the new one. The old dam was 61 ft high, had been built originally without the necessary precautions for cleaning the river bed perfectly, and the material that had slid down the side-hills had not been completely removed before the original dam was built. The old dam consisted of a loose rock-fill with a down-stream slope of 1 on 1, and up-stream slope of 1 on 2. The impermeable part consisted of a cement-bound masonry zone 6.5 ft thick at the bottom and 1.33 ft thick at the crown, and resting directly on the loose rock-fill. The defects of the original design, together with the foundation defects, caused severe settlements that almost made the structure fail.

In designing the new dam, for the purpose of raising the water surface elevation to a new height of 134 ft it was necessary to remove all the silt and loose materials and to deposit the new fill on the sound solid rock. This structure includes the old loose rock-fill dam and has been finished and in service since 1934 with very satisfactory results.

At the site of the Madero Dam, in the State of Hidalgo, the rock is also sound. It has been classified as a very resistant rhyolitic tuff in the river bed but the resistance gradually diminishes as the elevation increases. When the preliminary design was being prepared, it had been decided to use an arch dam, but on account of the poor resistance of the rock at the higher elevations in the site it was decided to build a rock-fill dam.

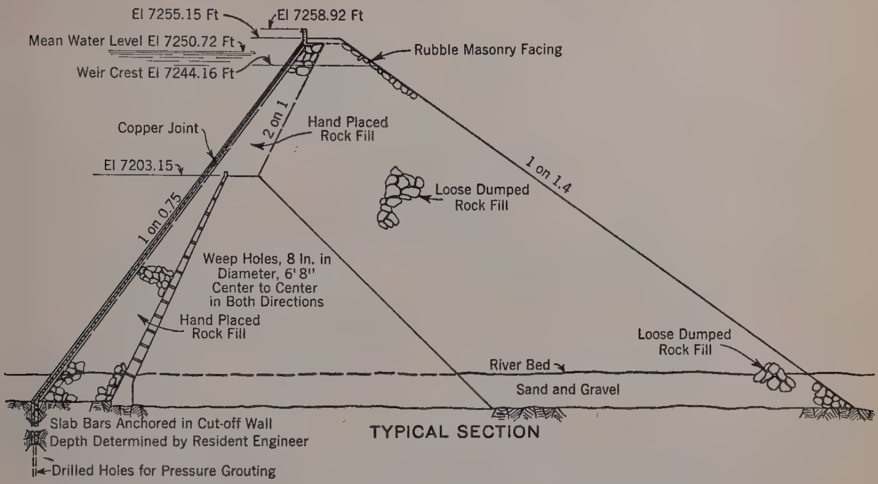

FIG. 33.—CROSS-SECTION OF TAXHIMAY DAM, STATE OF HIDALGO, MEXICO

The Material.—Almost always the material for the construction of rock-fill dams comes from near-by quarries. It may be of volcanic origin, it may be granitic, or it may be either sandstone, or limestone, but the main requirement is that it be sound.

The size of the rock used varies from about 4 in. to the largest size that is feasible to move with the equipment available. It is always better to use rocks of the largest dimensions possible, and to limit the volume of small fragments such as quarry spalls, to a quantity that partly fills the voids and is not so large as to prevent easy drainage through the rock mass, nor to favor local concentrations of the material which, because of the larger percentage of voids, may cause important settlements.

It is evident that water-jets make it possible to use a larger proportion of this quarry-run material, depending on the methods of placing the loose rock-fill. It is also evident that water causes the material to be slushed into the voids between large rocks, without endangering the easy drainage of the mass. It may be possible even to distribute the large rocks better by the use of water-jets.

Cross-Section.—The down-stream slope adopted in the dams built by the Commission has been 1 on 1.4, slightly steeper than the natural average slope for the loose rock-fills. Of course, this slope may vary within limits depending on the size of the rock used and mainly on the degree of arrangements obtained for the rock-fill.

In the aforementioned dams, in view of the low cost of labor, and in order to give them a better appearance, a "finish" layer has been built on the down-stream slope. In the Charcas Dam it consists of the formation of steps, and, in the others, in leaving a more or less smooth rubble masonry surface. This finish improves the appearance of the dam greatly at a relatively low cost.

The main function of the hand-placed rock-fill zone is to serve as a semi-rigid transition section between the outermost layer and the loose rock zone, as the latter is subject to uneven settlement. It gives the outer layer a smooth surface that corrects the irregularities of the loose rock-fill. It also stabilizes the slopes by using them as retaining walls to hold back the loose rock-fill making it possible to use such steep slopes as 1 on 0.75. Such an arrangement (which is recommended by several authorities) seems to offer some economy, but in Mexico this has not been the case on account of the limitations of the equipment generally available for moving rocks of large dimensions.

The original design of Charcas Dam included an up-stream slope of 1 on 0.75 with a zone of hand-placed rock. This zone varied in thickness from 61 ft at the base to 6 ft at the top. After construction had begun it was realized that it was not economical to build the zone of hand-placed rock-fill, which was almost a dry masonry, with 20 to 25% voids. An economic comparative study was made to determine the actual cost of a zone of hand-placed rock-fill, fulfilling all the aforementioned specifications, and the cost of making the slope less steep, adopting a ratio of 1 on 1.25. This was the average natural slope for a rock-fill that would not require perfect work to make it act as a retaining wall. It needed much less thickness, therefore, thus becoming reduced to a nearly superficial layer placed on the slope only for the purpose of supporting the slab.

The dam was built with this slope (1 on 1.25) without any particular attention being paid to the hand-placed rock zone, thus making it possible to obtain additional economy in the pouring of the concrete, because it was placed without the use of forms, and by using a concrete with s slump of 0.5 in. to 1 in.

The difficulties that were met in the construction of the hand-placed rock-fill zone at the Charcas Dam were attributed to the lack of equipment and it was on that account that Taxhimay Dam was built with an up-stream slope of 1 on 0.75 as designed, with a thick mass of hand-placed rock-fill (in which large rocks were used) but with a small percentage of voids. The construction of this type was considered economical because of the possibility of using steam-shovels, derricks, and railroad spurs that were available for its construction. However, it was also difficult to use this design since its cost threatened to be very great if it was to be built in accordance with all the required specifications; and, furthermore, it also required unusually high-class and permanent inspection. As a result of the foregoing experience the up-stream slope for the Madero Dam was made 1 on 1.2.

For the San Ildefonso Dam the slope adopted was 1 on 1.4 both up stream and down stream. When construction was begun it was foreseen that a large portion of the material available in the quarries would be wasted, because it consisted of alternate layers of sound basalt and very finely crushed basaltic breccia. This condition dictated a change in the design (see Fig. 34) and it

was decided to build the dam with a central zone to include the material that otherwise would have been wasted; the specifications required that this material be perfectly wetted and rolled, which seems to be in accordance with the tendency shown in the latest rock-fill dams built in the United States, such as Salt Spring and San Gabriel No. 2, both in California.

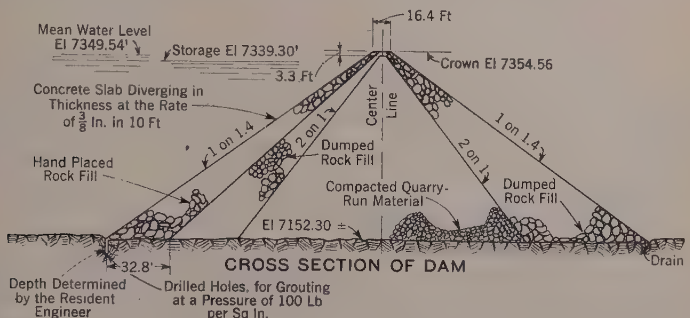

FIG. 34.—CROSS-SECTION OF SAN ILDEFONSO DAM, NEAR SAN JUAN DEL RIO, STATE OF QUERETARO, MEXICO

An exception to the aforementioned examples of slope is found in Tepuxtepec Dam, with slopes of 1 on 0.7 up stream and 1 on 0.8 down stream. This entire dam is practically a dry massonry structure with a percentage of voids varying between 20 and 30. It was built by the Southwest Mexico Power Company, and required masonry labor as skilled as would have been required if it had been built as an ordinary masonry structure. Large rocks transported by cableway were used, each individual piece being hand-placed; and all the voids were filled with spalls.

The cost was greater than would have been the case if the dam had been built with flatter slopes and with most of the rock dumped in place (except a thin zone of hand-placed rock to support the up-stream slab). The type was adopted because of the danger of earth shock in that region. It must be stated that the results obtained at the Tepuxtepec Dam have been splendid, and that no settlement has been observed.

Water-Proof Element.—Modern practice requires the construction of a water-proof element for rock-fill dams that consists of a rather thin concrete slab placed on the up-stream slope. In some rock-fill dams, a steel plate has been used instead of the concrete slab, but this has been the exception.

There is no doubt that the most important problem encountered in the design of the impermeable slab is to make it as water-proof as possible, and, at the same time, sufficiently flexible to follow the unavoidable settlements that occur in this type of dam, without excessive cracking. Just as difficult of solution are the problems relative to the joint at the place where the flexible slab connects with the rigid cut-off walls. It is also important to foresee the changes in length to which the slab may be subject due to changes in temperatures.

The type of slab built in sections, separated by elastic joints, seems to be the most satisfactory solution, and it is the one that has been adopted by the

Mexican National Commission of Irrigation. The up-stream element of the Taxhimay Dam consists of panels about 15 ft wide by 24 ft to 32 ft long, measured on the slope and connected by copper-plate contraction joints. The steel reinforcement placed on the center of the slab does not cross the vertical joints, but it does cross the horizontal joints in order to act as articulating units in the respective columns or strips in which the slab as a whole is divided (see Fig. 35).

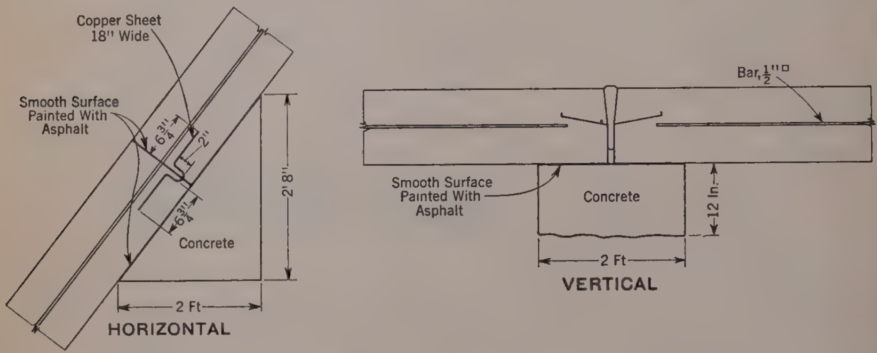

FIG. 35.—TYPICAL SECTIONS OF PANEL FACINGS, TAXHIMAY DAM

Whereas the tendency in the United States seems to be to divide the up-stream slab into rather large panels (about 50 ft or 60 ft square), the Mexican National Commission of Irrigation has adopted much smaller panels, even smaller than the ones used at Taxhimay, in order to make them more flexible, at the same time keeping them within economic limits.

At Madero Dam the panels were built 13 ft wide and 16 ft long and at San Ildefonso Dam they were made 10.5 ft by 17.5 ft. For these two structures neither the horizontal nor the vertical reinforcement crosses the copper-plate elastic joints. The size of the panels was determined by selecting the size of the pre-fabricated forms that would be simplest to build and move around on the slope.

As an exception to this rule Charcas Dam may be cited, in which, because the structure was low and long, and was built by a method that guaranteed a minimum of settlement, the slab was placed in alternate strips about 15 ft wide, leaving only construction joints through which the reinforcing steel placed in the central part of the slab was extended. The dam was inspected about two years after it was built, and it was observed that the joints had opened slightly, forming very fine cracks which, however, do not endanger the stability of the dam.

To make the up-stream slab even more flexible, in some dams it has been built in the form of thin slabs or layers of concrete placed one on top of the other so that the joints are staggered. The most interesting example of this type in Mexico is the Tepuxtepec Dam, in which two layers were used, each being 14 in. thick at the base and 10 in. thick at the top. The slabs were built in 50-ft strips, continuous from the river bed to the crown, and the joints between strips on the upper layer were provided with a copper, water-tight seal (see Fig. 36).

An improvement in this laminated type was also used at the Cogoti Dam in Chile. It is believed that the idea is fundamentally sound; but the cost is rather high and it is justified, therefore, only in special cases where important settlements are expected or where there is a possibility of earth movements, such as is the case at Tepuxtepec Dam, in Mexico, or at Cogoti Dam, in Chile.

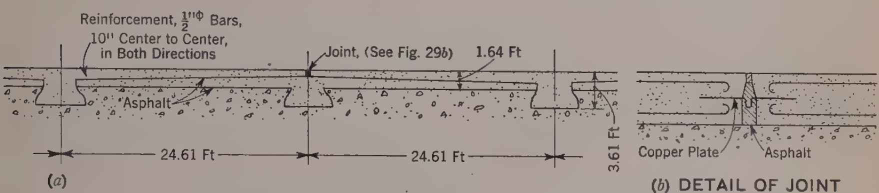

FIG. 36.—REINFORCED CONCRETE FACING, TEPUXTEPEC DAM

Thickness and Reinforcement of the Up-Stream Slabs.—The up-stream slabs in the dams built by the National Commission of Irrigation (see Fig. 37) are designed with a thickness of 8 in. at the top, with a batter of $\frac{1}{8}$ in. for every 40 in. measured downward along the up-stream slope. The reinforcement is designed for temperature variations and consists, in general, of a centrally located grid of rods placed in the center of the slab.

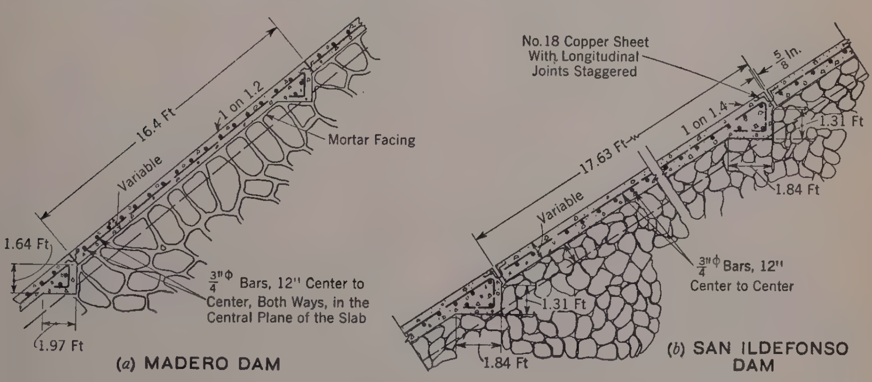

FIG. 37.—TYPICAL SECTIONS OF CONCRETE FACING

The slab is placed directly over the facing of the hand-placed rock-fill, taking care, however, to fill all the apparent hollow spaces with spalls so as to obtain a nearly smooth surface as a bed for the concrete face, thus preventing the mortar from seeping into the voids. The slab is attached to the rock-fill by means of 6-in. dowels, 3 ft to 4 ft long, one end of which is hooked. The hook is embedded in a concrete block; that is, it is left within the hand-placed rock-fill zone; the other end projects from the facing, and is also made into a hook that is left within the slab and tied to the steel reinforcement. These hooks are also used to hold the forms in case they are used for pouring the slab. Furthermore, in order to give the slabs a better support, steps or berms about 15 in. to 18 in. wide are left on the facing. Those steps are very convenient

for the use of workmen and help to build the slab in such a way that its upper part fills the step as shown in Fig. 37.

Bottom Joint.—To design the joint at the connection of the flexible slab with the rigid cut-off wall is another problem that requires careful consideration. At Madero Dam, the joint was made flexible by means of two copper sheets and

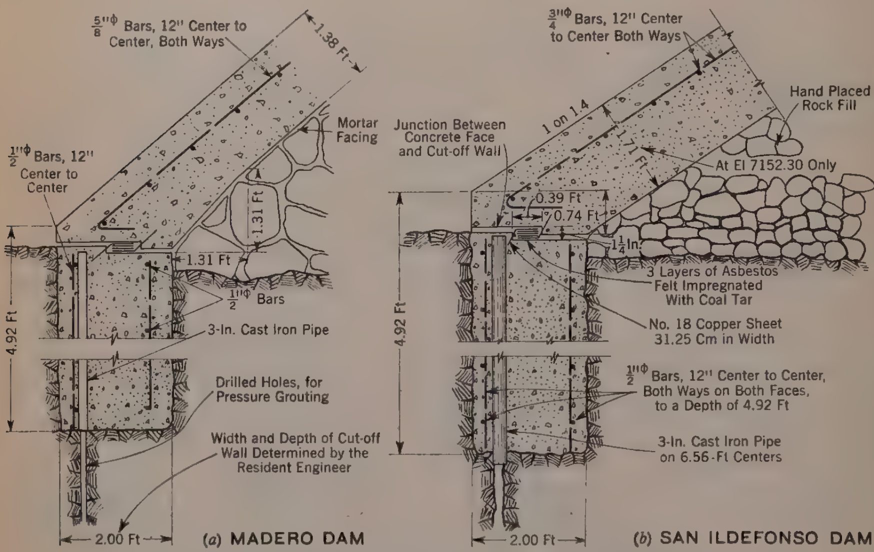

FIG. 38.—TYPICAL DETAILS AT CUT-OFF WALL

a layer of asbestos felt (see Fig. 38(*a*)). This same type of joint (see Fig. 38(*b*)) has been specified for the San Ildefonso Dam.

Placing Rock-Fill.—The more or less horizontal layers of a rock-fill are constructed by dumping the material. The height of such layers has varied between 6 ft and 12 ft (as in the case in Charcas Dam) to between 18 ft and 24 ft in the more important structures, such as Taxhimay and Madero. It is believed that the use of thinner layers tends to create a more uniform distribution of the rock; and, furthermore, by placing some of the rocks by hand, settlement is reduced quite considerably because the individual particles have a more uniform bearing pressure, one on the other. Deep layers require a more careful classification of material, a condition that is undesirable, because the large rocks roll downward, whereas the smaller ones remain on top. The use of water-jets makes deeper layers possible and tends to produce a better arrangement and a more compact rock mass.

The results obtained by the Mexican National Commission of Irrigation in building dams in rather thin layers and with a slight arrangement of the rock through the entire section, have been very satisfactory.

Conclusion.—The purpose of this discussion has been to present additional information that might not be easily obtainable and that is of interest to engineers connected with the design and construction of rock-fill dams, mainly

with the intention of showing how economic conditions influence both the design and the methods of construction.

In accordance with Mr. Galloway's desire that discussion be directed to matters of design, special reference has been made to the use of small rock fragments and also to the use of small concrete panels hung on narrow berms in the water-proofing element; the writers are in agreement with other conclusions of Mr. Galloway's paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

MULTIPLE-STAGE SEWAGE SLUDGE DIGESTION

Discussion

By C. E. Keefer, M. Am. Soc. C. E.

C. E. Keefer,[22] M. Am. Soc. C. E. (by letter).[22a]—A group of interesting experiments is reported in this paper, which should stimulate further investigations of the separate digestion of sludge. It would appear that the type of tanks used, which permitted the segregation of the solids into scum and settled material with facilities for seeding the raw sludge and digesting the material in several stages, was partly responsible for the unusually rapid digestion obtained. It is hoped that further experiments will be conducted with a similar type of tank in some other locality in order to compare and check the results.

A study was made of the heat added to the digestion tank for a period somewhat less than a month. The authors calculated that the heat added to the sludge was 13 900 000 Btu per day, and that the heat required for radiation and endothermic requirements was 1 200 000 Btu daily. Based on some laboratory experiments conducted by Herman Kratz, Jr. and the writer,[4] the authors calculate that the endothermic requirements alone would be 3 690 000 Btu, a quantity three times as great as the heat available for this purpose. They also state that the fall in temperature of the sludge from the second to the fourth tank was less than 0.5° F, whereas it should have been 2.3° F if the findings of Mr. Kratz and the writer were correct.

It is unfortunate that the authors have not given more of the data upon which their calculations are based. It would be interesting to know how the temperature in the tanks was obtained, how many thermometers were used in each tank, and the location of each of them. What type of thermometer was used? and to what degree of accuracy could they be read? The statement that

Note.—The paper by A. M. Rawn, M. Am. Soc. C. E., A. Perry Banta, Assoc. M. Am. Soc. C. E., and Richard Pomeroy, Esq., was presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 16, 1936, and published in November, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1938, by Edward W. Moore, Esq.; and March, 1938, by Messrs. Willem Rudolfs and H. Heukelekian, and Hermann Bach.

[22] Prin. Asst. Engr., Bureau of Sewers, Baltimore, Md.

[22a] Received by the Secretary April 8, 1938.

[4] "The Interchange of Heat During Sludge Digestion," by Messrs. C. E. Keefer and Herman Kratz, Jr., *Sewage Works Journal*, Vol. 5 (1933), p. 3.

the difference between the average temperatures of the second and the fourth tanks was less than 0.5° F would, in general, seem to be open to considerable question. It is doubtful whether the average temperature of the sludge in a digestion tank of any appreciable size can be obtained within 2° to 4° F as there is usually a considerable difference of temperature in various parts of the tank. Sludge in the zone immediately adjacent to heating coils will often be from 40° to 60° F hotter than the incoming cold sludge. Sludge near the walls and the bottom of a tank far removed from heating coils will be cooler than the sludge near the coils. To obtain accurate results a large number of thermometers would be required. The complexity of the problem can be appreciated when it is remembered that it is difficult to keep the temperature in specially designed and well-insulated constant-temperature rooms that are thermostatically controlled and provided with means such as fans to prevent the stratification of air within much less than 1° F. As it is much more difficult to determine the average temperature of sludge in a digestion tank, there is considerable question as to the advisability of drawing conclusions regarding the endothermic requirements of sludge digestion from large-scale experiments.

It would seem wiser to conduct experiments of this special nature in a calorimeter, in which the temperature can be determined within about 0.001° C. The disadvantage of such an investigation is that a small quantity of the sludge must be used.

In discussing the alleged difference in the results obtained in the laboratory at the sewage works at Baltimore and at the disposal plant near Harbor City, Calif., it should be stated that the Baltimore sludge was from a preliminary sedimentation tank, whereas the material from the latter plant contained 67% activated sludge. Furthermore, more seeding sludge was used at Baltimore. The fact that the Baltimore sludge digested much more slowly may have had a bearing on the difference in the results.

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DISCUSSIONS

LABORATORY INVESTIGATION OF FLUME TRACTION AND TRANSPORTATION

Discussion

By Messrs. J. E. Christiansen, and W. H. Huang

J. E. Christiansen,[51] Assoc. M. Am. Soc. C. E. (by letter).[51a]—The results of an interesting laboratory study of flume traction and transportation of sediment by suspension, combined with a review of literature on the subject, is presented in this paper. Since the writer has been interested primarily with transportation of silt by suspension, this discussion will deal chiefly with Part III, "Transportation of Material in Suspension."

Equation (49) is derived to express the bottom velocity required to lift a particle from the bed. This assumes that the full impact of the velocity, V_l, is applied in a vertical direction to the projected area of the particle. By definition, the author implies that V_l is the bottom velocity in the direction of flow (horizontal), and not the vertical velocity, or vertical components of the velocity. The derivation of this equation is similar to the usual derivation of an equation for settling velocity of spheres in a fluid,[52] except that the author makes no mention of the coefficient of resistance of the particle, which is unity for particles of only one size. Direct reasoning would indicate that to lift a particle from the bed would require the vertical components of the bottom velocity due to turbulence to exceed the settling velocity of the particle.

In the general formula for the distribution of suspended sediment (Equation (51)), the origin of y must be taken at the point of maximum velocity to be consistent. The Austausch coefficient is correctly defined by the expression,

$$\epsilon = \frac{\tau}{dv \div dy} \qquad (70)$$

in which τ is the shearing force per unit area. The definition of ϵ as given by the author applies only to a wide shallow stream for which the surfaces of equal

Note.—The paper by Y. L. Chang, Esq., was published in November, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1938, by Hans Kramer, M. Am. Soc. C. E.; and June, 1938, by Messrs. E. W. Lane, and Joe W. Johnson.

[51] Asst. Irrig. Engr., Coll. of Agriculture, Univ. of California, Davis, Calif.

[51a] Received by the Secretary June 22, 1938.

[52] "Distribution of Silt in Open Channels," by J. E. Christiansen, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, pp. 478–485, 1935.

velocity are essentially horizontal planes. The Austausch coefficient has the same physical significance for turbulent flow as the coefficient of viscosity has for laminar flow. Assuming that τ is a function of y, any selection for the origin of y, other than point of maximum velocity, indicates a shearing force at the point of maximum velocity and an infinite value of ϵ.

The writer believes that the method of computing the values of ϵ and τ used by Leighly[53, 54] could be used advantageously in the laboratory when the velocity distribution can be determined. This method involves the expression,

$$\epsilon = m\, g\, S \div \frac{dv}{dy} \qquad \text{(71)}$$

in which m is the mass of water between a unit surface of equal velocity and the region of maximum velocity, bounded on the sides by surfaces normal to the surfaces of equal velocity.

The author states (see "Experimental Verification") that "tests have been made on sediment distribution as described; the results plotted do not agree with the theoretical Equation (51) because this equation is presumably derived for a considerably wide channel suffering no side restrictions." This statement appears misleading because no such assumptions are made in its derivation.[55] The only assumptions concern the evaluation of ϵ. For a deep narrow flume, ϵ is not correctly expressed by the equation given by the author. From the subsequent remarks, it appears that the exponential curve did fit the observed data when a coefficient was applied to the exponent. Another factor that may have affected the value of the exponent, $\omega\, \rho \int_0^y \frac{dy}{\epsilon}$, is a possible error in the value of ω. Although Rubey's equation appears to fit Richards' experimental data[56] very well, it should be noted that Richards determined the size of the particles by means of sieves. Knapp[57] and Alexander and Jacob[58] have reported that settling velocities of particles computed from microscopic measurements of diameters are too high, and that they agree better with data on spheres when diameters are determined with sieves.

Concerning the size of particles, the author states (see "Part I," heading, "Mechanical Properties of Test Samples"), "for fine grains the volume of which was not measurable, D was assumed equal to the mean of D_1 and D_2." For Samples 6198 and 6197, however, the value of D as given in Table 1 is the same as for D_2. The writer has been unable to check the values of ω given from the formula and values of D as given in Table 1, or from the mean value

[53] "Toward a Theory of the Morphologic Significance of Turbulence in the Flow of Water in Streams," by John B. Leighly, Univ. of Calif., *Publications on Geography,* Vol. 6, No. 1, pp. 1–22, 1932.

[54] "Turbulence and the Transportation of Rock-Débris by Streams," *Geographical Review,* Vol. 24, pp. 453–464, 1934.

[55] "Review of the Theory of Turbulent Flow and Its Relation to Sediment-Transportation," by Morrough P. O'Brien, Assoc. M. Am. Soc. C. E., *Transactions,* Am. Geophysical Union, 1933.

[56] "Velocity of Galena and Quartz Falling in Water," by R. H. Richards, *Transactions,* Am. Inst. Min. Engrs., Vol. 38, pp. 230–234, 1908.

[57] "New Apparatus for Determining Size Distribution of Particles in Fine Powders," by R. T. Knapp, *Industrial Engineering Chemistry,* Vol. 6, pp. 66–71, 1934.

[58] "Mechanical Analysis of Finely Divided Natural Phosphates," by L. T. Alexander and K. D. Jacob, *Technical Bulletin 212,* U. S. Dept. of Agriculture, pp. 1–24, 1930.

of D_1 and D_2. It appears possible that the difficulty in making the equation fit the experimental data might be due, at least partly, to incorrect determinations of both ϵ and ω. In the writer's study[59] it appeared that Equation (51) was in very good agreement, without modification, with all the available published data that were analyzed.

The author's discussion of total load in suspension is interesting, but it must be kept in mind that these equations are valid only for silt particles with a uniform settling velocity. Silt found in suspension in natural streams includes particles of various sizes and settling velocities. For each size fraction, the concentration at any depth can be expressed by Equation (51) in terms of the known concentration at one depth, the turbulence of the stream as expressed by ϵ, and the settling velocity of the silt, ω. The total concentration of all size fractions cannot be expressed by a similar exponential equation, however, and, consequently, the total load in suspension is not correctly expressed by Equations (56) to (59).

Assume a general case of a natural stream for which the vertical velocity distribution is determined with a current meter, and a sample of silt is collected at one or more depths. The silt samples can be analyzed for distribution of size of particles, and a curve can be plotted showing this relationship graphically. The average settling velocity for each size fraction between rather narrow limits can be estimated. From the velocity distribution, $\frac{dv}{dy}$, and slope, S, values of ϵ can be determined by graphical integration. Curves can then be constructed showing the size distribution of silt in a vertical section for each size fraction. The total concentration at any depth is, $c_T = c_1 + c_2 + c_3 + \cdots$, in which c_1, c_2, etc., are the concentrations for each size fraction. A curve for c_T plotted against y can then be plotted, but the relationship cannot be expressed by a simple exponential equation. The total load, P, is correctly expressed by the relation,

$$P = \int_0^d c_T \, v \, dy \qquad (72)$$

and the integration can be performed graphically.

The writer questions the validity of Equation (61) and the reasoning that precedes it. He can see no more reason for assuming that the kinetic energy of a stream is independent of the silt carried, than for first assuming that the velocity is not affected. It would seem that the principal effect of silt on the velocity and kinetic energy would be due to a possible change in the smoothness of the bed, and the possible effect on the Austausch coefficient, ϵ. Few data are available on this point. Buckley[60] observed that the velocity increased during periods of high silt concentration, other factors remaining constant. In deriving the general expression for silt distribution, the basic

[59] "A Study of the Distribution of Silt in Open Channels," by J. E. Christiansen, Assoc. M. Am. Soc. C. E. Thesis submitted in partial fulfillment for the degree of Civil Engineer, Univ. of California, 1935. (Not published; on file in the Library, Univ. of California.)

[60] "The Influence of Silt on the Velocity of Water Flowing in Open Channels," by A. B. Buckley, *Minutes of Proceedings*, Inst. C. E., Vol. 216 (II), p. 183, 1922–23.

assumption is made that ϵ is not a function of silt concentration; and this assumption is probably justified for ordinary concentrations found in many rivers, but it may not be justified for the exceptionally high concentrations sometimes found.

Referring to the "Summary," Part III, the writer cannot agree with Conclusions (1), (3), (4), and (5). The reasoning leading to Conclusion (1) does not appear sound for the reasons previously mentioned. In connection with Conclusion (2), it must be kept in mind that Equation (51) expresses the relationship only for a single-size fraction, and that the distribution curve for a graded silt is not a simple exponential curve. Conclusion (3) is misleading. The first part of the expression, Equation (55), $c_m = \frac{1}{d}\int_0^d c\,dy$, is valid, but the latter part of the expression applies only to the specific case of parabolic velocity distribution, and maximum velocity at the surface, where it is seldom found. The statement concerning the location of the point of mean silt concentration is based on a calculation for specific conditions, and does not hold, generally. For the assumptions used by the author, a general expression for the position of the mean silt concentration in a vertical section can be obtained by equating,

$$e^{\alpha y} = \frac{1}{d}\left(e^{\alpha d} - 1\right) \qquad (73a)$$

from which,

$$y = \frac{1}{\alpha}\ln\left(\frac{e^{\alpha d} - 1}{\alpha d}\right) \qquad (73b)$$

The position of mean silt concentration, therefore, is a function of α as well as d. Furthermore, the use of the term, "mean silt concentration," to refer to the average silt concentration in a vertical section is misleading. Mean silt concentration is more properly defined by the expression,

$$c_m = \frac{P}{q} \qquad (74)$$

The total amount carried in a vertical section of uniform width is expressed correctly by the equation,

$$P = \int_0^d c\,v\,dy \qquad (75)$$

in which both c and v are functions of y. The first statement in Conclusion (4) is not verified by the data presented. The second statement is valid only for a single size fraction, and cannot be applied to the total silt concentration of a stream. Conclusion (5) is the result of an assumption that may not be valid, and is in direct conflict with certain previous observations.

Corrections for *Transactions:* In Equation (58) the first c should read c_0; and, on p. 1735, Line 18, change "Equation (57)" to read "Equation (58)."

In Fig. 6, transpose the two formulas; under "Summary: Part III," omit the last sentence of Item (3); on page 1212, June, 1938, *Proceedings*, after

Equation (77c) write: "In which G_1 = rate of sand movement, in pounds per hour per foot of width"; in the sentence that includes Equation (79), rewrite: "A plot of the data observed at the U. S. Waterways Experiment Station results in," etc.; and, in Fig. 18(b), the ordinates are "values of $\frac{G_1}{n_s}$." See, also, *Proceedings* for March, 1938, p. 604, and June, 1938, p. 1216.

W. H. Huang,[61] Esq. (by letter).[61a]—The lucid explanations which the author has given to some of the phenomena observed in his experiments are impressive. For example, he interprets Exponent β in Equation (20) and Equation (21c) as turbulence, and shows that sorting will take place in graded sands irrespective of the mode of motion of the particles, and that the coarser particles will move in greater quantities than the finer ones.

Attention is called to Equation (13), in which it appears that the term corresponding to the decrease in potential energy is incorrect. Mr. Chang takes the average of the drop of the water surface and that of the bed in the reach, dx, as the decrease in potential energy, and thus he obtains $w\,y \frac{\sin\alpha + \sin\alpha_0}{2} dx$. It is known, however, that throughout the depth at any section the potential energy is equal to that at the water surface. The decrease in potential energy, therefore, should be: $w\,y \cos\alpha_0 \tan\alpha\, dx$; and Equation (13) should read:

$$\frac{w\,y}{2\,g}[(V + dV)^2 - V^2] + T\,dx = w\,y \cos\alpha_0 \tan\alpha\, dx \qquad (76)$$

In deriving Equation (15) Mr. Chang refers to Fig. 5, which is also incorrect. Taking x as positive toward the right and y as positive upward, the depth of water at the left section should be taken as y, and the increment of depth at the right section should be taken as $(-)\,dy$. Using these correct notations, $\frac{dV}{dy} = \frac{-V}{y}$, $\frac{(-)\,dy}{dx} = \cos\alpha_0 \tan\alpha - \sin\alpha_0$, is obtained and,

$$T = \left[w\,y \cos\alpha_0 \tan\alpha + w\frac{V^2}{g}(\sin\alpha_0 - \cos\alpha_0 \tan\alpha)\right] \qquad (77)$$

For a sloping bed with uniform flow,

$$T = w\,y \sin\alpha_0 \qquad (78a)$$

and, for a horizontal bed,

$$T = w \tan\alpha \left(y - \frac{V^2}{g}\right) \qquad (78b)$$

Therefore, for great depth with comparatively small velocity, the tractive force on a horizontal bed is approximately equal to that on a sloping bed with the same water-surface slope (not 0.5 as the author has concluded). Therefore,

[61] Prof. of Hydr. Eng., National Central Univ., Chungking, China.

[61a] Received by the Secretary April 9, 1938.

the values of T_0 in Table 4 should be recalculated, and Equation (20) should be modified accordingly.

Equation (76) may also be considered as Bernoulli's equation with each term multiplied by the factor, $w\ y$; thus:

$$T\ dx = w\ y\ H_f \qquad (79)$$

in which H_f is the loss of head in the reach, dx.

Since,

$$H_f = S_e\ dx \qquad (80)$$

in which S_e is the energy gradient; therefore:

$$T = w\ y\ S_e \qquad (81)$$

By Manning's formula,

$$S_e = \frac{n^2\ Q^2}{2.21\ y^{3.33}} \qquad (82)$$

which, substituted in Equation (81), yields,

$$T = \frac{w\ n^2\ Q^2}{2.21\ y^{2.33}} \qquad (83)$$

or,

$$T = w\ y \sin \alpha_0 \left(\frac{y_0}{y}\right)^{3.33} \qquad (84)$$

in which y_0 is the depth of normal flow.

For channels other than the rectangular cross-section of infinite width as assumed, the average tractive force may be represented by the following formula: $\bar{T} = w\ R\ S_e$; or,

$$\bar{T} = \frac{w\ n^2\ Q^2}{2.21\ R^{\frac{1}{3}}\ A^2} \qquad (85)$$

Equation (85) may be used to calculate the tractive force in experimental flumes where the walls are roughened to give approximately the same roughness as that of the bed. When the roughness of the walls differs widely from that of the bed, as in the author's flume, more experimentation is needed to justify the use of Equation (4) and Equation (6).

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DISCUSSIONS

GRAPHICAL REPRESENTATION OF THE MECHANICAL ANALYSES OF SOILS

Discussion

By Charles H. Lee, M. Am. Soc. C. E.

Charles H. Lee,[35] M. Am. Soc. C. E. (by letter).[35a]—Two objectives are definable in this paper: First, to standardize graphical representation of grain-size distribution; and second, to simplify and systematize the interpretation of such graphs. Both undertakings are opportune and it is to be hoped that the first, especially, can be carried through to an early definite conclusion. The second is more a matter of individual experience, technical background, and judgment, and cannot be reduced to set rules. The author's method of simplification is an interesting one.

As for a graphical standard, the writer agrees with the author in favoring the one with the ordinate as percentage by weight of total sample plotted to natural scale, and the abscissa as particle size, in millimeters, plotted to a logarithmic scale, with small sizes at the left and large sizes at the right. In plotting the results of standard sieve and hydrometer analyses, however, it is found more convenient to have every number from 1 to 9 in each log cycle represented by a vertical line, and, in addition, to have the half lines between 1 and 2 and the standard sieve size lines. The use of log 2 and log 6 or any other combination, although giving a more open diagram, is not convenient for plotting or for picking off end points of soil fractions.

A form of graphical diagram which the writer has developed by trial through the ten years, 1928–1938, is presented in Fig. 3. It includes five cycles ranging in size from 0.001 to 152.4 mm which covers all ordinary requirements. Its one limitation is exclusion of smaller clay sizes extending into the colloids.

Fig. 3 also includes tabular spaces for entering standard Tyler sieve percentages that are held, and size classification according to the U. S. Bureau of

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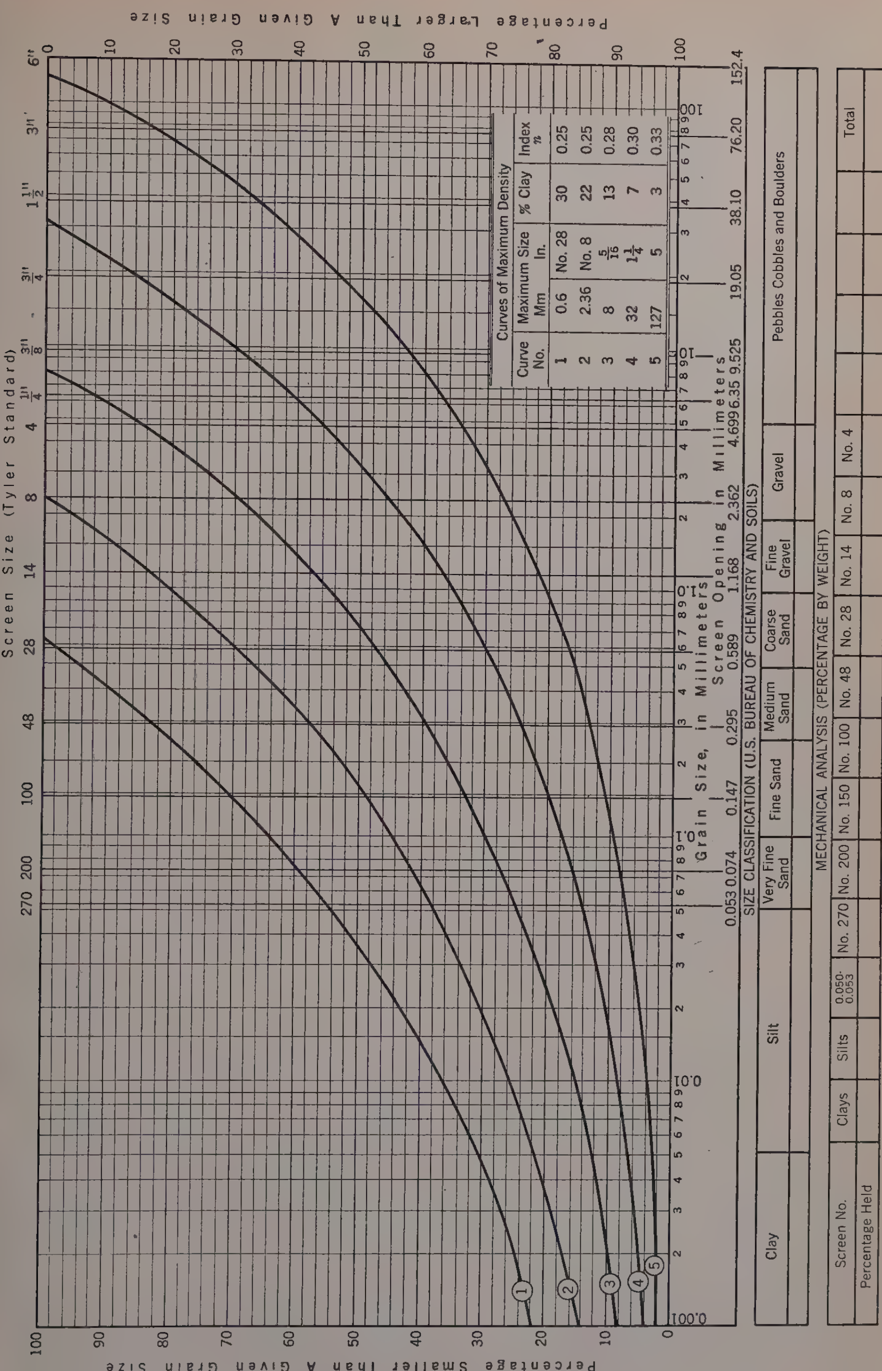
Note.—The paper by Frank B. Campbell, Assoc. M. Am. Soc. C. E., was published in December, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1938, by Messrs. Donald M. Burmister and A. J. Weinig, Jr.; March, 1938, by Messrs. Carl H. Kadie, Jr., Carlton S. Proctor, T. T. Knappen, Jacob Feld, and Howard F. Peckworth; May, 1938, by Messrs. Joel D. Justin, L. B. Olmstead, T. A. Middlebrooks, and Frank E. Fahlquist and Waldo I. Kenerson; and June, 1938, by Messrs. E. W. Lane, F. J. Sanger, and F. Knapp.

[35] Cons. Hydr. Engr., San Francisco, Calif.

[35a] Received by the Secretary June 3, 1938.



Curves of Maximum Density

Curve No.	Maximum Size Mm	Maximum Size In.	% Clay	Index n
1	0.6	No. 28	30	0.25
2	2.36	No. 8	22	0.25
3	8	$\frac{5}{16}$	13	0.28
4	32	$1\frac{1}{4}$	7	0.30
5	127	5	3	0.33

FIG. 3

Chemistry and Soils. This form has been found to have a wide appeal because it is easily understood, both by those who are familiar with separation by standard Tyler sieves and by those who use the U. S. Bureau of Soils classification. It is the writer's observation that the vast majority of American engineers are familiar with one or the other of these classifications, and that the engineering use of other classifications is confined to a relatively few specialists in soil technology.

The classification favored by the writer for an engineering standard is that of the U. S. Bureau of Chemistry and Soils. The reasons for such choice are: (1) It is generally recognized and widely used both by engineers and by scientific research workers; (2) the fractions conform reasonably well with distinctive physical characteristics resulting from particle size; (3) it has proved reasonably satisfactory in general use; and (4) it can be readily taken from a graph plotted from the results of any sieve and hydrometer, pipette, or similar analysis which a great variety of established laboratories are equipped to make.

The author's generalization that immature soils have size-distribution curves that are concave upward and mature soils convex upward is not supported by inspection of such curves from broad geographic areas. Curves that are concave upward represent a material in which the larger sizes predominate, whereas those in which small sizes predominate have curves that are convex upward. If the original rock contains non-weathering grains of relatively large size, such as quarts or silica, the curve may be concave upward even if weathering has reached a stage of maturity. Such a condition exists in regions of granitic rocks such as granite, diorite, etc. Rocks of this type comprise large areas in all parts of the world and contribute non-weathering constituents to both residual and transported earth materials.

Alluvial materials may vary from the uniform grain-size type to the well-distributed type, depending upon the velocity conditions at the point of deposition. Alluvial débris deposited by swiftly flowing streams at a point of grade change have size-distribution curves that are straight and sloping, or concave upward, rather than vertical. Flowing water is an excellent sorting agent only if the velocity is uniform, such as in streams traversing broad valleys or shore currents in lakes or in the ocean; but in streams that drain steep topography, the velocity changes rapidly, affording little opportunity for sorting.

The mean grain size, as defined by the author, is entirely arithmetical and has no physical meaning. Mean grain size, as a weighted geometric mean, can be determined readily from a size distribution diagram, and it is not a step in advance to adopt an arbitrary definition. The attempt to express gradation by the slope of the line passing through the 20% and 80% points on the curve is equally arbitrary, and will lead to confusion rather than simplification. The entire curve must be taken into consideration in studying the size distribution characteristics of a material.

The nomenclature proposed by the author for designating these arbitrary values is too complicated to be remembered or readily grasped mentally, and is impractical for use. The proposed size classification with log-2 and

log-6 division is also arbitrary and differs from those already in use. There are no compelling reasons advanced in its favor and its adoption would add to the confusion in an already over-confused field.

In classifying size-distribution graphs the writer gives consideration to physical characteristics more or less controlled by size distribution, such as porosity, permeability, and density. At one end of the scale are uniform grain-size materials, with high porosity and permeability, but low density. Such materials are sought after as sources of well water. At the other end are well-graded materials whose graphs follow closely a curve with an equation of the form:

$$p = \frac{(d)^n}{D} \qquad (1)$$

in which p = the proportion by weight passing a given screen or sieve opening; d = size of opening; D = maximum particle size; and n = an exponent ranging from 0.25 to 0.40. Such material has a low porosity and permeability, and high density, and is excellent for use in constructing earth embankments. An ideal group of five such curves with maximum sizes of particles equal to 0.6, 2.36, 8, 32, and 127 mm, percentages of clay equal to 30, 22, 13, 7, and 3, and values of n equal to 0.25, 0.25, 0.28, 0.30, and 0.33, are shown in Fig. 3.

Superimposed upon this sequence of grain-size distribution is the effect of size, which in fine silt and clay produces greater porosity than in coarser fractions, lower density, and far less permeability. In conjunction with water the fines also introduce the new characteristics of shrinkage and slow consolidation under load. The limiting curves of the group in Fig. 3 include the ranges of size distribution and of size, which experience has shown are satisfactory for construction of impervious sections of rolled-fill earth dams. It is to be noted that Curves a and f in Fig. 1 are both beyond the limiting curves of the group, and both were unsatisfactory for this type of construction.

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DISCUSSIONS

PRELIMINARY DESIGN OF SUSPENSION BRIDGES

Discussion

BY MESSRS. HARDY CROSS, AND A. A. EREMIN

HARDY CROSS,[23] M. AM. SOC. C. E. (by letter).[23a]—This paper is an important contribution both to the literature of suspension bridges and to the general theory of structural design. That it makes possible an immediate picture of the action of the truss and of the effect of variation of proportions of the structure is a rare virtue.

A case having important points of similarity to that discussed in the paper is presented by two parallel beams so connected that they deflect together. Assume that the dimensions of one of the beams (represented by the cable) is fixed and that its strength is of no further interest. Assume, also, that the second beam (stiffening truss) is added merely to stiffen the primary beam, whatever that may mean, and that structural failure of the secondary beam is to be feared only because it will then fail to stiffen properly. The shape of the deflection curve for a loaded beam and for a loaded cable are not alike; this does not destroy the usefulness of the comparison for present purposes.

The secondary beam may produce two effects: It may change the shape of the deflection curve; or, it may change the amount of the deflection. The authors present some evidence that the first effect is not important.

If the deflection of the primary beam is fixed, the angular distortions of the secondary beam are also fixed. Linear strains are then a function of depth and stresses are related to strains by the elastic modulus. Such a condition is represented in suspension bridges by cable stretch, expansion of cable, and movement of tower connections of cable due to the deformation of shore spans. These "deformation" stresses have no useful effect; they merely threaten the structural integrity of the secondary beam.

NOTE.—The paper by Shortridge Hardesty and Harold E. Wessman, Members, Am. Soc. C. E., was published in January, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1938, by Messrs. A. W. Fischer, and Jacob Karol; and June, 1938, by Messrs. Glenn B. Woodruff and Norman C. Raab.

[23] Prof., Civ. Eng., Yale Univ., New Haven, Conn.

[23a] Received by the Secretary July 7, 1938.

If the load on the combination is fixed, the presence of the secondary beam affects the moments on the primary beam and so affects the deflections. If the secondary beam is very flexible compared with the primary beam, this change in deflection is small, and the stresses vary as do the deformation stresses. Such a case is represented by such heavy bridges as the George Washington Bridge.

If the secondary beam is very stiff it will carry practically all the moment, and the deflection of the system under load will be primarily a function of the stiffness of the secondary beam. This is true of light suspension bridges; for these the elastic theory gives close approximations.

In most bridges the stress in the stiffening truss due to live load does not vary inversely as its strength. The truss is relatively inefficient as a stiffening member. The structure is "hybrid," acting partly as indicated by the elastic theory of suspension bridges where the moment to be resisted is fixed and partly as if its stresses due to load were of the nature of deformation stresses where the deflection produced is fixed.

One of the most important results of the paper may prove to be that more attention will be given to the question of the function of the stiffening truss. Stiffness may be discussed in terms of angular deformation, of deflection, of grade change. It may also be discussed in dynamic terms, and discussion of angular deformation, or of deflection, or of grade change, is not a completely adequate basis for estimating vibration characteristics. Probably stiffness needs discussion in all these terms. Engineers know that they want to stiffen the bridge, but they have not yet satisfactorily defined stiffness.

Valuable studies in the design of stiffening trusses, some of which are correlated to the thesis presented by the paper, have been made by Jacob Karol.[24]

A. A. Eremin,[25] Assoc. M. Am. Soc. C. E. (by letter).[25a]—In this paper the authors have developed a simple method of computing stresses in the preliminary design of suspension bridges. However, it has various limitations which should be considered by designers.

The error in computing stresses in stiffening trusses increases rapidly with a decrease in the span length of the bridge. From Table 1 it is evident that the error in computing the bending moment in the stiffening truss of the San Francisco-Oakland Bay Bridge by the preliminary method is about 1.77 per cent. The span length of the Bay Bridge is 2 294 ft. The error in computing the bending moment in the stiffening truss of the Maumee River Bridge, with its span length of 777.6 ft, is 7 per cent. The increased error in computing stresses in short-span suspension bridges is caused by the greater influence of approximate assumptions used in computing deformations of stiffening trusses in short-span bridges.

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[24] "A Partial Influence Line Procedure for Suspension Bridge Analysis by the Deflection Theory," by Jacob Karol. Thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Engineering in the Graduate School of the University of Illinois, 1938.

[25] Assoc. Bridge Designing Engr., Bridge Dept., Div. of State Highways, Sacramento, Calif.

[25a] Received by the Secretary August 12, 1938.

Equations (3) to (9), involving expressions for ordinates of suspension cables loaded with uniformly distributed dead and live loadings, were developed by the authors from equations in statics. Similar equations were developed in a slightly different manner in 1917.[26] It is to be regretted that, in computing the deformation of suspension cables, the authors considered only uniformly distributed loading. However, the maximum stresses in stiffening trusses of suspension bridges with moderate span length occur for a combined loading uniformly distributed, and concentrated forces at various points along the span length. Evidently, the computation of deflections and the deflection loops of suspension cables sustaining the combined loading is complicated and impracticable considering the simplicity of other available approximate methods of designing suspension bridges. The authors already have demonstrated the simplicity and other advantages of the design methods based on trigonometric functions. Approximate analyses of stresses in suspension bridges with moderate span lengths may also easily be made with model test, as shown by Mr. E. Rothenberg.[27] Likewise, an interesting preliminary study of the Bay Bridge was performed by the use of a model.[28] In the model tests various combinations of uniformly distributed and concentrated loading may be considered. Furthermore, the stiffening truss may be considered either with uniform or variable moment of inertia along the span length.

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[26] "Modern Framed Structures," by the late J. B. Johnson, and C. W. Bryan, Members, Am. Soc. C. E., and F. E. Turneaure, Hon. M. Am. Soc. C. E., Pt. 2, 1917 Edition, pp. 203–208.

[27] *Bautechnik*, Heft 53, 1929, p. 844.

[28] "Tests on Structural Models of Proposed San Francisco-Oakland Suspension Bridge," by G. E. Beggs and R. E. Davis, Members, Am. Soc. C. E., and H. E. Davis, Jun. Am. Soc. C. E., *Publications in Engineering*, Univ. of California, Vol. 3, No. 2, pp. 59–166, 1933.

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DISCUSSIONS

WATER-HAMMER PRESSURES IN COMPOUND AND BRANCHED PIPES

Discussion

By Messrs. F. Knapp, Martin A. Mason, Pierre F. Danel, and Antoine Craya, and A. A. Kalinske

F. Knapp,[17] Esq. (by letter).[17a]—All engineering theory is based upon certain fundamental hypotheses, and the degree of agreement between the theory and the observed behavior depends on the approximation with which the assumptions made describe the actual facts.

The analytical theory of water-hammer and the graphical method derived from it do not escape this necessity. Although the basic assumptions are of such a nature as to give excellent agreement between test and computation for the more common cases occurring in practice (and, as a matter of fact, this agreement is better than is actually required and hoped for in most hydraulic computations), it is well to remember, for exceptional cases, that the elastic surge theory assumes proportionality between stress and strain; that is, the validity of Hooke's law. Furthermore, without a more complete theory than has been published to the present, it is not possible to compute the surge conditions affected by cavitation of the water-column; that is, the subnormal pressures approximating the vapor pressure of the water.

The theory, furthermore, cannot account for the frictional losses as they occur, distributed along the pipes. Several approximate (and graphical) methods, however, have been devised in order to overcome this difficulty.

As far as the first mentioned hypothesis is concerned (the proportionality between stress and strain) the usual theory considers only cases with surges of such a magnitude as to remain well below the yield point of the pipes. There is no difficulty in completing the theory by replacing the observed stress-strain relation above the yield point of the pipe by a series of tangents

Note.—The paper by Robert Angus, Esq., was published in January, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1938, by Messrs. K. J. De Juhasz, and Harold A. Thomas.

[17] With the São Paulo Tramway Light & Power Co., São Paulo, Brazil; Assoc. Member, Waterhammer Committee, A. S. M. E., representing Brazil.

[17a] Received by the Secretary May 11, 1938.

or secants. The theory thus completed furnishes several important indications for the hydraulic and mechanical design of pipe lines.

The author's example of the "breach of the water column due to low pressures" is based on several assumptions that are neither corroborated by experimental verification nor by a careful examination of the subnormal pressure conditions in the pipe line. As stated by the author, the graphical method avoids the tedious tracing of the various pressure waves and the complicated study of various reflection factors, as these have all been included automatically in the construction. It is just this facility of the construction which may lead to considerable errors and for this reason the writer[14] treated, at length, the surge conditions in a pipe line by means of reflection and transmission factors combined with the "wave-plan." Although tedious, such a study represents an excellent basis for the complete understanding of the graphical method of surge computations.

Referring to Fig. 18 of the paper, the line connecting the points, $A_{0.6}$ and $C_{6.6-16}$, indicates a negative pressure wave with a magnitude of 100 ft; the author assumes this wave to travel up the flatter part of the pipe line without causing disturbances of any kind. He also assumes this wave to be reflected totally, but with opposite sign, at the intake. According to Fig. 18, the pressure head at the intake is only a small fraction of the magnitude of the negative surge wave and this means that the water column between Sections *B* and *C* is subject to cavitation conditions. It follows, also, that the maximum subnormal pressure in the upper flatter section of the pipe line for any section cannot be greater than the difference between the static water level and the elevation head of the section considered and an additional amount given by the prevailing barometric pressure minus the vapor pressure of the water. The wave thus travels up the line with diminishing magnitude in accordance with the profile of the pipe line. As a consequence, the reflection at the intake occurs with only a fraction of the original wave.

The column of water, subject to subnormal pressure conditions, releases, partly, the air contained in it. When the wave returns, the elastic properties are found to have changed to such an extent that the surge velocity is considerably less than before. Again, reaching Section *B*, the original pressure conditions are being restored, and it should be noted that during this process the atmospheric pressure plays an important part.

The author assumes that Section *B* represents a point of total reflection and, consequently, treats the two columns, $A\,B'$ and $B''\,C$, separately. This assumption is not justified; nor has it been corroborated by actual experimental verification. The conditions in the pipe line occur in accordance with simple physical considerations as described herein. Without anticipating the results of a probable future publication regarding this special aspect of the theory of water-hammer, the writer would remark that it is not surprising to find that an approach to a difficult problem such as that presented by the author (under assumptions that are fundamentally incorrect) gives results in accordance with these assumptions.

[14] "High Head Penstock Design," by A. W. K. Billings, M. Am. Soc. C. E., O. H. Dodkin, F. Knapp, and A. Santos, Jr., Assoc. M. Am. Soc. C. E., First Symposium on Waterhammer, 1933. Limited Special Edition; distributed by A. S. M. E.

In 1935, the writer called attention[18] to the incorrect solution of the problem under discussion, first proposed by O. Schnyder,[19] later, by M. L. Bergeron,[20] and now reviewed by the author.[21]

Referring to the text following Equation (40), it may be interesting to indicate the solution of branch-pipe problems in a different manner. Simultaneous equations for Branches (*a*) and (*c*) may be written as follows:

$$h_{At-2} - h_{aBt} = 2\,\rho_a\,(v_{At-2} - v_{aBt}) \dots\dots\dots\dots\dots (74a)$$

and,

$$h_{Ct-4} - h_{cBt} = -\,2\,\rho_c\,(v_{Ct-4} - v_{cBt}) \dots\dots\dots\dots\dots (74b)$$

From Equation (74*a*) it follows that:

$$v_{aBt} = v_{At-2} - \frac{1}{2\,\rho_a}\,(h_{At-2} - h_{aBt}) \dots\dots\dots\dots\dots (75a)$$

and, from Equation (74*b*):

$$v_{cBt} = v_{Ct-4} + \frac{1}{2\,\rho_c}\,(h_{Ct-4} - h_{cBt}) \dots\dots\dots\dots\dots (75b)$$

Introducing Equations (75) into the equation of continuity and considering, also, the relation expressed by Equation (35), the following is obtained:

$$\left[\frac{\rho}{\rho_a}\,h_{At-2} + \frac{\rho}{\rho_c}\,h_{Ct-4}\right] - h_{Bt} = -\,2\,\rho\,[\{v_{At-2} + v_{Ct-4}\} - v_{bBt}] \dots (76)$$

in which,

$$\rho = \frac{\rho_1\,\rho_2}{\rho_1 + \rho_2} \dots\dots\dots\dots\dots\dots\dots\dots\dots (77)$$

Equation (76) has exactly the same structure as the simultaneous Equations (26). The slope of the line, defined by $-\,2\,\rho$, is constant and equal to that of the line, $A_0\,M$, in Fig. 12. As a matter of fact, Equation (76) represents a momentary "end condition," replacing the two branches, (*a*) and (*c*), temporarily, by a single line. In practice, and in order to speed up the application, it is more convenient to start the line with the slope, $-\,2\,\rho$, at a point corresponding to the velocity in the surge tank at the wye with the pressure-ordinate, $h = 1.0$, instead of computing the starting point of this line by means of Equation (76). This relation, of course, may be extended so as to be applicable to any number of branches, meeting in one single point.

The treatment of the draft-tube problem, given by the author, is most interesting. The writer would only remark that, in practice, the change of the speed of the unit would also have to be considered in the computation.

As far as the "large and erratic" pressure variations at Points *A*, *B*, and *D* of Fig. 15 are concerned, it becomes evident from a closer study that this

[18] "Ueber eine allgemeine graphische Berechnungsmethode der Druckstoesse in Rohrleitungen," by F. Knapp, *Wasserkraft und Wasserwirtschaft*, 1935, p. 279; also, "Operation of Emergency Shut-Off Valves in Pipelines," by F. Knapp, *Transactions*, A. S. M. E., November, 1937.

[19] "Ueber Druckstoesse in Rohrleitungen," by O. Schnyder, *Wasserkraft und Wasserwirtschaft*, 1932, p. 49.

[20] "Pompes centrifuges e usines elevatoires," by M. L. Bergeron, *La Technique Moderne*, 1935, No. 5.

[21] The same criticism applies to the author's paper, "Air Chambers and Valves in Relation to Waterhammer," *Transactions*, A. S. M. E., November, 1937.

unsatisfactory behavior is due to the unstable characteristic of the valve at Section D, assumed by Professor Angus. Certain valves exist with such a head-velocity relation, but in practice the effect is not as pronounced as is assumed in Fig. 15. It is certainly not desirable, as proposed by the author, to make such an installation stable by the addition of a stand-pipe or surge tank, because the engineer who designs such a layout would choose a valve with a satisfactory characteristic.

In spite of the fact that he treats the basic relations in considerable detail, the author still uses the old-fashioned method of developing Equations (10) by means of differential equations. Their development, by simple physical considerations, was demonstrated by the writer in 1937.[22]

In spite of the writer's criticism of the paper, the author deserves the thanks of the profession for having brought to the attention of a greater circle of engineers a subject of lively interest, heretofore limited to a small number of specialists.

Martin A. Mason,[23] Jun. Am. Soc. C. E., Pierre F. Danel,[24] Assoc. M. Am. Soc. C. E., and Antoine Craya,[25] Esq. (by letter).[25a]—There are a few points of importance omitted from Professor Angus' excellent paper which should be discussed as concerning the accuracy of the graphical method and as showing a further liaison between it and the analytical method of studying water-hammer phenomena.

Consider, first, the case of a compound pipe consisting of a number of dicontinuities provided by changes in section, as that studied by Professor Angus in Fig. 11. At each of these discontinuities there is a loss of head due to kinetic energy losses, depending upon the sense and magnitude of the flow, and due directly to the change in character of the conduit. This loss may be evaluated[26] as $\left[1 - \left(\frac{d_1}{d_2}\right)^2\right]^2 \frac{V_1^2}{2\,g}$ in the case of an enlargement of section, and as $\left(\frac{1}{c} - 1\right)^2 \frac{V_2^2}{2\,g}$ in the case of a contraction. The effect of these losses, whether due to contraction or enlargement, is always the same in that the pressure indicated by the graphical method is not the true pressure at the point being studied for both sections of the conduit. It is the true pressure, plus or minus the kinetic energy correction, for one of the sections; for instance, in the case treated by Professor Angus, with flow from Points D to A (that is, from the reservoir to the gate), there is a change in pressure at Points C and B due to the change in section, which is not shown in Fig. 11. The true pressure may be found in the following manner: Consider Section $A\ B$ of the conduit, with a change in régime introduced by the gate closure. Then, the pressure-

[22] "O golpe de ariete: Theoria, confirmação experimental e applicaçœs praticas," by F. Knapp, *Boletin* da Inspectoria de Serviços Publicos, São Paulo, November, 1937.

[23] Freeman Scholar, Boston Soc. of Civ. Engrs., Grenoble, France; now Scientific Aide and Jun. Mech. Engr., National Bureau of Standards, Washington, D. C.

[24] Research Engr., Ateliers Neyret-Beylier et Piccard Pictet; Director of Hydr. Laboratory, Ecole des Ingénieurs Hydrauliciens, Univ. of Grenoble, Grenoble, France.

[25] Research Engr., Ateliers Neyret-Beylier et Piccard Pictet, Grenoble, France.

[25a] Received by the Secretary May 24, 1938.

[26] "Fluid Mechanics," by R. A. Dodge and M. J. Thompson, McGraw-Hill Co., 1937.

velocity relations in Section $A\ B$ at some time, i, later are shown by the régime points, A_i and B_i. In Section $B\ C$ of the conduit, however, the true pressure at Point B is not that shown by Point B_i since the loss of kinetic energy due to the sudden contraction at Point B has been considered. Therefore, to find the true

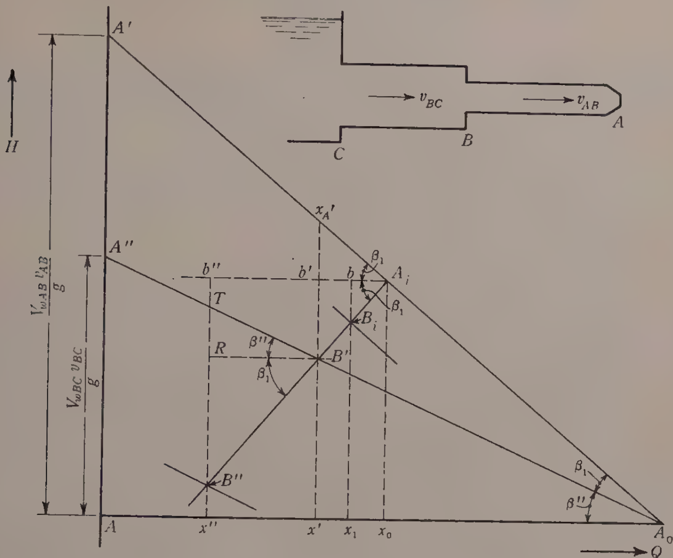

FIG. 24

pressure at Point B, Section $B\ C$, add to the pressure shown by Point B_i the head lost in contraction. In Fig. 24 this may be accomplished by extending Point B_i vertically upward a distance equal to $\left(\frac{1}{c} - 1\right)^2 \frac{V_{AB}^2}{2\,g}$ times the pressure scale of the graph. Then the characteristic lines representing the pressure-velocity relations in Section $B\ C$ should start, not from Point B_i but from the corrected Point B_i. The same correction is necessary for each change in section.

As noted by Professor Angus, and as more fully discussed elsewhere,[27] these terms may usually be neglected. If they are neglected, however, it should be remembered (particularly in regard to the pressure conditions at intermediate points in the conduit) that the results are not exact. Admittedly, the error in the final result is often very small. It is obvious that as the velocity head term, $\frac{V^2}{2\,g}$, increases in ratio to the pressure head the error becomes larger.

In connection with corrections of this kind it should be noted that conditions frequently occur in practice which demand care in their interpretation as to the necessity for applying the foregoing corrections. Professor L. Bergeron has shown[27] that sometimes a correction need not be applied for flow in one sense, but for flow in the other sense a correction is required. For instance, he

[27] Contributions to discussion of the papers by Messrs. L. Allievi, J. N. LeConte, and R. T. Knapp, at the Water Hammer Committee Meeting, A. S. M. E., 1937, by Prof. L. Bergeron and Martin A. Mason, Assoc. M. Am. Soc. C. E. (Publication pending.)

discusses the case of a nozzle opening into an air chamber, and the case of a simple pipe entrance. In the first case, the correction to be applied for reversed flow is four times that for normal flow; whereas, in the second case, no correction is required for flow in the normal sense (there being a negligible loss at the entrance); but, for flow in the reverse sense, correction must be made for the exit losses.

The study of water-hammer conditions in compound pipes by the graphical method offers a great advantage over the analytical method in that many of the factors that must be calculated in the latter method are automatically considered in the graphical method. Among these terms may be mentioned the so-called "coefficients" of reflection and transmission of the pressure waves, and in case the problem is being studied by the theory developed by Jaeger,[7] a third coefficient, α_i.

These terms may be easily found from the graphs drawn for a particular case, although their calculation is not at all necessary to the use of the graphical method. Fig. 24 is the graph for a simple case of a compound pipe consisting of two sections; the line, $A_0\,A'$, represents the pressure-velocity relation in Section $A\,B$, and Line $A_0\,A''$, the same relation as for Section $B\,C$. For simplicity, a pipe of only two sections has been chosen as the same proof applies equally well to compound pipes of any number of sections.

The construction of the graph by the graphical method shows the magnitude, F_i, of the direct pressure wave in Section $A\,B$, leaving End A at Time i, to be the length, $\overline{X_0\,A_i}$, in Fig. 24. Likewise, the magnitude of the reflected wave, f_i, from Point B, Section $A\,B$, is equal to the length, $\overline{-B'\,b'}$; and, the magnitude of the direct pressure wave, F', in Section $B\,C$, is equal to the length $\overline{X'\,B'}$, assuming that there are no reflected waves as yet in Section $B\,C$. Prolonging $X'\,B'$ to its intersection, X_A', with the line, $A_0\,A'$, $\overline{b'\,X_A'} = \overline{B'\,b'} = -f_i$; from which,

$$\overline{X'\,B'} = \overline{b\,X_1} - \overline{b'\,B'} = F_i - (-f_i) = F_i + f_i \quad \text{.........(78a)}$$

and,

$$\overline{X'\,X_A'} = \overline{b\,X_1} + \overline{b'\,X_A'} = F_i + (-f_i) = F_i - f_i \quad \text{.......(78b)}$$

Considering, however, the triangle, $A_0\,A'\,A''\,A$, cut by the parallel lines, $A\,A'$ and $X'\,X_A'$:

$$\frac{\overline{X'\,B'}}{\overline{X'\,X_A'}} = \frac{\overline{A\,A''}}{\overline{A\,A'}} = \frac{V_{wBC}\,v_{BC} \div \frac{1}{g}}{V_{wAB}\,v_{AB} \div \frac{1}{g}} = \frac{\frac{V_{wBC}}{A_{BC}}}{\frac{V_{wAB}}{A_{AB}}} \quad \text{.............(79)}$$

in which A represents the cross-sectional area of the conduit considered. Substituting the values of Equations (78) into Equation (79):

$$\frac{F_i + f_i}{F_i - f_i} = \frac{\frac{A_{AB}}{V_{wAB}}}{\frac{A_{BC}}{V_{wBC}}} = \frac{1 + \frac{f_i}{F_i}}{1 - \frac{f_i}{F_i}} \quad \text{..................(80a)}$$

[7] "Theorie General du Coup de Belier," by Charles Jaeger, Dunod, Paris, 1933.

and solving for $\frac{f_i}{F_i}$,

$$\frac{f_i}{F_i} = \frac{\frac{A_{AB}}{V_{wAB}} - \frac{A_{BC}}{V_{wBC}}}{\frac{A_{AB}}{V_{wAB}} + \frac{A_{BC}}{V_{wBC}}} = r \ldots\ldots (80b)$$

which is recognized as the coefficient of reflection used by Jaeger, Loewy, Comte de Sparre, and others.

Furthermore, since $\overline{X' B'} = F' = \overline{X_1 b} - \overline{b' B'} = F_i + f_i$:

$$\frac{F'}{F_i} = 1 + \frac{f_i}{F_i} = \frac{\frac{2\,A_{AB}}{V_{wAB}}}{\frac{A_{AB}}{V_{wAB}} + \frac{A_{BC}}{V_{wBC}}} = s \ldots\ldots (81)$$

which is the coefficient of transmission of Section $A\ B$ into Section $B\ C$, applied in a manner similar to Equation (80*b*).

Consider, now, that there is a reflected wave, f', in Section $B\ C$ (Fig. 24). For this case it is known by construction that Line $A_0\ A''$ has been displaced parallel to itself a vertical distance equal to twice the magnitude of the reflected wave; that is, $2\,f'$. Therefore,

$$\overline{B''\ b''} = \overline{B''\ R} + \overline{R\ b''} = \overline{B''\ R} + \overline{B''\ b'} = \overline{B''\ R} - f_i \ldots\ldots (82a)$$

and since $\overline{B''\ T} = -\,2\,f'$:

$$\overline{B''\ R} + \overline{R\ T} = -\,2\,f' \ldots\ldots (82b)$$

From the similar triangles, $T\ R\ B'$ and $A''\ A\ A_0$, $B''\ R\ B'$ and $A'\ A\ A_0$, one finds,

$$\frac{\overline{R\ T}}{\overline{B''\ R}} = \frac{\overline{A_0\ A''}}{\overline{A_0\ A'}} = \frac{\frac{A_{AB}}{V_{wAB}}}{\frac{A_{BC}}{V_{wBC}}} \ldots\ldots (83a)$$

and substituting Equation (83*a*) into Equation (82*b*):

$$\overline{B''\ R} = -\,f' \left(\frac{2\,\frac{A_{BC}}{V_{wBC}}}{\frac{A_{AB}}{V_{wAB}} + \frac{A_{BC}}{V_{wBC}}} \right) = -\,f'\,s' \ldots\ldots (83b)$$

in which s' is the coefficient of transmission of Section $B\ C$ into Section $A\ B$.

To find Jaeger's coefficient, α_i, it is necessary only to consider Equations (83*b*) and (80*b*) with respect to Equation (82*a*), or,

$$\overline{B''\ b''} = r\,F_i + s'\,f' \ldots\ldots (84a)$$

and, dividing by F_i,

$$\frac{\overline{B''\ b''}}{F_i} = r + s'\,\frac{f'}{F_i} = \alpha_i \ldots\ldots (84b)$$

in which α_i, Jaeger's coefficient, represents the ratio of $\overline{B'' b''}$ (the reflected wave in Section A B due to reflection in Section B C) to the direct wave in Section A B.

Professor Bergeron, who with Dr. O. Schnyder, is mainly responsible for the rapid development of the graphical method of study of water-hammer phenomena, has shown[28] that the method may be applied successfully in fields other than that of hydraulics. In fact, theoretically, this method of study of pressure-velocity relationships may be adapted to any case in which the disturbing waves are propagated as plane waves. Thus, it can be used to study the propagation of plane waves in a stretched chord, longitudinal waves in a metal bar, torsional waves in a rotating cylinder, and the study of transients in electric lines.

However, in many of these cases occurring in domains other than hydraulics (as well as in the field of hydraulics itself) difficulties are frequently introduced in the use of the method by the physical conditions under which the disturbance occurs. The principal of these difficulties is that of determining, at some time after the beginning of the disturbance, the characteristic curve of the pressure-velocity relations imposed by the disturbing device. In most cases this curve must be calculated by analytical means, as in the case of a surge tank consisting of an air reservoir, where, by means of Boyle's law, the pressure-velocity relations at any instant may be calculated. Unfortunately, this computation introduces, into the use of the graphical method for such cases, considerable additional work which it would be desirable to avoid. This phase of the problem of graphical solution of water-hammer problems is worthy of considerable study by those engineers who frequently use the graphical solution method.

A. A. Kalinske,[29] Esq. (by letter).[29a]—This paper, with previous writings by the author (which he lists in footnotes), provide, for American engineers, complete information on the solution of water-hammer problems by the graphical method. The writer suggests that any one wanting to become thoroughly familiar with this method of analysis should also consult the papers that are cited.

The presentation of a graphical method of analysis in a technical paper is extremely difficult as the author usually cannot get the reader to follow through, completely, the details of the graphical construction. Unless the reader does this, patiently and diligently, he will not grasp the principles of the graphical method. Furthermore, he is likely to use this method of analysis just as a tool, without any knowledge of the physical phenomena with which he is dealing.

The writer believes that the paper would be more lucid if Professor Angus had devoted additional space to showing what the graphical method of analysis really is; namely, a method for solving simultaneous equations. There is

[28] Mémoires de la Société des Ingénieurs Civils de France, *Bulletin*, July–August, 1937.

[29] Instructor in Hydraulics, and Research Engr., Coll. of Eng., State Univ. of Iowa, Iowa City, Iowa.

[29a] Received by the Secretary August 10, 1938.

nothing mysterious about it; it is simply one method of solving, step by step, simultaneous equations that relate various phenomena which occur together. In fact, these simultaneous equations can be solved analytically just as well as graphically. Furthermore, only after solving a few problems analytically by a step-by-step numerical solution will the physical phenomena be entirely clear to any one who is just beginning the study of water-hammer. For example, when using the graphical method outlined by the author for compound pipes, the physical phenomena occurring at pipe junctions are completely obscured. The graphical method is a useful tool and is very convenient for some problems; it is unnecessary and cumbersome for certain others, and it is practically never indispensable. Some problems may be solved most rapidly by a combination of the numerical and graphical methods. An engineer interested in this problem should be able to use either method.

If an engineer knows how to write the different simultaneous equations that relate and control the various phenomena occurring in water-hammer, he will have no difficulty solving the problem. He may use a numerical or a graphical method, but, in either case, he should first have a correct understanding of what he is doing. The writer believes that Professor Angus might well have explained in more detail how to set up the various simultaneous equations needed to solve the more complicated piping problems, and thus have generalized the method of attack to be used in solving any particular problem.

In general, the solution of any water-hammer problem involves the formulation of the following: (1) Relation between the head and the velocity at the control gate; (2) relation between the head and velocity at one end of a section of uniform conduit, and the head and velocity at the other end of the same section taken an instant earlier than the first (this interval of time being equal to the time required for the pressure wave to travel the length of the conduit section); and, (3) discharge relationships at the junction of pipe branches.

As Professor Angus states, if friction loss is not neglected, the solution of any problem is considerably complicated. However, he minimizes the number of problems in which friction must be considered. In most water-supply pipe systems and pump discharge lines, the head lost due to friction is far from negligible. In many such problems the flow head may be only a small fraction of the static head. This fact is of great importance in the design of relief devices. By considering the friction loss as concentrated at a single point or points, an approximate solution can be obtained. However, such a procedure is far from satisfactory.

Probably the best physical pictures of the effect of friction on the form of the pressure wave for instantaneous closures are those given by W. F. Durand[30] and H. A. Gibson.[31] A complete, satisfactory explanation of the

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[30] "Hydraulics of Pipe Lines," by W. F. Durand, D. Van Nostrand, New York, N. Y., 1921, p. 89.

[31] "Mechanical Properties of Fluids," by H. A. Gibson, Blackie & Sons, London, England, 1925, p. 213.

effect of friction for gradual gate closures does not seem to exist. In a paper published in 1937, Allievi[32] states:

"During the perturbed regimen, moreover, the velocity of each single liquid filament is varying with time, and, at the same instant, is different for each filament. Therefore, the effect of liquid friction is transmitted to the lowest filament which we are considering, according to very complex laws in which it is legitimate to assume that the velocity, V_i, of the lowest identical filament plays a predominant part."

Probably the only way to obtain a correct idea as to the part that friction plays in altering the results obtained when it is neglected, or when it is considered as localized at various points, is through experiments. In fact, the writer believes that any further knowledge that is to be gained regarding water-hammer will have to be obtained from carefully conducted experiments. Mathematical and graphical analyses have solved most of the general features of the water-hammer problem; it is now time to investigate some of the minor assumptions that have been made. Some other special features of the problem, which merit experimental investigation, are the effects of sharp bends in pipes, gradual and sudden pipe transitions, partly closed valves, and the characteristics of various relief devices, including those that store up energy and those that dissipate it partly.

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[32] "Air-Chambers for Discharge Lines," by Lorenzo Allievi, *Transactions*, A. S. M. E., Vol. 59, 1937, p. 655.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

ENGINEERING ECONOMICS AND PUBLIC WORKS

A SYMPOSIUM

Discussion

BY MESSRS. LOUIS E. AYRES, JOHN H. MEURSINGE, AND SAMUEL B. FOLK

LOUIS E. AYRES,[91] M. AM. SOC. C. E. (by letter).[91a]—It is unfortunate that no definite pronouncement, allocating expenditures for power purposes, has ever been made by the Tennessee Valley Authority (TVA). Lacking such pronouncement, Professor Mead calls attention to certain estimates in "testimony before the House Appropriations Committee"[11] (see heading, "Government Interference as a Permanent Policy: Power") in 1936 wherein the "estimated annual power expense" of \$12 191 000 was arrived at by taking 9% of \$135 450 000, the investment in "power houses and power facilities" only. Obviously, such an assumption will not generally be accepted as a basis for determining a power production "yardstick."

An engineer would normally analyze the basic data in quite a different manner. The usual method to obtain the investment base which may be supported by a given gross income is to divide the gross income by the percentage of annual charges. Those acquainted with water power costs will generally agree that a total of 10%, with interest at 6%, is usually adequate for annual charges in the case of large, privately owned water powers; and if interest is assumed at 3.5%, the annual charges become about 7.5 per cent. In the case, however, of such projects as those constructed by TVA, with possibly as much as one-half the project cost invested in reservoirs (including highway and railroad relocations, which involve very little maintenance and operation and no depreciation), one may use a lower percentage for annual

NOTE.—This Symposium was presented at the meeting of the Engineering-Economics and Finance Division, Boston, Mass., October 7, 1937, and published in February, 1938, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: March, 1938, by J. K. Finch, M. Am. Soc. C. E.; May, 1938, by Messrs. Elliott J. Dent, C. Frank Allen, Bradley G. Seitz, Alfred Allen Stuart, Pierce P. Furber, R. F. Bessey, Donald M. Baker, and Philip W. Henry; and June, 1938, by Messrs. H. K. Barrows, Harry A. Wiersema, J. D. Galloway, E. S. Martin, and K. Bert Hirashima.

[91] Cons. Engr. (Ayres, Lewis, Norris & May), Ann Arbor, Mich.

[91a] Received by the Secretary May 27, 1938.

[11] First Deficiency Appropriations Bill, March, 1936.

charges than in projects involving relatively larger proportions of the investment in structures and equipment. A value as low as 6.5% was used by H. K. Barrows, M. Am. Soc. C. E.,[92] in computing fixed charges on Boulder Dam, Bonneville, Grand Coulee, and the TVA projects. In the writer's opinion 7% is sufficient, under TVA conditions, to cover all charges, including interest, depreciation, taxes (as assessed under private ownership), insurance, operation, and maintenance. Furthermore, if the "estimated gross annual wholesale power revenue" of $23 120 000 is divided by 0.07, the investment base, or the production "yardstick" of TVA becomes $330 000 000.

However, Professor Mead questions the TVA estimates of annual income, first as regards the amount of output that may be utilized, and second as regards the average unit revenue that may be obtained therefrom, based on the rate schedules now in effect.

As to saleable output, Professor Mead states (see heading, "Government Interference as a Permanent Policy: Power") that the "estimated output is based on continuous efficiencies of more than 80%," and infers that "other losses due to plant use, transmission losses, transformer losses, and losses due to regulation for navigation and flood prevention" have been overlooked. This is not in accordance with the writer's understanding. In the TVA report to Congress, March, 1936, entitled "The Unified Development of the Tennessee River System," the estimate of 660 000 kw of continuous capacity was qualified by the sentence: "By continuous power is meant the dependable power which would be available 24 hours of the day and 365 days of the year after suitable deductions for expected and unavoidable water and energy losses." The writer understands that these deductions included restrictive assumptions as to flood control, allowances for navigation, evaporation, leakage, and seepage, and a final utilization factor of 90 per cent. In effect, all deductions, together with normal allowances for unit efficiencies, approximated an assumption of utilization at the switchboard of less than two-thirds of the theoretical power of all firm water, which is a somewhat more conservative basis than Professor Mead's 80% of 80% or 64%, including transformer and transmission losses.

As to average annual unit revenue, Professor Mead presents, incompletely, the TVA wholesale rate schedule and concludes: "From these rates it is evident that the annual returns, if and when markets are found for the entire output, cannot average as much as 3 mills per kw-hr." Such a conclusion can hardly be drawn, from the rate schedule considered, as the demand charge alone of 90 cents per kw per month amounts to 2.46 mills per kw-hr, at 50% load factor, and the lowest step in the energy charge is 2.0 mills per kw-hr. Hence, wholesale power, purchased in large blocks under this schedule, would cost more than 4.46 mills per kw-hr, if a load factor of 50% obtains, as assumed by Professor Mead. One must assume 100% load factor to obtain a value approximating 3 mills, and then only because energy purchased in any month in excess of 360 times the demand is subject to a reduction of 0.5 mill per kw-hr. Also, energy purchased at the Authority's switchboard is subject to a further reduction of 10 per cent. These conditions of the schedule could account for

[92] *Proceedings*, Am. Soc. C. E., April, 1938, p. 698.

the press dispatches of sales to "a chemical company," referred to by Professor Mead, of "firm power" at 2.74 mills per kw-hr. However, no one will assume that any considerable portion of TVA output will be sold at 100% load factor. If it is used to the best advantage eventually much of the output will be produced at 40% load factor or less, and will return a unit income of 5 mills or more. That firm power can be produced at 40% load factor or less may be judged from the fact that the total installed machinery contemplated is 1 922 000 kw, or slightly less than three times the estimated 660 000 kw of 24-hr firm power. Hence, it would seem that the TVA estimate based on 4 mills per kw-hr for firm output is conservative, having in mind the best ultimate use of this water power.

In addition to the firm output, however, there will be a considerable production of secondary power. Professor Mead is "aware that contracts have already been made to supply a considerable quantity of such power." Why should the income from secondary power be neglected? It would not be overlooked in a private undertaking. Professor Mead refers to press dispatches quoting sales of "12 000 kw of 'run-of-river power,' at 2.27 mills per kw-hr." Although there seem to be no published estimates of the amount of this secondary power, it would probably be not less than 2 000 000 000 kw-hr annually and would have a potential value of from 1 mill to 2 mills per kw-hr.

Assuming an average return of 5 mills per kw-hr for firm power and 1.5 mills per kw-hr for secondary power, the gross annual income amounts to about $32 000 000, which sum, capitalized at 7%, results in an investment base of $457 000 000; or, assuming 4 mills per kw-hr only for firm power and 1 mill only for secondary power, the annual income is computed to be about $25 000 000 and the investment base, $360 000 000, which is still in excess of Professor Mead's estimate of $353 150 000 as the proper "total cost of power installation."

An important hurdle in the path of any general agreement as to a proper power production "yardstick" is the allocation of total expenditures to the several purposes of navigation, flood control, and power. In determining his allowance for navigation, Professor Mead refers to the Army Engineers' estimate[15] of $74 709 000 for "navigation works with low locks and dams," agrees "that the higher dams will possibly improve navigation facilities," and concludes that "say, $90 000 000 for this purpose might be warranted." More recent estimates of the cost of a 9-ft channel, from Paducah, Ky., to Knoxville, Tenn., to the same standards as those used on the Upper Mississippi and Ohio Rivers, are much higher than the older estimates. In January, 1938, C. T. Barker, Assoc. M. Am. Soc. C. E., Head of the Navigation Section of TVA, testified in the constitutionality case in Chattanooga, Tenn., that to secure a navigable channel meeting present standards and requirements, by means of a series of low dams, would cost $144 072 700. If one assumes, therefore, that navigation may properly be charged with an amount substantially equal to what it would cost to provide the equivalent facilities by works constructed solely for that purpose, Professor Mead's allowance should be increased by at least 60 per cent. Obviously, the low-dam scheme would

[15] H. R. Doc. No. 328, 71st Cong., 2d Sessions, p. 5.

involve greater operating, maintenance, and dredging costs than the high-dam scheme which is being built, and would be less desirable for this and other reasons.

In the matter of flood protection, it may be recalled that the Army Engineers estimated that about \$45 000 000 would prevent the maximum possible damage on the Tennessee River. Professor Mead allows \$36 000 000 only, although he adds that "indirect damages might warrant some additional expense." In the "Comprehensive Report on Reservoirs in Mississippi River Basin,"[93] there is a list of 21 reservoir sites in the Tennessee Valley which would provide 10 188 100 acre-ft of flood storage at a cost of \$167 282 000, or \$16.40 per acre-ft. This is part of a list of 81 reservoirs in the Ohio Valley with a total storage capacity of 25 285 400 acre-ft at \$20.40 per acre-ft; part of a total of 106 reservoirs up stream from Cairo, Ill., with 52 853 400 acre-ft, at \$15.70 per acre-ft; and part of a grand total of 151 reservoirs, above the latitude of Red River Landing, La., with a total capacity of 98 677 900 acre-ft, at \$11.40 per acre-ft. If one then assumes a value as low as \$11.40 per acre-ft for the approximately 9 000 000 acre-ft of flood-control storage, provided by the TVA projects, the total is about \$100 000 000.

Professor Mead quotes Mr. Morgan as testifying[16] in 1936 that the "value of the flood protection feature will be worth \$100 000 000 to flood protection on the Lower Mississippi River." This value was based on the cost of raising Mississippi levees 3 to 4 ft, or the estimated lowering effect of TVA dams on a maximum Mississippi River flood. Carl A. Bock, M. Am. Soc. C. E., Assistant Chief Engineer, TVA, has stated[94] that the reservoir value of Gilbertsville alone "in protection to lands and improvements in the upper portion of the alluvial valley of the Mississippi River has been conservatively estimated to be approximately \$90 000 000." Professor Mead, however, discards these benefits as problematical and outside the "primary objective of TVA," although under the TVA Act, as amended, the Authority is empowered "to construct such dams and reservoirs as * * * will best serve to * * * control destructive flood waters in the Tennessee and Mississippi River drainage basins."

On the basis of the foregoing assumptions, one might evaluate the multiple-purpose project as follows:

Purpose		Total	Percentage of total
Power		\$360 000 000	56.2
Navigation		144 000 000	22.5
Flood control		136 000 000	21.3
Tennessee Valley (according to Professor Mead)	\$36 000 000	....	....
Mississippi Valley (according to Mr. Morgan)	100 000 000	....	....
Total		\$640 000 000	100.0

[93] H. R. Doc. No. 259, 74th Cong., 1st Session, p. 46.
[16] Testimony before Appropriations Committee, 1936.
[94] *Engineering News-Record*, April 7, 1938, p. 500.

The increase in the size of the Gilbertsville and Fontana Dams, since 1936, and added generating units at several plants have raised the estimated total cost of the eleven projects, exclusive of transmission lines, according to testimony before the House Appropriations Committee (Independent Offices Appropriations Bill for 1939), to about $505 000 000; and, if 56.2% of this sum is chargeable to power production, the power "yardstick" becomes about $285 000 000. In the writer's opinion, a proper allocation to power, exclusive of transmission line costs, should not exceed $300 000 000. Such an allocation would be $60 000 000 less than the investment base, indicated herein, upon which the gross estimated income, from a fully developed market, would pay all charges; and, hence, $60 000 000 might be assumed as available for interest deficits over a period of market development. In other words, on this basis, if the market could be fully developed by, say, 1950, the projects would pay all charges on a fair power "yardstick," assuming 7% for annual charges and the sale of all firm output at 4 mills and all secondary output at 1 mill per kw-hr.

Professor Mead states that: "Steam power can certainly be generated in the Tennessee Valley for not to exceed 4 mills per kw-hr." If this statement is premised on a load factor of 50%, as one may infer from the context, it is believed by the writer to be too low if all costs are considered. Units have been added to plants where the incremental cost was 4 mills per kw-hr at 50% load factor, and plants containing one or two large units, without reserve, may be as low as 4 mills per kw-hr; but a more representative cost of steam power in this area, with all charges considered and adequate reserve provided, can be shown to be 5 mills or more for energy at 50% load factor, construction costs at $85 per kw of installed capacity, and coal at 12 cents per million Btu.

Thus, the engineer may conclude that the Tennessee Valley projects are sound, based on the usual analyses. The construction work done is certainly excellent, as one might expect from the competent engineering personnel that has designed and constructed the projects. The cost of producing power is certainly comparable with, if not less than by, steam substitution; and, the value of the energy will be enhanced as it is utilized in a more fully developed market. The principal economic difficulty involved at present would appear to be one of market, and that in turn is largely dependent upon the settlement of a controversy. One may not condemn the TVA on the basis of switchboard costs. The real TVA controversy is not at the switchboard but at the consumer's meter. One deals in mills at the switchboard, but in cents at the consumer's meter. Most of the "yardstick" is beyond the switchboard. It would seem to be a safe assumption that if the TVA output were allowed to be sold at the switchboard to existing utilities, on the basis of existing TVA rate schedules and without "strings attached" to its use, substantially the entire output would soon be disposed of, to the mutual advantage of both the Government and the purchasers. However, TVA has not seen fit to stop at the switchboard. It insists on lower rates to the ultimate consumers, and its policy in this regard is the real cause of all the acrimonious controversy.

JOHN H. MEURSINGE,[95] ASSOC. M. AM. SOC. C. E. (by letter).[95a]—For a period of 300 000 yr human beings have tried to make a living by exploiting the natural resources of the earth to the extent that such exploitation did not tax the human intelligence beyond capacity. While this knowledge was growing in power, Man learned to develop new resources; but it was not until rather recently that he awakened to the fact that each newly developed contrivance meant another change in the social organization in which he happened to live.

Man has paid dearly for his insufficient knowledge by disastrous warfares and expensive social experiments. It was by the method of trial and error, and by reshaping his previous efforts that he was carried along the road of progress. The public works policy of the present Administration, for instance, adds another adventure to the long list of hazardous occurrences which mankind has encountered during his eternal journey. This policy will reflect upon the society of to-day.

Although this may seem very important to contemporaries, nevertheless the present public works policy should be considered as nothing but an infinitely small link in an inexhaustible chain of events which has carried technics through five phases (there will be more) to its present peak in the United States (see Table 9).

TABLE 9.—THE RELATION BETWEEN THE EVOLUTION OF TECHNICS AND OF SOCIETY

Phase	Power pressure	Material resource†	Social unit and center of production and distribution	Occupation	Civilizations
1	Seeds, roots, and fruits	Grasses, leaves, and leaf wood	Individual	Jungle man	Ancient tropical cultures
2	Men, animals, and fire	Manure, skin, and bones	Family	Hunter	Indians of North American plains; Magdaleniun cultures in Europe
3	Slaves, agricultural products, and domestic animals	Clays, bricks, and natural stone	City	Farmer	Aztecs, Mayas, Incas; Chinese, Babylonians, Assyrians, Arabs; Egyptians, Phœnicians; and, Greeks and Romans
4	Stone, water, and wind*	Cotton, fir wood, and glass	Nation	Business man	Portugal, Spain, Northern Italy, Holland, France, and England
5	Coal, oil, and natural gas*	Metals	World	Industrial worker	Austria, Germany, United States, and Russia

* Electricity is a power created from the resources of Phases 4 and 5.
† Material resources of Phase 6 are rubber, aluminum, and magnesium.

Phase 1.—According to the best available knowledge, Man's first ancestors lived in the tropics (the Java Man). Seeds (grains), roots (vegetables) and fruits were his food. Grasses, leaves, and leaf wood made up the materials for construction. The social organization consisted of "individuals." The women took care of the children. This type of social organization is rarely found any more. Even in the most remote parts of the jungle, newer phases have shown their reflection upon this kind of society.

[95] Structural Designer, General Petroleum Corporation of California, Vernon, Calif.
[95a] Received by the Secretary July 11, 1938.

Phase 2.—The hunter's period comes next. Man's technical ingenuity had a tremendous influence upon the society in which he lived. He invented tools with which to kill, and the social result was marriage, or the establishment of the "family."[96] To his vegetarian diet the jungle man began to add the meat of the game. Soon he was not confined to the tropics; Man migrated on to the plains where meat, quite often, became his exclusive food. Fire kept him warm and manure, skins, and bones were added to the construction materials. The women were physically unfit for the hunt, and, consequently, they became dependent upon the males. Technics had accomplished its first change in society.

The Indians of the North American plains, and the Magdaleniun cultures which appeared in Europe about 25 000 yr ago are representatives of Phase 2.[97]

Phase 3.—Technical accomplishment kept pace with the ever-growing population. Improved technics made possible the exploitation of plants, animals, and slaves. The new materials—clay, brick, and natural stones—were used for roads, bridges, and houses. An entirely new type of society, which revolved about the farmer, came into existence. Specialization in farming and the instinct of self-preservation gave birth to the "city." Mankind had stopped roaming. The Aztecs, Mayas, and Incas in the Americas; the Chinese, Babylonians, Assyrians, and the Arabs in Asia; the Egyptians and Phoenicians, in Africa; Greeks and Romans in Europe, were the great civilizations of this era (5 000 B.C. to 1 000 A.D.).

Phase 4.—The next step in this evolutionary process was the exploitation of wind and water for power resources; cotton, fir wood, and glass for construction materials. Portugal, Spain, Northern Italy, with Florence and Venice, Holland, France, and England, which countries happened to have the resources of this phase, carried civilization to new peaks. This era has been named eotechnics by Lewis Mumford.[98] Technical accomplishments could prove another change in society; the city had grown into a "nation"; and, as all the countries of this phase had acquired colonies, a new type of occupation had arisen; the trader or business man was added to the jungle man, hunter, and farmer.

Phase 5.—France and England had both shown the signs of a new phase. The underground resources were ready for extensive exploitation. A new type of technical complex, called paleotechnics by the late Professor Patrick Geddes, used coal, oil, and natural gas for power, zinc, copper, nickel, cobalt, iron, manganese, chromium, vanadium, and titanium for construction materials. Judging by contemporary developments, Austria, Germany, the United States, and Russia will make the most of these resources. Their technical accomplishments created the "industrial worker" as a new occupation, and his international relations point to the entire world, instead of to the nation, as his social unit.

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[96] "Kulturgeschichte der Menschheit in ihrem organischen Aufbau," by Julius Lippert. Verlag Ferdinand Enke, Stuttgart, 1886. Partly tr. by Peter Murdock, Assistant Professor of the Science of Society in Yale Univ., under the title, "The Evolution of Culture," Macmillan Co., New York, 1931.

[97] "The New Stone Age in Northern Europe," by John M. Tyler, N. Y., Charles Scribner's Sons, 1921.

[98] "Technics and Civilization," by Lewis Mumford, Harcourt, N. Y., 1934.

Technics developed Man from a jungle man into an industrial worker; it developed his individualism into internationalism. The relation between technics and social organization cannot be denied. Lewis Mumford[98] expresses this relation as follows:

"While each of the phases roughly represents a period of human history, it is characterized even more significantly by the fact that it forms a technological complex. Each phase, that is, has its origin in certain definite regions and tends to employ certain special resources and raw material. Each phase has its specific means of utilizing and generating energy, and its special form of production. Finally, each phase brings into existence particular types of workers, trains them in particular ways, develops certain aptitudes and discourages others, and drains upon and further develops certain aspects of the social heritage."

Although each new phase has always strongly reflected upon the civilizations in existence, the old civilizations were never able to regain their positions as world powers by adopting the methods of the newcomers. Most interesting, however, is the fact that they always reluctantly adopted the new ways, and hence always hastened their own decline. Another interesting phenomenon is the fact that the process has been evolutionary. No sharp lines can be drawn between the phases as they penetrate each other.

The typical worker of each phase believes in the economics of his own phase only. The man of the jungle claims that it is uneconomical to be hampered by women and children in his forays; the hunter claims that it is uneconomical to build the city; the farmer objects to the national policies of taxing imports; and the business man cannot understand why the worker should spend a part of his wages on international causes.

The five phases have not only developed five different societies and five different occupations; they have developed five kinds of economics. Although eotechnics (Phase 4) developed the economics of investment, interests, and profits, the jungle man (Phase 1), the Eskimo (Phase 2), the farmer who wrings his entire living from the soil (Phase 3), and the industrial worker (Phase 5), the world over, are rather indifferent to the economics of dollars and cents.

Here the conclusion may be drawn that these economics are limited to Phase 4. Economics is as much subject to the rules of evolution as anything else. Each of the authors of the Symposium overlooked this phenomenon.

The Symposium resembles the skillful work of a photographer. All the details of the subject have been shown with painstaking accuracy. The exposure has been well timed. Unfortunately, however, the camera has been placed so closely to the subject that only a limited field could be covered. It has been taken from the "eotechnical point" of view (Phase 4).

The science of economics is concerned with the production and distribution of the necessities of life. How has this been done since Man has been on earth? The jungle man searched the vicinity of his abode. The hunter could not leave his home for fear the fire might go out. Hence, the family assisted in production and distribution. In the next phase the specialization in farming made the city the center from which the goods produced were dis-

tributed. In the fourth and fifth phases, the city was replaced, respectively, by the nation and the world.

The production and distribution center could not have grown in size so continually, had it not been for the aid of technics. Engineering accomplishments made possible better and faster production and transportation. Hence, engineering economics should embrace the problem of making "necessary" additions to the production and distribution system. These additions should keep pace with the evolutionary growth of the system.

Unfortunately, the science of the evolution of economics is still so much in its infancy, and it is looked upon with so much suspicion, that it is rather fruitless to discuss engineering economics until some kind of an idea has been formulated as to what kind of economics and technics Phases 5 and 6 will eventually create. Hence, it is impossible to say whether or not the present public works policy (addition to the production system) is economically warranted. Those who criticize and those who acclaim it are simply giving vent to their emotions. It behooves the engineer to approach this problem from a scientific angle only.

Conclusion.—The civilizations of each phase have been dominated by the occupations created by that phase; but this does not mean that the occupations of the preceding phases had no place in their society. Their place was determined by the age of the phase to which they belonged.

To-day, for instance, in the United States the jungle man (Phase 1) and the hunter (Phase 2) have disappeared. The farmer (Phase 3), the business man (Phase 4), and the industrial worker (Phase 5) are still in existence, and, whereas the farmer's influence upon society is on the down grade, that of the industrial worker is on the up grade. The same can be said about the centers of production and distribution. The old evolutionary law is to-day as keenly felt as it was 300 000 yr ago.

During all this time the civil engineer has kept the road of progress in fairly good condition. Whereas the jungle man (Phase 1) was not in need of transportation, the hunter (Phase 2) needed trails; and the farmer (Phase 3) who traveled by wagon or on horseback, needed roads. The eotechnic (Phase 4) people who used sailing ships for hundreds of years needed the civil engineer to build and maintain canals and harbor works.

To-day, the paleotechnical (Phase 5) people are going forward in gasoline propelled engines, and the road of progress is in need of improvement. Grades and curves must be brought up to modern standards to eliminate obstructed views. Increased width should give ample room for the farmer (Phase 3), the business man (Phase 4), and the industrial worker (Phase 5) alike. Danger and speed-limit signs should keep the evolutionary traffic at a moderate speed; and swift justice should be dealt to those dangerous individuals who insist on driving in reverse "back to normal." They endanger the lives of millions of their fellow men.

If the civil engineer will build this "road" properly it will lead to a society in which the social rights of each occupation are determined by the importance of the relative natural resources to society as a whole.

SAMUEL B. FOLK,[99] ASSOC. M. AM. SOC. C. E. (by letter).[99a]—In succinct manner Mr. Fay has shown the advantages of orderly planning. The next logical step is to note what can be planned. The Administration's budget, as released to newspapers on January 5, 1938, lists:

General Public Works:	
TVA	$ 46 000 000
Commerce	1 850 000
Interior	55 973 100
Justice	2 041 200
State	3 400 000
Treasury	58 107 000
Public Works:	
Highways	$104 640 100
Rivers and harbors	36 168 000
Rural electrification	39 514 800
Other public works	200 049 300
Total	$545 743 500

No amount of orderly planning of $600 000 000 will keep the pendulum of prosperity swinging when the total construction in the United States drops from $12 000 000 000 in 1928 to $4 000 000 000 in 1934 (see Table 1). However, it can help it keep a small oscillation, when the essential factor is to keep it from stopping entirely. Hence, engineers should analyze the types of public works that can be planned in an orderly manner and they should propose a program designed to help keep the wheels moving to elevate the standard of living. Among the numerous public works projects that (*a*) can be planned in advance; (*b*) can be at least partly liquidated; (*c*) will add to the well-being of the nation; (*d*) will reduce the cost of crime; and incidentally (*e*) will assist in returning the engineer to a respectable place in the mind of society—is public housing. Since this activity does not compete with services already offered by private corporations it is not in the category of some of the other projects mentioned in papers of this Symposium. Early in 1933 the idea that private initiative could satisfy all housing needs was abandoned. A small beginning has been made on sound principles, which the authors of these papers must surely endorse:[100]

"The PWA housing program produced fifty-one sound, workable projects which afforded a total of approximately 22 000 living units. Base rents averaging $5.65 per room per month had been established on 23 of them when the program was transferred to the United States Housing Authority. This figure was the 'shelter' rent and an additional charge of $1.82 per room per month provided heat, hot and cold water, electricity for lighting and cooking, or gas for cooking if that fuel was to be used. At these low figures, former

[99] Associate Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

[99a] Received by the Secretary July 19, 1938.

[100] "In Defense of the PWA," by Harold L. Ickes, *New Republic*, March 30, 1938, p. 215.

slum dwellers obtained good shelter, ample light and air, modern equipment and generally salubrious surroundings.

"The average income of the families in the PWA projects is low, and every family admitted was taken from substandard housing. In one of the Montgomery, Alabama, projects, the average income of tenants is $12.50 per week. In Techwood, Atlanta, it is $22.11 per week; in Atlantic City, $26.46, Miami, $19.15."

When it is realized that this has been done in spite of the advance in materials prices, high costs, and high taxes, it is even more remarkable. In reference to the general property tax on urban improvements, Harold M. Groves, Professor of Public Finance, at the University of Wisconsin, has remarked:[101] "It seems odd that we should have singled out for especially heavy taxation under the property tax, an interest which we now seek to subsidize as 'low-cost housing'."

Regarding the high prices, the U. S. Bureau of Labor Statistics gives the relative wholesale prices of important building materials cited in Table 10.[102]

TABLE 10.—Relative Wholesale Prices of Building Materials (1926 Equals 100)

Average for year	Brick and tile	Cement	Lumber	Paint, etc.	Plumbing and heating	Structural steel	Other building materials	All building materials
1929	94.3	91.8	93.8	94.9	95.0	98.1	97.7	95.4
1932	77.3	77.2	58.5	71.7	66.8	80.9	79.5	71.4
1937	93.5	95.5	99.0	83.4	78.8	113.2	99.1	95.2

It will be observed that, although the national income in 1937 was only a little more than two-thirds of that in 1929, the prices of all building materials were almost exactly those of 1929 and land costs were still held at exorbitant values by real estate speculators who bought before the Government could get possession of the land for housing projects. These data are affirmed by reports[103] that construction costs are up 9% in 1938, as compared with 1937. Private initiative can do little against these odds to provide housing at the pocket-book level of the lowest quarter of the population. "No industrial organization is equipped to produce the kind and amount of houses we need, while conditions within our cities are such as to discourage rather than encourage industry in so equipping itself."[104]

To indicate further the complexity of the private building industry, a study of the Knickerbocker Village project in New York City accommodating about 1 600 "white collar" workers and their families, has been made.[105] The materials cost $2 636 000; labor, $2 020 000; overhead, $1 561 000; and land, $3 250 000. To reduce labor costs 25% (labor received an average of $1.15 per hr), as has so often been suggested, would have reduced the cost 5% and

[101] "What to Do About Taxes," *New Republic,* December 29, 1937.

[102] *Peoples Lobby Bulletin,* March, 1938.

[103] *Business Week,* May, 1938.

[104] "Can America Build Houses," *Pamphlet No. 19,* Public Affairs Committee, p. 4, New York, N. Y.

[105] *Fortune,* June, 1938.

would have reduced the rentals only 3.3 per cent. This does not indicate that any one factor is blameless, but that modern methods, reduction in land costs, materials, and overhead, Government subsidy, or outright Federal housing might be the best solution.

From these data it is observed that the engineer's attack on public works as a solution to prosperity must start much deeper than the Symposium indicates. Even after reducing the problem to fundamentals, pressure will be discovered from unexpected sources. Architecture is still very conservative, as it relates to public construction, and financial interests oppose any radical change of types of buildings, as may be noted in the following:[106]

"When recently the mechanized industries, particularly in metal, entered the housing field with the production of 'Prefabricated Houses,' they were met by the resistance of property holders, especially of the banks, who hold mortgages on about 58% of 1933 value of all urban real estate, and who fear that an influx of cheap modern dwelling would subtract substantially from the market value of existing structures."

Professor Mead has mentioned the interference of Government in business (see "The Record of Government in Business") but has neglected the interference of business in Government. Much legislation is lobbied by trade associations that want, as John T. Flynn, the Economist, has stated:[107] "Less Government in business for me; more Government in business for the other fellow." All want tariffs, subsidies, and laws. Contrary to Professor Mead's opinion, Government in industry has not always been so unsatisfactory. In a brief discussion, data cannot be cited of the success of public water-works, electric plants, highways, bridges, and ferries. There are projects more extensive in scope and area, however, than any one State could undertake and expect success. On the other hand, to be fair, one should note that many private ventures are failures. The railroads have been built and maintained with constant subsidies from the Public Treasury. The story of the financing of the systems is not entirely creditable. Private initiative could not undertake a comprehensive policy of regional development of the Tennessee Valley. Even if it could, many think it is better to have it developed by Federal authority in order that the social shocks can be absorbed.

The social effect may be as important as that of many past technical inventions. History has recorded the opposition to nearly every milestone in progress. Note the law sought to prevent the use of coaches in Hungary in 1523,[108] the opposition of the turnpike companies who profited by tolls, and the objections of canal owners to the railroads. The list reads like a catalog of inventions—air-brakes opposed; objections to electric locomotives on account of capital equipment loss, to subways, interurbans, and trucks. Steamships were opposed by the British Navy. The telegraph companies had a chance to buy the telephone, but chose to fight it. Bath-tubs were opposed, legislated against, and taxed. The newspapers fight the radio news. Just now History

[106] "Technological Trends and National Policy," National Resources Committee, June, 1937, p. 59.

[107] Syndicated column in *The Columbus Citizen* and other newspapers, February 5, 1936.

[108] "Technological Trends and National Policy," National Resources Committee, June, 1937, p. 39 *et seq.*

is writing the opposition to TVA, predicated on the idea that honest people have invested in public utility securities. Fate, however, is not cognizant of the honesty of men when new inventions cause obsolescence of old ideas and old methods or when financial jugglers out-manipulate the investor by pyramiding holding companies. Progress cannot be halted even if bankruptcy is in order for some industries. "In all previous depressions," states Professor Mead, "the responsibility for recovery has been met largely by the business interests of the country" (see "Government Function in Business Recessions and Recovery"). He should further note, as all business leaders did, that in all previous depressions there was not the large debt structure that burdened the country as in 1929, and that business beat a path to Washington demanding that Government "do something" about the sorry state of affairs. Previous depressions were ended when debts were written off, and agriculture and industry started anew to spend for consumptive products, including new inventions which employed a large amount of help. All efforts in this depression were in the direction of saving the debts and creating new ones, thus ear-marking nearly all income for interest. This is neither bankruptcy nor progress.

With his further statement that "The Administration has also urged the engineering schools to teach their students to consider not only the design and construction of engineering works but also to study and determine their ultimate effects on society" the writer, as an educator, agrees wholeheartedly. Most graduates leave college ignorant of the effect of science on culture, and new developments "hatch" so rapidly that the graduate never catches up, but remains saturated with the narrowness of his specialty. One required reading of the "Social Effects of Inventions" and "Resistances to the Adoption of Technological Innovations"[109] might open the doors to a wider knowledge of the effect of their own engineering work on society.

With regard to his opinion that "power development as an objective is not a legal function of the Federal Government" (see "Government Interference as a Permanent Policy: Power"), reference should be made to the Ashwander *v.* Tennessee Valley Authority case (56 U.S. 466) in 1936 when the Supreme Court held that, "the Wilson Dam and its power plant must be taken to have been constructed in the exercise of constitutional functions of the Federal Government." Further, in Tennessee Electric Power Co. *v.* Tennessee Valley Authority (Fed. Supp. 947 District Court Tennessee E.D.) the Court held:

"The Court is of the opinion that the relative value of these various plans is immaterial, since it has been established that the TVA project is reasonably adapted to use for combined flood control, navigation, power, and national defense, and that in actual operation the creation of energy is subordinated to the needs of navigation and flood control. * * * The dams and their power equipment, both constructed, under construction, and authorized, must be taken to have been authorized, constructed and planned in the exercise of the constitutional functions of the Government. * * *"

"While the Government, in selling property of the United States, performs many functions, that would be performed in the operation of a private business

[109] "Technological Trends and National Policy," Sections III and IV, National Resources Committee, June, 1937, pp. 24–66.

trading in similar property, inasmuch as the energy sold is created at dams lawfully erected within the Federal power, the Government in performing these functions is not entering into private business. It is merely using an appropriate method of disposing of its property. The Government may sell land belonging to the United States in competition with a real estate agency, carry parcels in competition with express companies, and manage and control its thousands of square miles of national parks even as a private company. The Government has an equal right to sell hydroelectric power, lawfully created, in competition with a private utility. * * * These complainants have no immunity from lawful competition, even if their business be curtailed or destroyed."

However, the essential problem is to develop a modern and progressive viewpoint by engineers, especially civil engineers, who by training and employment are more closely affiliated with public problems than others whose work is allied with large industries not so closely tuned to the common weal. The civil engineer ought to be the first to recognize that "control of power is a social as well as an engineering and economic problem."[110] This being true, perhaps some new methods of analyzing public projects must be devised. The price criteria set up by Mr. Morris L. Cooke might be used as a start. He suggests[111] that the primary principle underlying a national power policy, both as to wholesale and retail prices, should be service at the lowest reasonable cost to consumers, with the benefit of any favorable conditions attached to Federal ownership and operation. Factors involved in establishing the "lowest reasonable cost" are: (1) Rational allocation of the costs of multiple-purpose projects; (2) amortization of the allocated cost of construction during the life of the property; (3) interest; (4) taxes, as under the TVA, where a percentage of gross is paid to the States; (5) cost of maintenance; (6) cost of operation (administrative, commercial, technical); and (7), cost of direct promotion of more intensive use.

Quoting Mr. Cooke:

"Exceptions to this governing principle, in the form of sale at less than cost as computed by conventional accounting methods, should be allowed only on special occasion in a particular area as a means toward achievement of some objective important to the national welfare, to gain which would effect a proper relation between costs and benefits in terms of social accounting. * * * In place of maximum money profits accruing to private ownership, the objective of public enterprise is maximum social benefits widely and equitably distributed."

Regarding the system of high dams, which Professor Mead states "is believed to have been built almost exclusively on account of power development," the testimony presented in the various engineering reports led the District Court, E.D. Tennessee, to state[112] its judgment that the improvement of the Tennessee River, from its mouth to Knoxville, by a series of low movable dams without power development, would have practically no effect on floods.

[110] Vice-Chairman Basil Manly, of the Federal Power Commission, quoted report of the National Resources Committee, June, 1937, p. 278.

[111] "Design for National Power Policy," *New Republic*, March 2, 1938.

[112] 21 Fed. Supp. 947, District Court, E.D. Tennessee.

Referring to low-lift dams, it also declared that:

"Such a waterway would be inferior to the high-dam developments and would not permit the economical development of power. * * * The board of engineers pointed out that in addition to having no value whatever for flood control, the 32 low dams, although less expensive to construct than the high dams, provided a navigation channel inferior to the high dam plan. * * * It concluded that 'it is evident that the full utilization of the resources of this river for the public benefit requires its development of power and navigation.'"

No engineer should object to a yardstick. It may not be "clearly dishonest," as Professor Mead states, but only a question of the type of yardstick required to measure the objectives sought. Mr. Cooke has defined it in clear terms:[113]

"The advantages of public enterprise as a yardstick cannot be realized by attempting to set up a public enterprise in such a manner as to duplicate a private enterprise in every detail, and then comparing the two. They are essentially different kinds of institutions, with different objectives. Comparison of the costs of identical component parts of technical operations by a uniform system of accounting can indeed be useful; but comparison of the costs of a public enterprise in its entirety is neither realistic nor practicable because of the variables involved. For instance, public agencies obtain their money at low rates and almost without exception amortize their investments. The private utilities, by their failure to amortize capital-debt, laid themselves open to the recent collapse of equity values.

"It is a recognition of, and experiment with, the variables that is important. A public enterprise can make experiments and comparisons on many fronts that private enterprise cannot or will not make. It can explore such things as the advantages and disadvantages of mass consumption at low rates, the extent to which electricity can be used advantageously on farms, and the influence of productivity and the costs of good wages and superior working conditions. It can explore the part to be played by electricity as the 'coordinating agent' in the promotion of a comprehensive conservation program."

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[113] "Design for National Power Policy," *New Republic*, March 2, 1938.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FLOOD ROUTING

Discussion

By Messrs. E. L. Myers, and Ralph W. Powell

E. L. Myers,[34] M. Am. Soc. C. E. (by letter).[34a]—In a definite and concise manner, this paper describes a method of arriving at probable flood discharges and gage heights in the Tennessee Valley. This is a most constructive piece of work and should prove of great assistance to engineers making flood studies of other streams. Until recently, little information was available as to the manner of arriving at flood stages due to assumed floods. An excellent report of á flood study of the Illinois River was published in 1929 by the Division of Waterways of the State of Illinois. It was used largely by the State Reclamation Department of Texas in 1930 in a study of floods in the Trinity River.

The conditions on the Trinity River and the Illinois River, no doubt, are quite different from those on the Tennessee River; however, the general formulas used are applicable in either case. The study of the Illinois River was complicated by the existence of many levee districts which also applies to the Trinity River in a lesser degree. The effects of reservoir storage entered into the Trinity River study, but not into that of the Illinois River.

A noteworthy feature of the study of the Tennessee River as outlined in the flood-routing report is the small effect that reservoir storage has upon the magnitude of floods in the Lower Tennessee Valley. This is characteristic of most studies of flood control by reservoirs. In the case of the study of the Trinity River, it was found that prevention of floods by reservoirs was impracticable and much more expensive than protection by levees.

Ralph W. Powell,[35] M. Am. Soc. C. E. (by letter).[35a]—Since the subject of flood routing is so important not only for flood forecasting, but also for the rational design of any system of control by reservoirs, the authors should be

Note.—The paper by Edward J. Rutter, Assoc. M. Am. Soc. C. E., and Quintin B. Graves, and Franklin F. Snyder, Juniors, Am. Soc. C. E., was published in February, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1938, by Messrs. Cecil S. Camp, and R. D. Goodrich.

[34] Cons. Civ. Engr. (Myers, Noyes & Forrest), Dallas, Tex.

[34a] Received by the Secretary May 31, 1938.

[35] Associate Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

[35a] Received by the Secretary June 16, 1938.

thanked for placing the methods they have developed at the disposal of the profession. "It is not difficult to point to cases where primary decisions on levee systems or retarding-basin systems of the first magnitude have been [in the past] based on flood-routing computations of doubtful reliability." [36] When the writer compares the elaborate studies summarized in this paper with the crude methods he used when working on the same problem of the Tennessee River, in 1927, it is striking how much progress has been made in ten years.

This is, as far as the writer knows, the first published method for routing floods through reaches affected by back-water from navigation dams, and it seems to solve the problem satisfactorily. The only questions the writer wishes to raise concern reaches not affectd by back-water—what the authors call "natural conditions." In this case, cannot the method be simplified further? And cannot satisfactory routings be made without the cross-sections at frequent intervals upon which this method is based?

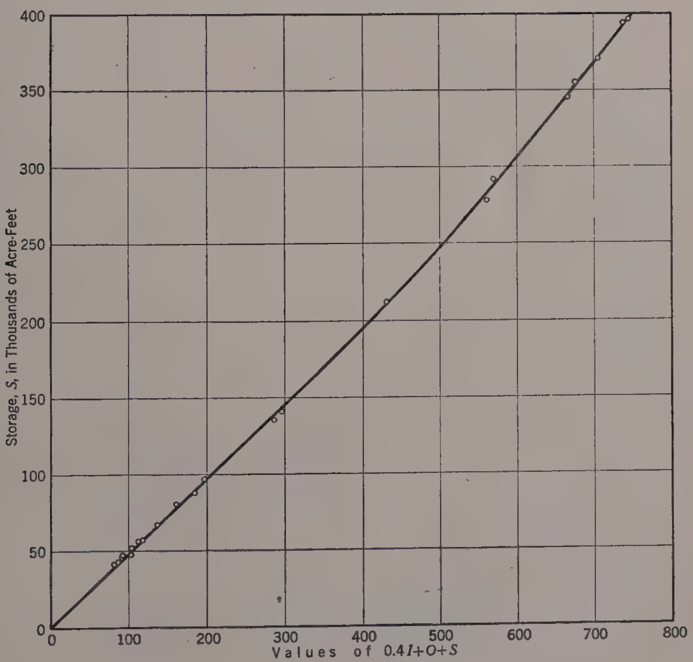

FIG. 11

In an attempt to answer these questions the writer secured from the authors the continuation of Table 3 through January 12, 1927, and plotted S against $O + S$ and then against $I + O + S$. As was to be expected, in the first curve, S was definitely higher on the rising stage than on the falling stage for the same value of $O + S$. In the second curve, S was greater on the

[36] "The Hydraulics of Flood Movements in Rivers," by Harold A. Thomas, M. Am. Soc. C. E., *Engineering Bulletin*, Carnegie Inst. of Technology, 1934 (revised 1937), p. 46.

falling stage than on the rising (for the same value of $I + O + S$), and the difference was larger than before. Trials were then made of $0.5\,I + O + S$; $0.4\,I + O + S$; and, $0.3\,I + O + S$, and it was found that the use of $0.4\,I$ gave the best agreement. The values are shown in Fig. 11; that is, in this case, in determining the storage in the reach, the up-stream flow seems to have a weight of 0.4 compared to 1.0 for the down-stream flow.

TABLE 5.—VALUES OF $I_1 + 1.4\,I_2 + S_1 - O_1 = 0.4\,I_2 + O_2 + S_2$, AS COMPUTED* FROM FIG. 11

Day: December, 1926	Total inflow, I_1 (1)	$0.4\,I$ (2)	Chickamauga discharge, O_1 (3)	Storage, S_1 (4)	$I_1 + 1.4\,I_2 + S_1 - O_1 = 0.4\,I_2 + O_2 + S_2$ (5)	$O_1 + S_1$ (6)	Day: January, 1927	Total inflow, I_1 (1)	$0.4\,I$ (2)	Chickamauga discharge, O_1 (3)	Storage, S_1 (4)	$I_1 + 1.4\,I_2 + S_1 - O_1 = 0.4\,I_2 + O_2 + S_2$ (5)	$O_1 + S_1$ (6)
(a) COMPUTATIONS FOR DECEMBER, 1926							(b) COMPUTATIONS FOR JANUARY, 1927						
21	45	18	37	48	185	...	1	129	52	166	205	294	371
22	92	37	59	89	289	148	2	90	36	117	141	202	258
23	119	48	102	139	405	241	3	63	25	80	97	161	177
24	178	71	142	192	565	334	4	58	23	60	78	136	138
25	241	96	184	285	678	469	5	43	17	54	65	116	119
26	240	96	228	354	715	582	6	44	18	42	56	114	98
27	249	100	238	377	753	615	7	40	16	43	56	102	98
28	261	104	248	401	744	649	8	36	14	39	49	92	88
29	236	94	255	395	676	650	9	33	13	35	44	87	79
30	214	86	238	352	566	590	10	32	13	33	41	82	74
31	170	68	213	285	423	498	11	30	12	30	40	81	70
..	...	..	...	...	...	...	12	29	12	30	39	..	69

* As in Table 3, flows and storages are in units of 1 000 cu ft per sec and 1 000 acre-ft, respectively

Using this curve, the values in Table 5 were computed. All the values of I_1 (Columns (1), Table 5) are taken from Table 3 (and its continuation supplied by the authors); O_1 and S_1 for December 21, 1926 (Columns (3) and (4)) are also taken from Table 3. From these, are computed $I_1 + 1.4\,I_2 + S_1 - O_1 = 45 + 129 + 48 - 37 = 185$ (Column (5)) which also equals $0.4\,I_2 + O_2 + S_2$ for December 21, and $0.4\,I_1 + O_1 + S_1$ for December 22, 1926. Subtracting $0.4\,I_1$ for December 22 (= 37) gives $O_1 + S_1$ for December 22 (= 148). Entering Fig. 11 with $0.4\,I + O + S = 185$ gives S_1 (Column (4)) for December 22 (= 89). Subtracting this from $O_1 + S_1 = 148$ (Column (3)) gives $O_1 = 59$. The remainder of Table 5 was computed by a similar process. It should be noted that the values given are not those plotted in Fig. 11, except for the values of I; but they differ from them only slightly. The maximum difference between any value of O and that computed by the authors for the corresponding day was four units on January 2, 1927, which is less than 4% and well within the probable error of the data. In fact, on 15 of the 22 days computed, the difference was one unit or less. It seems, therefore, that if this sample is typical, the channel storage can be computed with sufficient accuracy by this method wherever accurate hydrographs of one large flood have been obtained, without the necessity of a complete flood profile

and field surveys of cross-sections. All that is necessary is to make a rough estimate of the storage in the reach before the beginning of the rise (from the known length and assumed average width and average depth). Then, tabulating the known values of I and O, and starting with the assumed S_1, the other values of S can be computed by the formula, $S_2 = S_1 + I_1 + I_2 - O_1 - O_2$, and a curve like Fig. 11 plotted; or, if preferred, one may start with an assumed storage at the end of the period and work backward. In cases where the data are insufficient for making an accurate estimate of the local and unmeasured tributary inflow, this backward order would be preferred, and it is believed that it will be possible to construct a fairly accurate curve of the type shown in Fig. 11 by using the falling stage data only. For this the unmeasured inflow will form such a small part of the whole that even the roughest approximation of it will introduce little error. As has been shown by Robert E. Horton,[37] M. Am. Soc. C. E., beyond the point of inflection of the hydrograph the flow is essentially the draw-down from channel storage, so that it should give a fairly accurate measurement of it.

Of course it should be understood that the coefficient 0.4 applies to this particular reach. For other reaches a trial would have to be made to determine whether that value or some other gave the most reasonable results. It should also be understood that no single value of the coefficient can be expected to fit the facts of all floods perfectly, or even all parts of the same flood, even on the same reach. This is only intended to be an approximate method.

As stated by the authors, Professor Thomas [9] has given a method of accomplishing this same result, but it is believed that the method herein given is simpler, and that it partly meets his objection to what he calls the first approximate method, that change of slope is "the very essence of the behavior of a flood wave," [38] and is entirely neglected in the older methods. To meet this objection more fully, the writer would recommend that if the method herein proposed is to be used for a reach in which actual gagings are available at each end, the rating curves should be drawn for a steady stage and then corrected for rising or falling stage. Perhaps a few words on this correction would not be out of order.

Professor Thomas' equation [39] can be written:

$$\frac{dy}{dx} = \frac{S\left(1 - \frac{Q^2}{Q_n{}^2}\right)}{1 - \frac{(u - v)^2}{g\,d}} \qquad (3)$$

in which S = slope of the water surface at the station; Q = actual flow at the station (in cubic feet per second); Q_n = flow as given by rating curve for steady flow; u = velocity of a flood wave, in feet per second; v = mean velocity of water at the station; g = acceleration of gravity, say, 32.16; and d = mean

[37] "Surface Run-Off Phenomena," *Publication 101,* Horton Hydrological Laboratory, 1935, p. 34.

[9] "The Hydraulics of Flood Movements in Rivers," by Harold A. Thomas, *Engineering Bulletin,* Carnegie Inst. of Technology, p. 65.

[38] *Loc. cit.,* p. 53.

[39] *Loc. cit.,* p. 25, Equation (22).

depth of stream at a station (in feet). Now if j = rate of increase of gage height, in feet per hour (negative for falling stage):

$$\frac{dy}{dx} = \frac{-j}{3\,600\,u} \quad \text{(4)}$$

and,

$$\left(\frac{Q}{Q_n}\right)^2 = 1 + \frac{j}{3\,600\,S\,u}\left[1 - \frac{(u-v)^2}{g\,d}\right] \quad \text{(5)}$$

As Professor Thomas states, this applies strictly only to cases in which the velocity is so low that velocity-head effects can be neglected, but this limitation would not forbid the use of Equation (5) at most gaging stations; and, ordinarily, the values of u and v will be close enough so that the last term will be unimportant. The maximum correction for changing stage occurs when $u = v$ and,

$$\frac{Q}{Q_n} = \sqrt{1 + \frac{j}{3\,600\,s\,v}} \quad \text{(6)}$$

This is the same as the Jones formula used by the U. S. Geological Survey [40] except that v is the average velocity rather than the surface velocity. The use of the latter somewhat reduces the correction and, therefore, may give a better result, but actually the surface velocity depends largely on the wind, and is not a real factor in the problem. The writer is not suggesting that all flood problems require that all gage readings be converted into flow by Equation (6), but that if the data are available, the correction should be applied to the hydrographs used in obtaining the channel storage correction, as outlined herein.

It might be added that the upper part of most rating curves (if uncorrected for changing stage) are generally weighted to give falling rather than rising stage quantities, because gaging parties more often arrive at the station after the peak has passed and get a measurement on falling stage.

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[40] *Water Supply Paper No. 375-E.*

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DISCUSSIONS

WATER-SOFTENING PLANT DESIGN

Discussion

BY MESSRS. PHILIP BURGESS, A. ELLIOTT KIMBERLY, L. R. HOWSON, CHARLES P. HOOVER, M. H. KLEGERMAN, ROLLIN F. MACDOWELL, AND D. E. DAVIS

PHILIP BURGESS,[2] M. AM. SOC. C. E. (by letter).[2a]—To a striking degree, the valuable data contained in this paper should be very helpful to engineers who are concerned with the design of water-softening plants. It is to be regretted that more data could not have been presented which would substantiate some of the author's conclusions in a helpful manner.

Mr. Knox states that he is emphasizing only the special features of water-softening plants that are not features of filtration plants. However, his discussion contains valuable data with respect to many features that are included in both types of plants. It is not an uncommon experience for an engineer to be called upon to convert a rapid sand filtration plant into a water-softening plant. Such a problem includes generally the addition of more storage and of devices for the application of much larger quantities of chemicals, and the disposition of the resulting large quantities of sludge entailed by the softening process.

Furthermore, the author states that the pneumatic equipment is preferable for handling chemicals at large softening plants if the chemicals must be elevated. In general, pneumatic equipment is desirable because it eliminates the dust problem and permits ground storage of chemicals in bulk; at the same time it provides means for elevating the chemicals from ground storage to elevated tanks, or hoppers, through which the chemicals are fed by gravity as required. The writer would suggest also the use of automatic weighing machines as an adjunct to the dry-feeding equipment.

That feature of a water softening, as well as of a modern rapid sand filtration, plant which has been least appreciated, is the necessity for proper mixing of chemicals after their application with the raw water. Old plants frequently

NOTE.—The paper by W. H. Knox, M. Am. Soc. C. E., was presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 21, 1937, and published in May, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[2] (Burgess & Niple); Pres., The Burnip Constr. Co., Columbus, Ohio.

[2a] Received by the Secretary May 26, 1938.

contained baffled mixing devices which, as the author states, are unsatisfactory because they do not permit the required flexibility of operation. Furthermore, it is difficult to secure proper distribution of the water as it leaves such tanks and enters the settling tanks. Modern devices of the flocculator type permit flexibility of operation and also proper distribution of the water to the settling tanks.

Reaction tanks should permit flexibility of operation at proper velocities, which the author states should be between 1 and 1.5 ft per sec. Experience indicates that slow velocities are not effective and higher velocities are harmful because they break up the coagulated floc.

The writer is interested in the design of two plants, each of 10 mgd capacity, wherein it is not now proposed to soften the water but which are to be designed so that softening may be added at minimum cost. This means that suitable devices are to be provided for storing and handling the larger quantities of chemicals and that the basins are built to allow sludge removal devices to be added when desired. No other changes are essential.

Rapid sand filters designed for filtering softened water should be equipped with devices which will permit washing at unusually high rates. This tends to decrease the deposition of after-deposits on the sand and is helpful in eliminating sand troubles and cost of maintenance. Furthermore, the filter sand may be comparatively coarse and inexpensive and need not be more than 24 in. deep. The filters are not required to improve the pathological quality of the water, but act rather in the nature of finishing devices wherein the coagulating reactions are completed, and some suspended matter is removed from the treated water. They seldom accomplish any substantial bacterial removal.

On the contrary, most of the solids in a softening plant are removed in the settling basins and it is of major importance that the engineer design these basins so as to permit proper displacement of the water with continuous flow through them. Inlet and outlet devices should be efficient and the basins should be properly designed with respect to depth, surface area, and velocity of flow. The author has not discussed these features of design, and it is hoped that he may add some data pertaining to them in his closing discussion.

The matter of providing covers for the basins and for the reaction tanks is of importance. It is noted that frequently engineers will cover the mixing tanks and will not cover the settling basins. In cold climates, the reverse is necessary because the settling basins must be covered to prevent trouble from ice which would not be experienced in the reaction tanks where the water is flowing at comparatively high velocity.

In designing re-carbonation devices, the engineer should appreciate that the gas supplied is of a toxic or poisonous nature and the features for handling it should be designed accordingly.

The author might have stated in his comments on the design of zeolite softeners that the wash water required in such plants may be 15% to 20% of the total quantity of water softened as compared with 3% to 4% of wash water generally required for the usual water purification plant. On the other hand,

the disposal of sludge is the unsolved problem in connection with the operation of lime softeners and there is no such problem in connection with the zeolite plant.

The writer is not in accord with the author to the effect that zeolite plants should be designed by the manufacturer who is now in a position similar to that of the manufacturer of filtration-plant equipment some years ago. The engineer should inform himself so that he is capable of designing such plants and should not be dependent upon the manufacturer.

In conclusion, the writer believes that the author has rendered a distinct service to the profession in presenting much of the basic data required for the proper design of water-softening plants.

A. Elliott Kimberly,[3] M. Am. Soc. C. E. (by letter).[3a]—Engineers interested in water-softening plant design will appreciate Mr. Knox's timely and informative review of current design practice. The author has had the unusual opportunity of examining plans for water-softening plants submitted during the twelve years, 1925–1937, to the Department of Health of a State in which ground and surface waters greatly exceed acceptable economic limits of hardness. This paper emphasizes the important, essential differences between water-softening and water-purification plant design, especially as regards the application of chemicals, the substitution of mechanical agitation for baffling, and the maintenance of continuously effective settling tank capacities through effective mechanical means for continuous sludge removal.

Experience has shown definitely that effective softening and the economic use of softening agents at lime-soda softening plants are dependent upon proper mixing at non-depositing velocities, generally obtained by the use of mechanical devices, assisted by a certain percentage of sludge return. Such a plan insures a maximum softening effect without waste of chemicals.

An important unsolved problem in lime-soda softening at municipal plants is the matter of sludge disposal. Doubtless, many engineers recall the difficulties which attend efforts to flush from a settling basin a compacted sludge containing from 13% to 15% of solids, the accumulation of many months, awaiting sufficient stream flow, and the serious problems which arise when no stream is available for dilution, or when no quarry-pits or other places of disposal are available. Due to the lime-sludge disposal problem, lime-soda softening is frequently rejected in favor of zeolites, even in cases where the former may be definitely more adaptable to raw-water conditions. The solution of this problem lies in lime recovery.

In 1915, Charles P. Hoover, Chemist-in-Charge of the Columbus Water Works, conducted extensive experiments at Columbus, Ohio, and actually operated the Columbus plant for a period of 4 hr with 5 000 lb of lime produced at a practical lime recovery plant. The softening results are stated as normal and satisfactory. The recovery plan was as follows:[4] The settling basin precipitate is pumped into a thickener from which the material is fed directly into a rotary kiln. The kiln used in these tests is 20 ft long and 34 in. in

[3] Cons. San. Engr., Columbus, Ohio.

[3a] Received by the Secretary May 26, 1938.

[4] Forty-fifth Annual Rept., Div. of Water, City of Columbus, Columbus, Ohio, 1915.

diameter and was oil fired. The sludge contained 18% of solids and from it was produced quick lime containing from 63% to 73% of water soluble calcium oxide.

A lime recovery plant, in successful operation in England, comprises duplicate agitated mixing tanks into which the sludge from the settling tanks is pumped.[5] The mixed sludge containing about 25% of solids is pumped to a vacuum filter, which reduces the water content to about 50 per cent. The filter is 8 ft in diameter by 8 ft wide and operates under a 16-in. to 20-in. vacuum. The filter cake discharges by gravity into a rotary kiln 86 ft long and 6 ft in diameter, operating at the rate of 1.33 rpm. From 12 to 14 hp are required to drive the kiln, which is oil-fired.

The capacity of the kiln is about 17 tons per day. The oil consumption is stated as 27% of the lime produced. The maximum temperature reached is about 1 500° C. Provisions are made for preventing dust by passing the evolved gases through a suitable dust collector. The results obtained at this plant are said to be highly satisfactory.

In the fall of 1935, studies were made by the Nichols-Herrshoff Company to determine the possibilities of incineration of lime-sludge reduced in volume by thickeners and vacuum filters, following the treatment successfully applied in the incineration of sewage sludge.

The need for a satisfactory practical method for the disposal of lime-sludge is evident. The successful application of filtration and incineration may solve the problem, obviously, to effect marked economies in water-softening plant operation. Many of the Ohio water supplies softened with lime and soda ash contain substantial quantities of magnesium (average 9% of the total hardness of Ohio ground-waters) and, consequently, sludge calcination results in the production of an admixture of calcium and magnesium oxides.

In a private communication with the writer, Mr. Hoover points out that, by a change in design which makes possible a two-way application of lime (additional mixing basins and clarifier units), it is possible to precipitate lime sludge containing practically no magnesium. This result is accomplished merely by holding the potential of hydrogen (pH) of the water at a point below that required to precipitate the magnesium. Since, ordinarily, considerably more reclaimed lime is produced at a plant than is required to soften the water, the two-way lime dosage plan suggested by Mr. Hoover should have definite application in practice, in cases where lime recovery is planned at softening plants treating water relatively high in magnesium salts.

Lime-soda softening was used in 1936 to effect a reduction of fluorides.[6] Briefly stated, it has been determined that, in the presence of magnesium salts, the application of lime to a water containing fluorides effects a fluoride reduction which, between the limits of 1.5 and 3.5 ppm, initial fluorides, varies as the square root of the magnesium removal. Clinical data indicate that fluorides on the order of about 1.0 ppm have no deleterious effect on teeth enamel. Mottled enamel is endemic in about twenty-eight States and, therefore, the advantages of lime treatment, with or without complete softening, appear definitely important.

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[5] *The Industrial Chemist*, July, 1933.

[6] *Journal*, Am. Water Works Assoc., January, 1937, Vol. 29, pp. 9–25, inclusive.

Quite recently, a new type of zeolite has been developed which produces either carbonic, sulfuric, or hydrochloric acid, depending upon the composition of the raw water. Where carbonates predominate, a definite quantity of carbonic acid is produced. Using this zeolite, a certain percentage of the raw water is by-passed through a filter and, later, is admixed with the lime-softened water to adjust the potential of hydrogen. The process has the advantage that the quantity of carbon dioxide produced is constant and that no phenols or other objectionable materials are introduced into the water.

Prior to 1937, installations were limited to commercial practice, but in the latter part of May of that year a small unit went into operation at a softening and fluoride reduction plant at Mt. Victory, Ohio. The efficiency of this so-called "hydrogen zeolite" unit has been investigated by the Division of Engineering of the Ohio Department of Health. The results are said to justify the statement that the equipment is satisfactory.

Recarbonating with "hydrogen zeolite" is of much interest to the water-works designer since, with the use of zeolite, the quantity of carbonic acid is fixed so long as the composition of the well water remains unchanged, whereas using flue gases or burning gas or coke, the percentage of carbon dioxide produced is variable, thus preventing a close control of recarbonation.

L. R. Howson,[7] M. Am. Soc. C. E. (by letter).[7a]—The fact that approximately one-half the municipal water-softening plants of the United States are found in Ohio offers more than passing tribute to the contributions of the Ohio State Department of Health in acquainting the citizens of Ohio, not only with the necessity of securing a bacterially satisfactory water, but with the advantages to be secured from a reduction in the hardness as well. Without question, water softening in Ohio has paid large economic dividends.

The engineer who undertakes to design water filtration works is frequently aware of the ultimate, if not the present, desirability of softening, as well as filtering, the supply. When that is the case, the design should be made so as, economically and practicably, to install sludge removal equipment and other facilities necessary to the adaptation of the filtration plant to water softening. In the Milwaukee, Wis., plant (capacity 200 mgd), mixing design and periods follow softening practice. The settling basins are designed so that sludge-conveying equipment can be installed economically at a later date. It is believed that within the next decade, more or less, many plants treating Lake Michigan water with a hardness of 125 ppm will include softening.

With respect to aeration, it is the writer's belief that when used with surface waters for the removal of tastes and odors, some of the more positive and complete methods of control, such as activated carbon, are to be preferred. An experience has come to the writer's attention in which one of the inspirator type of aerators was installed for carbon dioxide removal before the lime treatment of a well supply. Red water troubles resulted, due to the presence of oxygen absorbed by the water. By-passing the inspirator type of aerator and using a simple splash type overcomes the red-water trouble in this plant.

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[7] Cons. Engr. (Alvord, Burdick & Howson), Chicago, Ill.

[7a] Received by the Secretary May 31, 1938.

In the moderate-sized softening plants chemicals, in general, are stored overhead. In the larger plants, elevated storage becomes costly and it is usually desirable to provide ground storage with only a limited overhead storage, transfers from ground to elevated storage being made at frequent intervals through dust-proof conveying equipment. Vacuum fan exhausters are frequently installed to remove dust from dry-feed machines during the charging operation.

Mr. Knox properly calls attention to the fact that mechanical mixing devices must be provided for water softening, and that the mixing period must be relatively long. Experience with stirring devices of both the horizontal-paddle, vertical-shaft type and the horizontal-shaft, flocculator type, leads to the opinion that the latter is much to be preferred. It has been found difficult to prevent the accumulation of sludge deposits where the vertical-shaft, horizontal-paddle type of equipment is used.

Where flocculators are used in softening, the peripheral velocity is ordinarily from 2 ft to 3 ft per sec, compared to from 1 ft to 1.5 ft per sec where alum is used for plain filtration.

Where softening is practiced in conjunction with the filtration of a sometimes turbid surface supply, provision for series operation is desirable. At times of high turbidity the first stage is used for the removal of turbidity with the second operated as single-stage softening treatment. Two-stage recarbonation is also being frequently adopted.

As Mr. Knox states, sludge disposal is one of the big unsolved problems in water softening and on that account is probably one of the greatest deterrents to the adoption of lime-soda softening in some cities. It has been found practicable at some localities, by continuous removal of the sludge in small quantities, to discharge it into the sewer system. Although the volume of sludge is considerable the effect of the lime in sweetening the sewage and the sewage sludge is beneficial providing the quantity of sludge is not out of proportion to the sewage into which it is discharged.

In the smaller and moderate-sized softening plants the building heating system quite frequently consists of oil-fired or gas-fired boilers. With oil, two boilers are commonly provided—one of sufficient capacity to provide all the carbon dioxide requirements, and the other of sufficient capacity to furnish all the heat required. There have been a few installations in which two burners have been installed under only one boiler, but the difficulty of securing good combustion and of controlling the carbon dioxide content of the gases under widely varying demands is difficult to overcome.

Filtration rates as high as 3 gal per min per sq ft are commonly used in softening plants treating well water. With proper pre-treatment the sand should not require replacement for many years.

Mr. Knox is to be congratulated on having condensed his broad observations on water softening into such a concise, yet comprehensive, discussion.

CHARLES P. HOOVER,[8] ESQ. (by letter).[8a]—The lime-soda ash water-softening plants that have been built in the past were not designed to produce softened

[8] Chemist in Chg., Water Softening and Purification Works, Columbus, Ohio.
[8a] Received by the Secretary June 3, 1938.

water of any predetermined alkalinity or pH-value. This is probably due to the fact that, to this time, no definite values have been agreed upon for the calcium carbonate alkalinity and the pH-value of the softened water.

What should they be? Results of experimental work, conducted during 1935 and 1936 at the Columbus, Ohio, Water Softening and Purification Works, lead to the conclusion that if the softened water is to be in contact with galvanized plumbing, the potential of hydrogen should range from about 7.8 to 8.2, and the calcium carbonate alkalinity should be sufficient to produce saturation at these pH-values. In other words, the water should be in chemical balance to calcium carbonate. A pH-value of 7.8 to 8.2 means a water free from normal carbonates.

Low-alkalinity water containing all, or nearly all, normal carbonates (especially in hot water) is more corrosive to galvanized metal than water containing no normal carbonates. Results of recent experiments indicate that normal carbonates (calcium and magnesium) deposit an uneven scale, in hot water tanks from $\frac{1}{32}$ in. to $\frac{1}{16}$ in. thick, in a year's time which is only somewhat adherent; whereas, the bicarbonates of calcium and magnesium, although they produce a thinner scale, produce one that adheres much better to the metal. The bicarbonates, therefore, produce a much better protective coating on galvanized surfaces than the normal carbonates.

When recarbonation of lime-softened water was first adopted and accepted as an essential part of water softening, about 1925, it was thought that its use would eliminate all filter-sand troubles, and all incrustation and corrosion problems in the distribution system.

Water-softening treatment, it is believed, can be made to eliminate distribution troubles, but thus far no practical means has been found to eliminate incrustation of filter sand entirely and, at the same time, produce the desired alkalinity. Tentative results from incomplete experiments indicate that the following procedure seems promising as a means for removing or crystallizing colloidal precipitates, and thus prevents them from crystallizing on, and coating, the sand grains:

(*a*) The addition of precipitated calcium carbonate, water-softening sludge, or other finely divided inert substances to the softened carbonated water, then stirring and settling before filtration;

(*b*) Blanket filtration of the softened carbonated water through precipitated calcium carbonate, water-softening sludge, or other finely divided inert material; and,

(*c*) If water is carbonated and then allowed to settle for a long period of time before being filtered, sand troubles will be eliminated, but the settling time must be measured in hours and days rather than in minutes.

The difficulty is that precipitates resulting from water-softening reactions are least soluble at a high pH-value (about 9.4) and if enough carbon dioxide is added to bring the potential hydrogen below this value more of the precipitate stays in solution, and colloidal precipitates are dissolved. The alkalinity or the carbonate hardness is thus increased beyond that desired. Therefore, the addition of carbon dioxide to softened settled water just ahead of the filters in

sufficient quantity to stabilize it (so as to prevent incrustation and cementing together of the sand grains) cannot be practiced if it is desired to produce water of low alkalinity. This means that until a satisfactory procedure for removing colloidal precipitates is developed, sand filters will have to be re-sanded every ten or twelve years. Proper adjustment of the pH-value by the addition of carbon dioxide to lime-softened water should be made following filtration.

A careful study of Mr. Knox's paper by engineers who design water-softening plants should prevent them from repeating mistakes that have been made in the past. There are many items that might profitably be discussed; at least four of them should not be ignored:

(1) If a plant is designed to soften surface water that becomes extremely muddy during flood times, provisions should be made to coagulate and remove the bulk of the mud before the softening chemicals are added. The flow line would be to: (*a*) Add coagulant; (*b*) mix; (*c*) settle; (*d*) add softening re-agents; (*e*) mix; (*f*) settle; (*g*) carbonate; (*h*) filter; and, (*i*) adjust pH.

(2) Too much emphasis cannot be registered against the use of pressure filters for filtering lime-softened water. At a water-softening plant, it is necessary to spade and break up the sand in the filter beds frequently, and it is almost impossible to do this where pressure filters are used.

(3) The importance of accurate chemical feeding devices decreases in almost direct proportion to the size of the plant; that is, the smaller the plant, the more necessity there is for accurate control. Automatic controls are especially desirable in small plants. Devices for weighing the softening re-agents are preferable to measuring devices. The necessity for making provisions that will prevent arching of chemicals in hoppers is also very important.

(4) During the summer, there will be sweating from pipes and exposed concrete in plants treating cold well water. The difficulties from this source can be minimized by providing proper insulation and drainage; and it is especially important that the elevation of the filters be low enough that the level of the water in them may be kept below the elevation of the operating floor.

M. H. KLEGERMAN,[9] ASSOC. M. AM. SOC. C. E. (by letter).[9a]—Preliminary testing for the purpose of providing basic design factors is probably more readily possible in the field of water supply conditioning than in many other fields of engineering design. Such preliminary testing, or pilot-plant installations, may provide facilities varying from a simple laboratory jar-mixing set, to a model plant handling a substantial volume of water and equipped with diverse mechanical aids entering into the different steps in the treatment. The desirability of such procedure should be emphasized, if only but to confirm, in any particular instance, the suitability of accepted design factors for treatment of the water supply in question.

In a recent investigation for the treatment of the water supply of a Southern municipality, involving "excess lime" followed by recarbonation, a small pilot

[9] Associate, Alexander Potter, New York, N. Y.

[9a] Received by the Secretary May 31, 1938.

plant was installed which provided principally for: (*a*) Automatic chemical feed; (*b*) mechanical mixing with means for varying mixing speed and time of mixing; (*c*) "stage mixing" (that is, varying the speed of mixing from "fast" to "slow" as the flow proceeded from inlet to outlet of the mixing compartments); (*d*) settling, with provisions for varying the detention period; and (*e*) sand filters.

Without entering into a detailed discussion of the various factors derived from this investigation (which were principally of value to the particular water supply under consideration) it is of interest to illustrate at least one of the items noted which has application to the paper. That item pertains to the question of speed and time of mixing and time required for settling. The author has stated that where mechanical devices are used (see heading, "Lime-Soda Water Softening: Mixing Devices"), a 30-min to 40-min mixing time is desirable and that velocities provided by mixing devices should be between 1 and 1.5 ft per sec. As for the settling period, he states that a detention of 2 hr is generally sufficient to secure a satisfactory clarified effluent (see heading, "Lime-Soda Water Softening: Clarifiers").

Although these factors are generally confirmed by the tests conducted by the writer, these tests, nevertheless, indicated that mixing speed, mixing time, and settling time, were not independent elements, but bore a relationship; that is, various combinations of these three elements were capable of producing equivalent results.

In illustrating this relationship, the writer will follow the policy adopted by the author of avoiding discussion of the chemistry of the process, and will use "turbidity" which is an index of the clarity of the settling-tank effluent as a basis of comparison. (The water supply in these tests emanated from limestone springs; it was clear and colorless so that any turbidity present in the settled water resulted only from the reacting chemicals.)

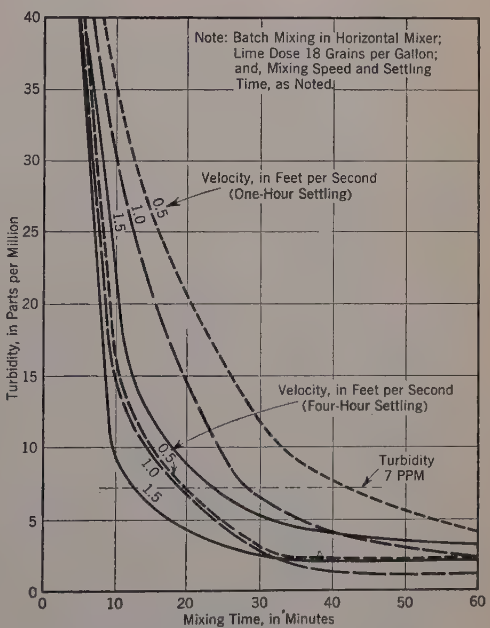

FIG. 1.—RELATION BETWEEN MIXING SPEED, MIXING TIME, AND SETTLING TIME

Fig. 1 indicates the average results of a series of "batch" tests, made in a rectangular mixing tank provided with power-driven mixing paddles mounted on a horizontal shaft, mixing being at right angles to the direction of flow. Means were provided for mixing at three different speeds. (The speed of mixing refers to the paddle-tip speed.)

The test procedure involved filling the mixing tank, introducing the previously determined lime dose (the same for each test), then operating the mixing device at a speed of 0.5 ft per sec for one series of tests, 1.0 ft per sec for another, and 1.5 ft per sec for a third. Samples were dipped from the mixing tank at intervals of 10 min for each test series, thus providing samples representing 10-min mixing at a given speed, 20 min, 30-min, etc. Two samples were taken at each interval, one being permitted to settle in a jar for 1 hr and the duplicate sample for 4 hr.

As will be noted from the curves, the same "end-point" as determined by turbidity measurements was obtainable with a number of variations. For example, assuming that the desired turbidity of the clarified effluent is, say, 7 ppm, the combinations in Table 1, in so far as this water supply is concerned,

TABLE 1.—COMBINATIONS OF MIXING SPEED, MIXING TIME, AND SETTLING TIME TO PRODUCE A TANK EFFLUENT OF IDENTICAL TURBIDITY

(For This Illustration, Seven Parts per Million Was Selected as the Turbidity)

Speed of mixing, in feet per second	(a) SETTLING TIME, 1 HOUR		(b) SETTLING TIME, 4 HOURS	
	Series	Time of mixing, in minutes	Series	Time of mixing, in minutes
0 5	*a*	42	*d*	20
1 0	*b*	28	*e*	19
1 5	*c*	24	*f*	13

were found possible. These data indicate that, in general, for the same speed of mixing, cutting the mixing time in half requires quadrupling the detention time in the settling basins for equivalent clarity of effluent (compare Series *a* and *d* and Series *c* and *f*). It will also be noted that substantial reduction in time of mixing is possible at increased mixing speeds.

For example, the same result is obtained with 24-min mixing, at a speed of 1.5 ft per sec and 1 hr settling, as is obtained with equivalent mixing time (that is, 20 min) with only a speed of 0.5 ft per sec but with 4-hr settling (compare Series *c* and *d*). The cost of structures would be far greater in the latter case (Series *d*) than in the former (Series *c*). Similarly, sludge removal equipment would be more costly in the tanks providing for longer detention. Mixing equipment costs would be about the same, except that the drive units for the faster speed of mixing (Series *c*) would have to be powered substantially higher than that of the lower speed (Series *d*). Likewise, power consumption in the faster mix would obviously be much greater than that in the slower, the power requirements varying as the cube of the mixing speed. Thus, selection of final factors involves economic consideration.

It is the writer's opinion that most conditions involving mixing of chemicals with water for coagulation warrant the provision of variable speed devices, particularly in view of the changing characteristics of the water in many instances, requiring, in turn, varying dosages of lime, soda ash, etc., for soften-

ing. The ease with which most mechanical mixers may so be equipped merits their adoption over baffling and hydraulic methods not readily controllable.

The author has given the profession a valuable "bird's-eye view" of standard practice in the field of water-softening plant design. It would be of additional value if data could be presented relative to filter-washing practice in which lime or lime and soda ash are the treating chemicals. Does incrustation of filter sand with lime warrant differences in filter-washing provisions not required in ordinary rapid sand-filter practice?

ROLLIN F. MACDOWELL,[10] M. AM. SOC. C. E. (by letter).[10a]—Very concisely, Mr. Knox has covered the main considerations in the selection of the type and the design of municipal water-softening plants. The writer offers a few additional suggestions from his experience in this field.

Municipal water softening is a relatively new field in public service, but has already proved to be practical and economically sound. It is no longer considered sufficient to provide only a clean water, free from pathogenic bacteria, and thus the number of water-softening plants constructed is accelerating rapidly. As of January 1, 1938, there were 362 municipal plants installed, or under construction, in the United States, of which Ohio had 90, or approximately 25 per cent.

Mr. Knox states (see "Sources of Supply") that, if the hardness is more than 700 to 800 ppm, another source of water supply should be sought. Although this is true from a standpoint of cost, it does not mean that it is impractical to soften water containing 800 ppm, or more, of hardness. It is sometimes impossible to find another and softer source of supply. At Elmore, Ohio, the hardness of the raw water is about 1 060 ppm. This supply is softened to 70 ppm without difficulty and a satisfactory, palatable water is produced. The cost has not proved to be a burden. In fact, it was not necessary to raise the water rates in order to finance the cost of operation.

Aerators.—The writer agrees that it is not necessary to provide patented aerators in order to secure satisfactory results. Furthermore, non-patented aerators can be designed so as to be sightly. The patented aerators are usually entirely or partly of metal construction, which corrodes rapidly under the operating conditions that are usually most favorable to oxidation, thus causing the metal to corrode. Coke aerators can be made of concrete and cypress wood construction which is more permanent and requires less maintenance. Incidentally, if aerators are provided following softening, aluminum should not be used in the construction as the soft water pits this metal badly.

It is certainly true that sludge disposal "is one of the chief difficulties" of lime-soda water softening. This is especially the case in those municipalities, usually small, which are not on or near a sizeable stream into which the sludge can be discharged without sight nuisance. In such cases, and because of the sludge problem alone, the zeolite method of softening is usually advisable.

Type of Plant.—For every proposed plant careful consideration should be given by the designing engineer to all the factors involved in the determination

[10] Civ. and San. Engr., Cleveland, Ohio.

[10a] Received by the Secretary June 1, 1938.

of the type of plant to be provided. There are some factors, such as the sludge problem with lime-soda plants, which may make zeolite advisable aside from other considerations. On the other hand, if the raw water is from a surface source, or otherwise subject to pollution, or if the raw water has an objectionable fluoride content, the lime-soda method must be utilized; but, usually, the problem is largely an economic one, including first cost as well as cost of operation.

With the small plants having capacities as great as, say, 0.5 mgd, the saving in labor and the cost of technical control in the operation of zeolite plants quite often make this method advisable. This is especially true if automatic regeneration can be provided, thus eliminating the necessity for regular and frequent attention to the plant.

Troughs and Piping.—In gravity zeolite plants, because of the highly corrosive character of the salt brine used for regeneration, special attention should be given to the materials used for troughs and piping. For this reason, steel troughs are unsatisfactory, but concrete and asbestos cement have been used with success. For the collector and brine distribution systems, cast-iron pipe or material containing chromium and nickel is satisfactory.

Control Equipment.—With both gravity and pressure zeolite plants certain control equipment is found desirable. Automatic proportioning of the by-passed hard water is worth while, especially if the hardness of the raw water varies or two sources of raw water, having different degrees of hardness, are used.

With pressure zeolite plants in which the water is pumped from the source of supply or from a collecting basin directly through the pressure softeners, or through pressure filters and softeners, to the distribution system and elevated storage, provisions should be made to eliminate the surge in the pipe lines due to rapid starting of the pumps. For this purpose a slow opening discharge gate-valve, of the hydraulic type, operated electrically through the pump starter, is found to be effective.

Salt Storage.—The most economical method of purchasing and handling salt for regeneration is in bulk carload lots. A minimum carload is 40 tons. The wet salt storage basin, therefore, should have a capacity of at least 40 tons of dry salt, with ample free-board for float-valve control of the feed-water. Preferably, however, the basin should be divided into two compartments, each of which holds 40 tons of salt, in order that one compartment may be filled at any convenient time while the other is in service.

Brine pumps, which are usually of the centrifugal type, should be entirely brass-lined and mounted. If they cannot be located conveniently as to elevation to provide a positive head on the suction side, satisfactory self-priming units can be secured.

D. E. Davis,[11] M. Am. Soc. C. E. (by letter).[11a]—This paper will repay the careful study of those contemplating the construction of a softening plant. It

[11] Engr. (The Chester Engrs.), Pittsburgh, Pa.

[11a] Received by the Secretary July 5, 1938.

brings into sharp focus a long first-hand experience of the essential design features that should be incorporated if success is to be attained. Communities have been slow to recognize that softening plants are a good investment; but at chemical costs of 1 cent per 1 000 gal per 100 ppm of hardness removed, as quoted by the author, combined with reasonable fixed charges, it is usually found that soap savings alone more than justify the expenditure for a plant.

With good reason, the author stresses the desirability of short, direct lines for conveying chemicals to the mixers, with adequate provisions for cleaning, since lime in particular tends to build up a dense coating. No plant feature is more important than the mixing chamber, preferably of the mechanical type in which the raw water and the chemicals are brought into intimate contact at a speed of rotation less than that which will break the floc, but fast enough to permit the building up of a large aggregate which will quickly settle in the clarifiers. The flow of water may well be in an upward direction, thus maintaining the floc longer in suspension while it is being built.

Perforated baffles at the entrance and exit contribute to even flow through settling basins, and provide the "fly-wheel" or "cushion" of storage for meeting fluctuations in softener rates, the advisability of which is stressed by the author. The combination of a clarifier followed by a settling basin should give better and more flexible operation than a clarifier alone.

The disposal of the sludge, as the author shows, is one of the distinct limitations to the use of the lime-soda softening process under certain conditions, and more positive methods of meeting this situation may well be one of the design features in future plants.

Coke used for generating carbon dioxide for recarbonation should be low in sulfur content in order to prevent corrosion in compressors and lines. The author's suggestions relative to the design of piping which conveys water with a normal carbonate content is well taken, since it will usually be necessary to clean the deposit from these lines at intervals of several years.

The most careful study of the hydraulics of a zeolite plant is essential if head losses and sand rise are to be properly handled. The relationships between varying hardness, sand volumes, storage capacities, and wash and rinse waters require most careful study if the design is to prove successful.

In listing the disadvantages of zeolite, the author states that "objectionable proportions of iron and manganese are not suitable for zeolite treatment unless these minerals are removed prior to softening." This scarcely holds true for manganese in quantities as great as 7.0 ppm, and averaging 5.0 ppm for several years, because zeolite has been shown capable of removing these quantities continuously by base exchange and without damaging the green sand. However, manganese is not removed beyond the end of the period of zero hard water, but appears immediately with the first sign of exhaustion of the zeolite.

Professor W. F. Langlier has shown that changes in the calcium content of a water may have a material effect on its corrosive qualities.[12] Since in zeolite softening, calcium is removed or reduced, the resulting soft water is

[12] "The Analytical Control of Anti-Corrosion Treatment," by W. F. Langlier, *Journal, Am. Water Works Assoc.*, Vol. 28, No. 10, p. 1500.

likely to be corrosive. This is sometimes offset by an increase in pH which accompanies the action of some zeolites.

When dealing with a pure, clean, but hard water, a zeolite plant offers many advantages, including low first cost and "fool-proofness" in operation. The choice of type of softening plant for a given situation can only be determined after a careful examination and weighing of the various determining factors.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

MOTOR TRANSPORTATION—A FORWARD VIEW
A SYMPOSIUM

Discussion

BY MESSRS. F. LAVIS, EDGAR DOW GILMAN, GEORGE HARTLEY, ROBERT KINGERY, R. L. MORRISON, AND ROY F. BESSEY

F. LAVIS,[15] M. AM. SOC. C. E. (by letter).[15a]—The writer has been much interested in the paper by Mr. MacDonald, whose point of view is based on actual practice; but so much has been written during the past year or two about the "Highway of the Future" that the layman will expect this to develop at once into a clear, uninterrupted, wide pavement with cars "sailing along" at 100 miles per hr, or faster, with no stops for traffic lights or interference by traffic police, no grade crossings, no cars stopped on the pavement or traveled way, all traffic lanes separated, and perhaps (the Utopia) no noticeable expansion joints to mar the smoothness of the pavement.

With this ideal situation, pipes, sewers, and conduits will be laid outside the traveled way, so that pavements need never be disturbed and these latter will be so durable that they need never be relaid. There will be wide shoulders and rights of way, and the roadside will be beautifully landscaped.

In 1937, a speaker was reported by the Associated Press to have told the National Planning Conference in Detroit, Mich., that, "The American street and highway system must be scrapped and rebuilt unless the automobile is to become a malignant growth." "We cannot," he said "look with tolerance upon shackles which fetter this newest servant of mankind"; and then he also blamed the highway structure for the great toll of accidents on it.

Apparently, it is to be taken for granted that in this Utopia, the motor vehicles themselves will be of supreme design and construction, built of materials that cannot fail, that will have no mechanical defects, that will, perhaps, take their fuel out of the air so as to avoid stopping, and will be driven by supermen from Mars. The latter is the most essential because the frail humanity of

NOTE.—This Symposium was presented at the meeting of the Highway Division, Detroit, Mich., July 21, 1937 and published in June, 1938, *Proceedings*. This discussion is printed in *Proceedings*, in order that the views expressed may be brought before all members for further discussion of the Symposium.

[15] Cons. Engr., New York, N. Y.

[15a] Received by the Secretary August 2, 1938.

this day and age has apparently not cultivated the skill and stamina to take advantage of these transcendental highways and vehicles, and as one reviews the slow progress in the development of the human machine it scarcely seems likely that succeeding generations will attain the ability and skill to operate these super-motor vehicles about which so much is being written.

The writer is entirely serious in attempting to create this vision because it must be given consideration. It is being preached in many quarters, not by laymen alone but by engineers—engineers who claim that most of their fellow craftsmen have no thought for the morrow, no vision of the future. These statements must be given consideration, but consideration based on some sanity.

The writer is not unmindful of the developments in both cars and highways of the past thirty years, since the time that he bought his first car in 1908 and thought a 100-mile drive in a day was a feat, a perilous adventure, and a difficult, grueling task. Without effort, he can compare that car with the car of to-day, which will go the 100 miles in a few hours with almost no apparent effort either on the part of the car or of the driver; and he has no doubt that similar or even greater progress will continue.

After looking at these visions, however, engineers should look at the realities of this long-distance planning and attempted provision for the future and visualize the real status of their attitude toward the highway of the future (which is being built to-day), and think about these problems with sense and with their "feet on the ground."

It may be fairly claimed that highway engineers and highway authorities, without having indulged in any extravagant visions of the future, have contributed their share to this advance, and their progress to date has fully kept pace with that of the automotive engineer. There is no reason why they may not be expected to continue going along together.

The writer is not opposed to planning. For many years, while engaged in the planning and design of railroads, his work has been nothing but a long attempt to forecast the future properly and reasonably, and to build for the future, but always with the brake on his enthusiasm of having to "make a dollar earn the most interest." It seems to be a most desirable training for engineers to have been connected with commercial enterprises. Engineers building highways with public funds would do well to think, for some years to come, of the need of making public dollars earn the most interest rather than of providing special facilities for the few or of building highways for excessively high speeds.

It is trite to state that one should do the greatest good to the greatest number; but trite or not, it is one of the great duties of the engineer who spends public monies, and perhaps particularly that of the engineer who directs the spending of them for highways. This does not gainsay the fact that returns from the expenditures of public monies may reach the dollar stage, through health and happiness from recreation; but engineers should be reasonably certain that there is a real return some way or other, and that this return should be a return to the people at large and not to a special few.

Writers on highway subjects during the past year or two have very generally stressed two things: (1) The great number of highway accidents; and (2) the need of "building safety into highways." Strangely enough, many who have deplored the accidents and urged engineers to "do something about it," talk about designing highways for speeds of 100 miles per hr, or even more. They point to the fact that the motor vehicle is being improved every year and that when a highway is built it must remain unchanged for a long time. Consequently, one must build to-day the highway that will meet the requirements of 10, 15, 20, or 25 yr hence, and that, at first thought (and maybe on second thought) seems a very valid argument, particularly if the human element behind the wheel of the car is ignored.

The highest types of modern pavements may be expected to have a life of 20 or 25 yr, even under heavy traffic. There is no possible reason, therefore, for denying the responsibility of the engineer in laying them out so that the most use may be obtained from them during their life.

Accidents and accident prevention are primary topics of consideration today, and accidents on highways are a growing menace to the entire body politic. Furthermore, the prevalence of accidents is now tending to be greater in the open country, in the rural districts, than in cities and congested areas. British experience in this respect, coincides with that of the United States, and a reasonable inference from this information is that excessive speeds are a predominant cause.

It has been quite generally stated that 85% of all accidents are caused by the failure of the human element, the driver of the car. On the other hand, it is claimed that accident reports do not show the extent or the occasions in which the highway structure itself is the cause. Admitting that this may be true, to some extent, and that, in some cases, faults of the highway structure may contribute to, or aggravate, accidents and possibly increase their severity, it still remains a fact that highway accidents are very largely the result of incompetent and careless driving and excessive speed. Here, again, British statistics and British observers are in accord with those in the United States. A New York State report[16] published in 1937 states: "So long as the motor vehicle is employed as a means of transportation the driver will remain the chief factor in the accident problem."

When writers refer so glibly, therefore, to designing highways for speeds of 100 miles per hr, they not only ignore this very definite fact of human fallibility but they probably are definitely increasing the accident risk. The writer does not believe that design for high speed provides a margin of safety for drivers at lower speeds. Few drivers are capable of driving faster than 40 to 50 miles per hr, with safety to themselves and to the other users of the highways.

This question of speed as it affects the design of highways and their cost is a most important one. Designs based on speeds greater than 50 to 60 miles per hr are not warranted to-day, or by any reasonable consideration of the future.

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[16] Annual Rept., New York State Bureau of Taxation and Finance, New York *Herald-Tribune,* July 18, 1937.

Most engineers visualize the highway of the future as something in the nature of the trunk-line military "autobahen" created in Germany, the so-called "freeways," or the parkways near New York City, N. Y., in Westchester County, and on Long Island. These are typical of high-capacity highways in open country. There are also the great arterial highways through congested areas such as New Jersey Route 25, from the Holland Tunnel to Elizabeth, of which the Pulaski Skyway is a part, the great West Side Highway in New York City, and others of a similar type.

In general, such highways have good alignment, gradients that do not require trucks to reduce speed rates, say, of less than 5%, and four to six traffic lanes with traffic in opposing directions definitely separated by a raised curbed strip, perhaps 3 to 5 ft wide.

All these very expensive roads, however, have been built to meet heavy traffic demands, and they carry large numbers of vehicles. It is well known, of course, that maximum traffic capacity is attained at comparatively low speeds, probably less than 30 to 35 miles per hr. There seems to be little reason, therefore, for assuming speeds much greater than this, certainly not greater than 50 to 60 miles per hr, in designing such roads.

Probably the most important task that confronts highway authorities and highway engineers to-day is the provision of sufficient highway capacity to meet traffic demands. This means that new highways, built to-day, should have capacity for meeting the traffic demands of some years to come. Ability to estimate such demands with reasonable accuracy is a necessary qualification for all responsible highway engineers.

Such engineers also should have some reasonable appreciation of the economic relation of expenditures to highway capacity and highway use. Expenditures in this case mean total annual cost—that is, interest on investment, plus annual cost of upkeep. These factors should have a definite relation to traffic volume and operating costs. Of course, highway engineering cannot be entirely evaluated in dollars and cents but the nearer highway engineers can come to such evaluation, the nearer they will approach reasonable economic justification for the large expenditures they make.

The writer is inclined to believe that where there are legitimate criticisms of highway design and highway structures this is due to incompetent engineering rather than to any lack of engineering knowledge by those who keep abreast of the developments of the times. This, however, is not a failing confined to highway engineering alone.

Because of the general nature of highway work as part of a public works program, by which the appointment of engineers and the methods of their functioning may, in some cases, is dictated by political needs and exigencies, there is probably some incompetence in road design and construction and this is especially true in the smaller governmental units and rural districts. Such defects as result from this source cannot be enumerated or specified herein but the writer suspects that these are the defects that cause writers and commentators to refer to certain roads as obsolete before they are completed and to

place the blame for accidents on such defects. The remedy is better engineering by properly qualified engineers.

Most important highway officials and engineers are fully alive to the necessity of meeting future needs, and know how to do it; but they evaluate these needs with some sanity and common sense. The writer, however, would like to mention, very briefly, some of the features of modern road construction that should receive general attention.

Most of the States to-day have reasonably well conceived plans of main and secondary highways and are fully alive to the need of adequate surveys both of terrain and traffic. The importance of securing ample rights of way is fairly generally recognized, but there should be a reasonable balance between providing for the future and the permanent alienation of taxable values. It is probably difficult to evolve any formulas that will guide the engineer in adjusting alignment and gradients to the topography. The old-time railroad locating engineer had his Wellington and values for curvature, distance, rise and fall, and rates of gradient. The highway locating engineer of to-day has little to guide him, but experience and judgment. It is here that communities and governments would be wise and well advised in procuring the widest experience and best judgment. Those responsible for highway location should probably err (if they err at all) in laying out the best alignment and gradients they can, to fit the topography and terrain reasonably well. It is suggested that sight-distance diagrams should be developed, at least in all cases of doubt.

The present practice of separating the traffic on four-lane roads by a center strip may well be extended. There is some tendency to build traffic lanes wider than 10 ft. This is a question that should receive careful consideration as to its economic justification. Probably one of the most serious defects of some modern roads is their obstruction by stopped vehicles. Roadside filling stations and eating places probably serve a public need and help taxable property, but provision should be made for stopping vehicles off the main traveled way instead of on it. It is undoubtedly desirable that consideration be given to the provision of sidewalks, at least on some highways.

The provision of reasonably wide hard shoulders and proper forms of side ditches is now generally recognized both as a necessary safety measure and as an aid to increased capacity. It would seem advisable to have all roadside structures set back a reasonable distance from the pavement or traveled way.

It should not be necessary to refer to narrow bridges, except for the fact that so many exist as interruptions to the free flow of traffic on otherwise good roads. It should be a recognized necessary attribute of road improvement that the general newly established character of such roads should be maintained throughout the length of the improvement.

It seems generally desirable that new roads should be designed so that they maintain certain general characteristics of alignment, gradients, width of pavement, etc., for considerable distances; but there seems to be no valid reason, why if proper signs and signals are displayed, changes in these characteristics may not be made if adequate considerations exist to warrant them and drivers

are properly notified. A certain standardization of warning signs and the method of their display is most desirable.

All these features are well within the purview of the competent, informed highway engineer of to-day. In view of the needs of the vast network of roads all over the United States, it is doubtful whether highway planners can afford to be too visionary. Grade crossings with other highways and with railways can only be eliminated in special cases. The construction of expensive "free-ways" to facilitate certain traffic must be weighed against the needs of the entire highway net and particularly their effect on taxable values of adjacent lands.

Perhaps the time has come when the construction of toll highways for trucks may be considered, but this involves consideration of the economic effect of maintaining the railway system, which seems to be gradually drifting toward Government control and possibly support by the taxpayers. Both railways and highways are tending to become part of one general network of land transportation.

Viewing the entire problem as a whole, it seems that engineers cannot go much further to-day than to adopt the best current practice and to continue improving it as the need arises. What is really needed is to make certain that highway problems are placed and kept in the hands of engineers competent to solve them.

No one expects to stand still, because standing still is really going backward; but, on the other hand, highway engineers are not to be stampeded into attempting to build highways for super-cars, driven by super-men who do not exist.

Engineers (that is, competent engineers) have always been as forward looking and alert as other people—perhaps more so. They should continue to justify that tradition, but should continue trying to make their dollars earn the most interest.

Edgar Dow Gilman,[17] M. Am. Soc. C. E. (by letter).[17a]—The Symposium entitled "Increasing the Traffic Capacity and Safety of Thoroughfares"[18] raised many of the same questions that the present Symposium now raises.

First, one wonders if, in giving consideration to traffic problems, the word, "traffic," does not mean the movement of all people and not just the movement of motor vehicles. Sidney J. Williams, M. Am. Soc. C. E., in his comment[19] on the Symposium, stated that "much attention has been given to the time lost by automobilists on account of pedestrians, but little attention has been paid to the time lost by pedestrians on account of automobilists." This is a "ray of light" that should be brightened. Traffic regulation should be for masses of people and not for individuals.

Second, although the writer's natural instinct would be to emphasize the importance of the Engineering Profession in traffic problems, studies of hun-

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[17] Director, Dept. of Public Utilities, Cincinnati, Ohio.

[17a] Received by the Secretary June 22, 1938.

[18] *Proceedings*, Am. Soc. C. E., November, 1937, p. 1741.

[19] *Loc. cit.*, February, 1938, p. 395.

dreds of accidents prove quite conclusively that only a small proportion of accidents are traceable to conditions in which pure engineering is a factor. In fact, there are data to indicate that as engineering improves conditions at certain points, the accidents increase rather than decrease. Some of the points in the city that are obviously the most dangerous have the fewest accidents. The obviousness of the danger seems to be one of the best deterrents to accidents that has been found. As roads are straightened, automobile drivers take more chances and are not as attentive. As a matter of fact, the situation can be summed up in this statement: "If people drove as they should drive, they could drive perfectly safely even in unsafe places."

Studies of events leading to an accident show that the vast majority of accidents are caused by influences that no act of engineering, short of complete separation and isolation of traffic lanes, could avoid. They are caused primarily by foolishness and carelessness, selfishness, lack of courtesy, and lack of consideration on the part of drivers of cars. No mechanical devices or physical rearrangement of streets can be substituted for the manner (or lack of manners) in which people handle automobiles.

The full question of engineering, although important, has been emphasized to the point where automobile drivers use the theory as an alibi for their own actions. This does not mean that the engineering should not be made as beneficial as possible, but engineers are leading themselves into "the hole" of building up a point of view on the part of the public that accidents are the result of something they have done wrong or something they have failed to do, and that drivers themselves cannot be held responsible unless and until the engineering is something highly idealistic and hypothetical and beyond all practical means of attainment. The engineering should go ahead, but the emphasis should be placed on the drivers and not on the engineering.

Mr. Kettering states (see heading,' 'Lighting—Road or Vehicle?"), "authorities agree on one fact: Night-driving conditions are far behind day-time conditions, and for one reason—lack of adequate lighting and glaring headlights." Many data have been published showing that accidents at night are more numerous than accidents in the day time and in many of the papers including such data the definite conclusion is drawn that this is due to the absence of light, without any consideration of the many other factors in night driving. Of late, a few writers have attempted to proportion these factors properly. Certainly there are enough other factors to justify some question as to whether lack of adequate light on streets and highways is the one reason for a greater number of accidents at night than in the day time.

Nearly all day-time driving is done by people who are soberly intent on the particular accomplishment of a business in hand. It is after the day's work is done that social functions and parties are held. Practically all the driving under the influence of liquor is done at night. Accident records show that this is a big factor. The element of fatigue enters into night driving to a considerably greater extent that it does in day-time driving. Many police records of accidents at night have the notation "Driver fell asleep." All the young men working, and in most cases those not working, use the evening to drive with

their "best girls," and their attention is not always on their driving under these conditions.

A mere tabulation of the time of day that accidents occur does not present a justification for the conclusion that night-driving hazards can be reduced at least to day-time driving hazards by lighting the highways. Furthermore, some serious thought should be given to the question as to what would be the effect of illuminating certain highways and not others. It is admitted that it would be impracticable to light all. What will be the effect of lighting some to an extent that will make it safe to go at certain speeds, upon the speed of drivers on other highways where it would not be safe to go at anywhere near those speeds?

If the mental attitude of the mass of drivers could be "built up" to a point where good judgment could be counted upon, this would not be a problem; but in view of records of the immediate past is one justified in assuming that the drivers of cars in mass will exercise good judgment in the manner in which they drive? It is a strange commentary on the entire situation to say that if the streets were studded with cast-iron posts set in the road, requiring automobiles to be driven a tortuous and difficult path, there would probably be no accidents and no traffic deaths. Of course, there would be no speed and no mass movement in short times, and this is not desirable.

It is good engineering to pull out these cast-iron posts from the middle of highways so that automobiles can travel in straight lines, and engineers, very properly, study and eliminate these physical obstructions to movement; but it is a fair question to ask whether they are not improperly designating such things by saying that they are eliminating "hazards" when actually, due to the psychology of the situation, they may many times be adding an intangible type of hazard when they are removing a physical obstacle.

Sometimes, it seems that the public has placed the engineer in a very unfavorable position, that in which engineers are taking the responsibility even where it is not an engineering problem. Coincident with the study of engineering in traffic, engineers should constantly emphasize the fact that the effect of engineering can be but minor on accident records and that deaths and injuries and accidents are largely a result of the action of the people themselves. Although engineers will work to try to get greater utilization of motor vehicles, there is a responsibility on the part of the people who use motor vehicles over which the engineer can have no control.

This feeling that the engineer is responsible for the accidents has progressed to such an extent that there is a one-page article in a popular magazine of recent date rather strongly setting forth the claim that it is the duty of the engineer to separate roadways and crossings and make it impossible for children and pedestrians to get in the path of automobiles. It is a strange mind that can take such a position that this is the sole duty of the engineer. The engineering involved would present no great difficulties. The big problem in the accomplishment of such plans is that of financing, and this means taxes. Engineers can submit billions of dollars worth of plans for eliminating intersectional highway crossings in high-sounding terms of medial and other frictions. The

plans will involve a multitude of details, but none that presents insurmountable engineering obstacles. The obstacle to such plans is in the financing and the payment by the public for such idealistic construction. The public has not been able to provide the necessary money with which to eliminate the relatively few railroad grade crossings, and yet in the article referred to the statement is made that it is "up to the engineer" to provide road separations and other tremendously expensive structures.

Engineers should not complacently permit themselves to be thrust into a false position in the traffic situation. They can do much to facilitate traffic movement if people will use their plans as they are supposed to be used. They can do something to add to traffic safety. They cannot do anything that will be a substitute for good judgment, common sense, and decency on the part of the man who has his hands on the wheel and his foot on the accelerator. The attitude of the public itself is the major problem in the safety angle of traffic. The public says it wants safety, but it resents enforcement and there has actually grown up a predominating belief that the problem of traffic safety is a contest in which the police and other enforcement agencies are on one side and the public itself is on the opposite side, fighting against each other. The problem of traffic safety is a problem of mass psychology—the establishment in the minds of the masses of people of the true factors that cause accidents. Engineers are too readily letting this fact sink into oblivion.

George Hartley,[20] Esq. (by letter).[20a]—In the group of papers comprising this Symposium a valuable contribution to the art of planning future highways is ably presented. In his interesting résumé, Mr. Kettering has illustrated, splendidly, the possible future of highway travel. The entire Symposium would have been rendered more complete by the inclusion of a few additional details.

The major part of Mr. Kettering's paper evidently represents careful editing of replies to the questionnaire (Items (1) to (18) under "Introduction") by the various contributors listed under "Acknowledgments." It would have been of interest had it been possible to include typical replies or edited replies to show the range of opinion expressed by the various contributors. Views not directly in accord with those of the authors might have been discarded or considered insignificant.

Each of the authors attacks the problem from a theoretical engineering standpoint with only a slight reference to the importance of motor vehicle laws and their relation to engineering design. What good will a highway or motor vehicle, designed for, say, 80 miles per hr, be if some provision is not made to bring the law into such form as to allow speeds approaching this value. Engineers are constantly thinking in terms of increasing speed but, to-day, even on well traveled secondary highways, posted speed limits of 15 and 20 miles per hr are not uncommon and the main express highways, rigorously patrolled, keep travel at a speed representing only a fraction of that for which the highway was designed. Safety proponents are always demand-

[20] With Robinson & Steinman, New York, N. Y.

[20a] Received by the Secretary July 15, 1938.

ing slower speeds and these bodies are constantly increasing in number and importance. Any serious attempt to revise present highway laws so as to allow increased speeds will surely meet organized opposition.

Some mention should also have been made in regard to some form of standardized financing. Funds for any one structure may come from a number of sources with the subsequent duplication of services. Surely, in a properly planned highway, the financing should be arranged in such form as to permit steady growth and permanence.

Messrs. Smith and Kettering discuss the parking problem in terms of a future Utopia without referring to the parking-meter system. Intended originally as an added tax-gathering system these meters are being installed at a rapid rate in crowded business districts. They create convenient short-time parking in shopping districts where once it was practically impossible to park.

Mr. MacDonald condemns the design practice of the past without due consideration of the fact that these highways were built along single standards, because a certain minimum of width and construction were required regardless of service requirements. That many of these structures are still in service after a score or more years is truly a tribute to their designers. The older bridges in the United States (some of them more than fifty years old) also carry modern traffic. Fifty years ago when the horse and buggy represented the acme of transportation on the highways, who might have forecast or planned future traffic requirements? With their planned programs and improved materials, can engineers to-day design structures that will serve throughout the life of a planned program? May not the entire viewpoint be completely revised in only a few years by new inventions or even the technique of planning itself? If so, the present generation of engineers must, in turn, await the criticism of its successors. The development of express highways to by-pass crowded centers of population may be one of the major factors in causing a revision of planning methods in Germany where the express highway established on a larger scale than in the United States, is already attracting commercial establishments with the attendant growth of adjacent residential districts.

ROBERT KINGERY,[21] ESQ. (by letter).[21a]—As Mr. Smith states (see heading, "Introduction") no identical pattern or design of a road facility can be transplanted directly from one metropolitan area to another. However, the good designs and successful operation of a given type of highway facility may be adapted to conditions in most metropolitan centers. Without question the original boulevard system in Chicago, Ill., laid out and acquired in 1869 and 1870, has been imitated in many parts of the world. Perhaps the original was an imitation of some other system. Members of the Regional Planning Association of Chicago have profited not only from the Wayne County Highway, and the Detroit Rapid Transit policies, but have studied and have incorporated, in major thoroughfares and super-highway designs, some of the parkway fea-

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[21] Secy., Chicago Regional Planning Assoc., Chicago, Ill.

[21a] Received by the Secretary July 21, 1938.

tures from Westchester County, New York, and from the Mount Vernon Parkway, near Washington, D. C.

In the ten years since, in 1928, the Chicago Regional Planning Association originally laid out a program of major highways, super-highways, boulevards, or parkways, almost exactly one-fifth of that program has been completed. It is now (1938) advancing at a much more rapid rate. Of the 603-mile system in the 15-county area, 127 miles of right of way from 160 ft to 240 ft in width have been acquired, and 114 miles of pavement have been built, all outside the City of Chicago itself.

Within the city about 14 miles of major trunk traffic artery of the parkway type have been completed along the lake shore from a point 6.5 miles north of the center of the city to a point 7.5 miles south. Entrance to the city from other directions is more difficult, there being no existing broad rights of way. Many studies have been made by the Chicago Plan Commission and by other official and unofficial bodies and now it seems more hopeful than ever before that a decision upon a comprehensive plan will soon be agreed upon, and construction begun by the co-operation of several highway building agencies.

However, one note of caution might be injected into the subject of Mr. Smith's paper. At many meetings of city planners and in many papers about metropolitan highway planning and about zoning one will hear or read of the conception of the dreamer who pictures cities with buildings and towers a thousand feet high, with miles of two-decked, three-decked, and multi-decked highways in centers of population concentration. To some planners this type of thinking is in the wrong direction. There is adequate evidence of the result of excess concentration of population and of commerce in small centers. One example among many is the fact that closely surrounding such an intensive concentration of high buildings is almost always a broad belt in which disintegration is far advanced. The City of Chicago has such a belt, which is 10 miles long and more than 3 miles wide. It covers 36 sq miles, or one-sixth the area of the city; and more than 260 000 people have moved from it during the past twenty years. Many of the tax delinquencies are in this area; and it includes a substantial number of both juvenile and adult delinquencies which form the principal police problem; as well as a substantial majority of fire-protection problems. Tax receipts from this area do not nearly support the governmental services to it. In slightly modified form, the same can be said of every large metropolitan area.

Although high buildings with multi-decked highways—with masts for the anchorage of dirigibles, with landing fields for the clouds of aeroplanes that are seen in photographs of the "City-of-the-Future"—are all stirring to the imagination, by no means do they involve solvency of the municipal pocket-book.

Construction of highway facilities, of course, will not solve all these problems, but it can come nearer to it. Penetrating the centers of these large cities and surrounding the business centers of them in the so-called blighted areas, the development of exceptionally broad rights of way or parkways with adequate space for pavements—on the ground level, elevated, or depressed—with adequate space for grass and trees will, in the judgment of many, do more to

reconstruct the city at its core than almost any other single influence. With the construction of such radial or belt line boulevards (or both), and with the addition of adequate parks and playgrounds in such a zone, and with courageous and intelligent zoning or re-zoning of the neighborhood, property values will be restored for residential occupancy of much of that area.

For some decades too much attention has been given to the construction of buildings in the development of cities in the United States, and too little to their setting and their surroundings. The introduction of more open space in the form of boulevards and parkways is one of the greatest services that the highway officials in metropolitan areas can perform, not only toward facilitating traffic movement, but toward salvaging and even enhancing the value of the metropolitan area as a whole. There would follow the further introduction, by park authorities, of the parks and playgrounds with their trees and grass, and the intelligent, unselfish and courageous zoning or re-zoning of large units of the metropolis. The result will be a better rounded community and sounder and safer distribution of traffic volumes.

R. L. Morrison,[22] M. Am. Soc. C. E. (by letter).[22a]—One of the most outstanding items in Mr. Kettering's interesting paper is the constantly increasing maximum speed of automobiles from about 58 miles per hr in 1926 to about 87 miles per hr in 1937. This is an increase of 50% in 11 yr, and the curve is almost a straight line, with practically no indication that a top limit is even being approached.

Mr. Kettering states that "people, on the average, do not want to travel much faster than 60 miles per hr," and perhaps that is a reasonably safe speed on a good clear road in the daytime. Investigations made by the Michigan State Highway Department in 1936 showed average speeds of less than 50 miles per hr, but the fastest 15% of vehicles were driven more than 65 miles per hr and, on one route, 70 miles per hr.[23] This means that many of the drivers must have been using the entire speed capacity of their cars. Darkness slowed down this fastest 15% only 2 or 3 miles per hr although, even with good brakes and lights, they could not possibly stop before hitting a dark object on the road.

While the maximum speed of vehicles was increasing 50% between 1930 and 1937, the fatalities on rural roads (the only ones on which maximum speeds can be used) increased about 37%, and the rural night accidents increased 60 per cent.[24] The increase in automobile registrations during the same period was less than 5 per cent. A close relationship between increased speed capacity of cars and increased rural night fatalities is evident.

Mr. Kettering states that the roads "where it is necessary should be above or below the ground. * * * Crossings should be eliminated * * *," and other safety features, which he lists, should be provided. Such roads would certainly cost at least $300 000 per mile. If 10% of all motor taxes could be spent

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[22] Prof. of Highway Eng. and Highway Transport, Univ. of Michigan, Ann Arbor, Mich.

[22a] Received by the Secretary July 22, 1938.

[23] "Graduated Speed Regulations," by D. Grant Mickle, *Proceedings*, 1937 Michigan Highway Conference, pp. 14–25.

[24] "Accident Facts," 1938 Edition, pp. 28 and 36.

entirely on the construction of these superhighways (which is very unlikely), it would be possible to build about 300 miles per yr. Therefore, at the present rate of revenues, it would take nearly 200 yr to build the 50 000 or 60 000 miles of superhighways which are assumed to be needed. It is impossible for the writer to see how superhighways can ever make more than "a small dent" in the accident rate, assuming that they do reduce accidents where they are built.

The paper discusses two items, the vehicle and the road, both of which are constantly being improved. The cars go faster each year and the roads are gradually being changed to facilitate fast driving; but, unfortunately, there is a third factor, and a very important one. It is very doubtful whether this third factor—the driver—had any more inherent ability to handle cars at high speeds in 1937 than he had in 1926. He doubtless developed a little additional skill during that time, but most accidents are caused by lack of judgment rather than by lack of skill, and certainly there is little evidence that the judgment of the average driver has improved. In fact, the less judgment he has the faster he goes. In other words, the automotive engineers and civil engineers are constantly producing cars and roads with speed capacities more and more beyond the ability of average drivers to use them safely. Of course, this is done to meet the desires of the motorists whom they both serve.

To state the problem is much easier than to offer a solution. It can be said, however, that regardless of the responsibilities of the automotive and civil engineers, public officials have a very definite responsibility to use all reasonable methods to reduce accidents, and this is a responsibility that is very far from being met in many localities. For instance, there are scarcely more than 100 traffic engineers in the entire United States, and when a State or city does decide to have one the usual procedure is not to find and appoint an experienced traffic engineer, but to appoint to that office some engineer who has no knowledge of traffic engineering. Residence in the State or city concerned is usually considered of much greater importance than experience in the work which he is expected to do. A number of States still do not require standard driver's licenses, and, in many places, both traffic laws and their enforcement are far below what they should be.

Although Mr. MacDonald does not make the exact statement in his paper, it seems to be his guiding principle, that highways should be built so as to be of maximum service to the motorists who use and pay for them. Maximum service to the motorist through highway improvement includes decreasing his operating costs, saving his time, and increasing his safety, comfort, and general enjoyment of travel. It also includes making new areas available to travel for recreation or for other purposes. Of course, there are many other "benefits of good roads" of a more or less incidental nature. The main problem before highway administrators is to determine how available funds should be expended in order to render maximum service to the greatest number of highway users.

Unfortunately, it is the spectacular that is likely to appeal most to the general public and to politicians in high places. One school bus is struck by a train and immediately almost a billion dollars is appropriated for railway

grade separations, although the same sum of money spent for such things as crossing gates, better street-lighting, the flood-lighting of street obstructions, and more efficient policing would probably save a vastly greater number of lives. Other billions are proposed for building four or six-lane roads across the deserts to accommodate a largely imaginary coast-to-coast or border-to-border traffic although millions of dollars are wasted annually by motorists traveling over untreated gravel roads and poor earth roads, and in crawling through congested city streets for the lack of by-passes around the cities.

Broad vision of future needs is required for real progress in any field, but it is possible that sound economics is more needed than dreams and visions in planning highway improvements for the next few years. Perhaps the greatest need of all is a determination of what constitutes sound highway economics. Even in that field there seems to be a great temptation to pile up vast imaginary savings. This is done by taking the difference in operating costs over pavements and over earth roads and assuming that every mile operated on a pavement results in this saving. This involves the assumption, of course, that all the millions of vehicles now using pavements would be traveling on earth roads if the pavements were not there, which seems palpably absurd. If these imaginary savings could be used to finance the super-highways on the desert all would be well.

The very real savings that can be made to present highway users through sound improvements justify the kind of road-building program outlined by Mr. MacDonald.

Roy F. Bessey,[25] M. Am. Soc. C. E. (by letter).[25a]—This Symposium is thought-provoking, timely, and valuable. The flattening out of motor vehicle curves shown by Mr. Kettering does not indicate the end of the great national task of modern highway building, but rather the transition from the era of pioneer construction to the era of reconstruction, readjustment, and improvement for greater efficiency, safety, and beauty.

Those responsible for highway design and construction have done an excellent job under difficult conditions, trying to keep pace with very rapidly developing or changing needs—with the growth of population, cities, industry, commerce, distribution, and technics, and, particularly, with the mushroom expansion of the automotive industry in both technics and production. In their thinking, many of those responsible have done much better than keep pace with public opinion in appreciation of many important requirements—and, consequently, have been far ahead of adequate public financial support in meeting conditions.

Although a great national network of good roads has been created, it may be concluded that transition from the pioneer stage of motor highway development is still under way. Advances in technology generally, and particularly in the field of automotive equipment; increasing density, diversity, volume and speed of traffic; and growing needs in safety, convenience, comfort, time-

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[25a] Received by the Secretary July 25, 1938.

saving, and recreation—all call for further, probably radical, improvements in highway design and construction.

The pressures upon highway design and construction have been so great until recent years that there has been little time and means for refinement; but the present is none too soon to complete a new estimate of the situation—to re-evaluate needs and practices and to overhaul standards and plans. For this work the current nation-wide highway-planning survey is providing the essential data.

A number of desirable changes and adjustments are indicated in the Symposium. More will be disclosed by the analyses of the planning survey. Others will be found in city, metropolitan, county, district, State, regional, and national planning studies. Obviously, highway plans will have to be adjusted: (*a*) To the shifts of population and industries out of the centers of cities; (*b*) to other changes in population and industrial patterns; (*c*) to changing land use; (*d*) to activities relating to the conservation or exploitation of natural and economic resources (including flood and erosion control); (*e*) to other means of transportation (including rapid transit); and (*f*) to various other physical, economic, and social changes.

Obviously, too, it will be desirable to transfer some of the emphasis in highway design and construction from questions, such as line, grade, and pavement, in which the art is relatively far advanced, to others, such as surface, lighting, parking, rights of way, and roadsides, which (perhaps unavoidably) have had less attention in the past. All the latter features are increasingly becoming of importance to highway efficiency and safety. Wide rights of way and suitable roadside control will be particularly important factors in delaying absolescense.

From a regional planning viewpoint, the new highway era seems very definitely to involve three outstanding problems and programs:

(1) Trunk highway systems—modernization;

(2) Secondary road systems—more complete co-ordination with such resources and facilities as forests, recreational areas, mineral areas, present and probable future land use, schools, etc.; and,

(3) Co-ordinated transportation—establishment of more adequate relationships of one means of transportation with another, and with the entire system.

Requirements for trunk highway modernization will vary considerably (with geographical location, density of population, kind of traffic, etc.); but there are many requirements common to various conditions, such, for example, as location, alignment, grade, roadway and pavement standards to permit fast and continuous travel; freedom from the threats to movement; the safety and convenience involved in too-frequent crossings and points of ingress and egress, in roadway or roadside parking, in improper or excessive roadside use and obstruction, in conflicts with city and town congestions; provision for pedestrians; and provision for parking on sites away from the roadway. Fortunately, an outstanding requirement of recreational highways—the control of

roadsides from the aesthetic standpoint—harmonizes generally with the requirements of commercial traffic and involves broad economic values as well.

The "freeway" or "limited motorway," with its wide rights of way, separation of opposing traffic lanes, grade separations, rotary intersections, border controls, limited points of access, etc., seems to provide the general solution and to indicate the directions in which the most effective improvements can be made in trunk highways.

A very intimate connection between highway-planning and land-use inventories and plans would seem to be the key to the most rational and efficient development of secondary road systems.

The advantages of co-ordinated transportation need not be elaborated. Not the least is elimination of much of uneconomic or needless duplication and competition. Some of the directions for study and planning of co-ordinating features in the highway field might be exemplified: Feeder lines for railroads, rapid transit lines, waterways, ports; integrated terminal facilities; interchangeable freight bodies or containers; roadside emergency airplane landing fields; and arterial express highways co-ordinated with airport locations.

In passing into the new era of automotive transport, a higher degree of co-operation is essential. Now that some of the urgencies of the pioneer period have diminished, it is to be assumed that close co-operation is more practicable. Of course, this co-operation must be based upon a mutual willingness to strive to keep design in automotive equipment and design in highways in close harmony. A closer co-operation is also required as to highway location, design, construction, and operation between various levels of government, between adjoining governmental jurisdictions, and between those responsible for highways and those concerned with other means of transportation.

Finally, if broad highway planning is to reach maximum practicable rationality and effectiveness, quickly, it should be very closely related to national, regional, State and local planning programs, and to functional planning in such fields as land use, zoning, industrial location, transportation, resource development, and recreation. Of course, the comprehensive highway planning work will be reciprocally valuable in the general planning programs.

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DISCUSSIONS

ECONOMIC ASPECTS OF ENERGY GENERATION
A SYMPOSIUM

Discussion

BY MESSRS. JOEL D. JUSTIN, AND PHILIP SPORN

JOEL D. JUSTIN,[51] M. AM. SOC. C. E. (by letter).[51a]—In view of the fact that there has always been considerable discussion as to the relative cost of transporting energy long distances, as coal by railroad, and of transporting it as electricity over transmission lines, a concrete case may be of interest.

Several years ago it was proposed to construct a large steam plant on the Susquehanna River adjacent to the anthracite mines of Pennsylvania, utilize low-value anthracite (very nearly a waste product), and transmit the electric energy to Philadelphia, a transmission distance of about 150 miles.

However, a thorough investigation of the proposal proved that it would be very much more economical to locate the proposed steam plant at, or near, Philadelphia and ship the energy as coal over the rails to the plant. This kind of transmission, by which power is transmitted long distances without take-offs, might be termed "express transmission," and it is generally uneconomical except in the case of certain low-cost hydro-electric projects. This does not necessarily mean that long-distance transmission is uneconomical, however. Often, a minor load center reached by transmission could not alone support a large, low-cost, efficient steam plant, but several of these centers inter-connected may make such a plant economically advisable. There are also other factors which sometimes justify long-distance transmission as discussed by the writer (see text beginning "Reserve Capacity Savings Due to Inter-Connection").

In considering the desirability of installing additional capacity, Mr. Sprague calls attention to the operating saving that may sometimes be effected through

NOTE.—The Symposium on Economic Aspects of Energy Generation was presented at the meeting of the Society and at the Joint Meeting of the Power and Engineering-Economics and Finance Divisions, Pittsburgh, Pa., October 14, 1936, and published in the December, 1937, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1938, by Messrs. W. S. Finlay, Jr., R. M. Riegel, Ralph Bennett, J. M. Mousson, Ben C. Sprague, V. M. Marquis, R. L. Sackett, D. J. McCormack, and I. E. Moultrop; March, 1938, by Messrs. M. M. Samuels, and E. W. Kramer; and May, 1938, by Messrs. Paul E. Gisiger, F. A. Dale, Ray S. Quick, F. Knapp, and D. S. Jacobus.

[51] Cons. Engr., Philadelphia, Pa.

[51a] Received by the Secretary August 19, 1938.

the installation of a new steam plant, and shows by an example that such an annual saving may sometimes cover a considerable part of the fixed charges on the new plant. The statement is correct, of course, but the weight that should be accorded to this factor is sometimes quite uncertain.

Thus, in the illustration given by Mr. Sprague, a new steam plant would operate at a capacity of 7 000 hr per yr, or on a capacity factor of 80 per cent. No steam plant operates for many years at such a high capacity factor. As shown by Mr. Samuels, the average annual capacity factor of all steam plants in the United States is about 34 per cent.

In an economic study for the purpose of determining the advisability of a capital expenditure for a new steam plant, erroneous conclusions may be reached by assuming a capacity factor of 80% as in Mr. Sprague's illustration. Although such a high capacity factor may rule for the first few years, experience indicates that the average annual capacity factor of the steam plant during its life is more likely to be something of the order of 35 per cent. Hence, such savings as those indicated by Mr. Sprague should, more properly, be considered as temporarily contributing to the carrying of the fixed charges on the plant until such time as the full capacity of the plant is required.

The foregoing is only one of the reasons why, in common with many other engineers, the writer finds it advisable, in economic investigations of this character, to make a study of the entire system, together with set-ups showing estimated results over a term of years in the future.

Based on the best available load predictions, the object is to arrange a program to meet the increasing load with increments of capacity in such a manner that the total annual operating cost (for power supply) plus annual fixed charges on new money is always at a minimum as compared to what it would be with any other plan of development. This is a "cut-and-try" method and is necessarily laborious. Conclusions thus arrived at are often at variance with short-cut methods of analysis, but are much more reliable. If load growth does not proceed as estimated, the soundness of the conclusions are not usually affected, as, generally, this merely means that the program is stepped back or accelerated, as the case may be, to meet the new load estimates.

PHILIP SPORN,[52] M. AM. SOC. C. E. (by letter).[52a]—It is interesting and gratifying that all the discussions are free from any great amount of "heat" and acrimony which prevail altogether too frequently in any discussion involving economics. For his own contribution to the Symposium, in particular, the writer anticipated much critical review because it dealt with that uncertainty —cost—and what is more controversial—comparative cost. He is particularly delighted, therefore, that the discussions are so generally commendatory, leaving so little to discuss in the closure.

The writer is not a partisan of either steam power or hydro power but he is an advocate of generating power in that manner which will be most economical and, therefore, in that manner which is certain to be most productive both from a social and from a business standpoint. During the five years

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[52a] Received by the Secretary August 17, 1938.

since 1933 it has been assumed altogether too frequently that the former (that is, social benefit) could be obtained at the expense of the latter. The writer, however, does not believe that it can. If a situation to satisfy the true economic criteria demands hydro power, it should be used for that situation, but if the situation demands the use of steam power, the choice should be steam power. The writer makes this statement because underlying some of the discussions he detects a feeling (not openly expressed) that he has not been entirely impartial in his presentation and that he has, perhaps, "tipped the scales" in favor of steam power. This, of course, is not the case. Furthermore, the writer very definitely has refrained from adopting a regional viewpoint and has been unwilling to look at power as something that should be handled from a narrow viewpoint or from the viewpoint that what is locally available is of necessity the best simply because it is "on the spot." On the contrary, he has always considered as the basic outlook for the proper engineering and social economic solution of the power problem, the principle that, regardless of where in the country the fuel or the materials must come from, as long as they are within the boundaries of the United States, in the long run the national economy would best be served by using those materials and that equipment which result in the lowest cost power. For some time to come it may be very uneconomical to develop many hydro sites and, with the grave situation in which the soft coal industry finds itself, the national interest would best be served by adhering to steam power, provided steam power gives the net over-all lowest cost of power—and by that the writer does not mean cost after deducting any subsidies either of a direct or of an indirect nature.

Again, this means that the writer is perfectly willing to look with equanimity upon the immediate present loss of white coal or hydro power, but he does not consider that a loss in the true sense of the term; nor is it of any grave consequence, provided the national economy is better served by developing some other form of power even if that involves the burning of fuel.

Mr. Sprague's point regarding certain limitations and qualifications necessary to be considered in connection with the data presented in Fig. 34, on the relative costs of freight and electrical transmission of energy, are well taken and are generally understood by every engineer who has anything to do with the power problem. That is, it is well understood that any study of the type made as the basis for Fig. 34, of necessity, fails to give consideration to the distribution angle and that from a distribution standpoint the economics of short transmission distances become more favorable to the electrical side as against the freight side. Hence, in spite of the apparent better economics obtainable by freight for transmission distances even as short as 30 miles, the mean weighted distance of transmission on the inter-connected system of the American Gas and Electric Company is actually close to 60 miles under normal conditions.

However, Fig. 34 seems very much worth studying. Taking to heart the lessons it shows would obviate the bad effects of the large mass of misinformation propagated almost daily by those who should know better as regards the feasibility or economy of long-distance transmission. Thus, quoting from a

United Press Dispatch as printed in the *New York World Telegram* of July 29, 1938:

"Mr. Ross suggested that the Columbia River on the west coast might even supply electricity to New York, 3,000 miles away, and the Niagara or St. Lawrence Rivers might send electricity to homes in Florida or for irrigating the Southwest.

"Mr. Ross expressed his ideas to the Engineers' Club of Seattle. He has made a special study, at the request of President Roosevelt, he said, of the possibilities of transmitting electricity from one part of the country to the other.

"The results of such a system, Mr. Ross said, would be cheaper electricity, more abundant use of it and improved service to consumers. He said the price would be reduced because a purchaser now getting electricity from a distance of 100 miles could get it as cheaply from a distance of 1,000 miles.

"Direct current transmission would be used between the major plants suggested by Mr. Ross, but the present alternating current system would not be disturbed."

The writer has followed very closely every major transmission development for the past twenty years and has been intimately associated with the expansion and development of the transmission art during that period. He has particularly followed at close range the work that has been done and that is being done in the development or the attempted development of direct-current transmission, and all that he can say in connection with any opinion that economical 1 000-mile or 3 000-mile transmission of power is imminent, is that it is pure "poppycock," and that on the whole does not warrant any serious lay, let alone engineering, consideration.

Mr. Samuels seems quite unhappy about the purported inaccurate use of some of the terms such as "load factor," "use factor," and "plant factor." The writer cannot help but agree with him, but he feels that it will be necessary to wait for a younger generation to grow up and be taught before any degree of perfection is possible. The writer does not agree that 15 yr should be considered the average life of a steam plant because he does not believe that any such conclusion would be correct even if statistics may make it appear so. For 20 yr (1918–1938), the power industry has grown very rapidly. As a matter of fact, until the depression that started in 1930, regardless of the changes in the business cycle, there never was a year in which the output of electric energy by utility industries was not greater than the preceding year's output. Changes in the art of steam power generation moved forward just as rapidly. This led to a very high rate of obsolescence on steam plants, both from inadequacy and change in the art, but more so from inadequacy than change in the art. To-day, the electric utility industry has definitely passed the period of very rapid growth and is subject to the same fluctuations of the business cycle almost to the same degree as other businesses; and, although the art of steam power generation is continuing to improve, the effects of obsolescence and inadequacy probably will not be such in the next 20 yr as to limit the plants that are now being built to a life of only 20 yr. As a matter of fact, many of the developments that have taken place during the past several years have been such as to arrest, definitely, the sharp obsolescent trend of

plants built in the decade, 1916–1926. Therefore, the writer believes that Mr. Samuels' selection of 15 yr for the life of a steam plant is ultra-conservative.

Mr. Kramer takes exception to some of the statements in the paper with regard to the status that hydro-electric energy holds in the present power "picture" of the United States, and specifically points out that its position in the Northwestern States is better than would be indicated by the statements that hydro plants in general as a source of power, are out of the economic range. He may be correct as regards certain sections of the Northwest; but the writer does not believe he is right as regards California, and he is quite certain that Mr. Kramer is wrong as regards the Southeastern States. Furthermore, as regards the Northwest, the writer is not at all certain that if consideration is given to the definite static conditions of hydro developments (that is, to the fact that anticipated improvements in that field are almost negligible, whereas the improvements that can be looked forward to in the steam generation art are many), the economics would not be again in favor of steam generation. However, even if Mr. Kramer were correct, it is a fact that 70% of all the energy generated and used in the United States is produced in the area east of the Mississippi River, and the writer's statement, even in its full broadness, is definitely applicable to that region.

Mr. Kramer also takes issue with the use of 10.75% as a fixed-charge rate for hydro plants, and he shows a build-up whereby he gets a value of 6.87 per cent. Even disregarding the fact that there is no general agreement on this point (and the writer, definitely, does not agree to the basis followed by Mr. Kramer in building up his fixed charge of 6.87% by the use of the sinking-fund method of depreciation), there is the further fact that what is really important is the relative value of fixed charges. Mr. Kramer does not make clear that where the writer has used 10.75% as a fixed charge for hydro-electric power he has used a fixed charge of 12.75% for steam plants and that, if a charge of 6.87% is the proper one for hydro plants, then logically 8.87% would be the proper one to use for a steam plant.

Again, Mr. Kramer takes exception to the statements that the most economical sites were developed long ago and states that this is not true if applied to such developments as Boulder, Bonneville, or Grand Coulee, or to the undeveloped project on the St. Lawrence River. His conclusion is that in many cases such projects have not been undertaken heretofore on account of their size. This disregards the fact that one of the principal reasons that they have not been developed is that they are located such a distance from a market. There is much undeveloped hydro-energy in Alaska but, for the present, at least, it will have to remain undeveloped until local markets for it are found. It cannot possibly be brought to market economically in the United States. There are many people who believe that much of the power that is now undergoing development is uneconomical power for the reason that the cost of bringing it to the market makes the combined delivered cost so much greater than power that could be generated within reasonable distances from the market.

A number of other points have been raised which the writer would like to discuss but space will not permit. They are so important and so technical

that it would involve the preparation of a discussion of a length perhaps equal to some of the papers in the Symposium. For instance, Mr. Samuels quotes Mr. Knowlton as having recently presented in the *Electrical World* a tabulation of large plants from which it appears that, above a certain point, the saving in fuel brought about by a higher steam pressure is offset by higher fixed charges. This merits considerable discussion. There is the aspect, for example, that what is special practice as regards pressures and temperatures to-day, becomes standard practice five to ten years from the time such temperatures or pressures are first brought within a practical operating range. There are many other phases of this problem that need to be discussed. The discussion of subjects such as this one would be interesting and informative but probably would not add sufficiently to the value of the Symposium to warrant the time and the space. It may very well be apropos for discussion at some other point.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

PIN-CONNECTED PLATE LINKS

Discussion

By Harold D. Hussey, M. Am. Soc. C. E.

Harold D. Hussey,[9] M. Am. Soc. C. E. (by letter).[9a]—This paper is of much value in its field of structural design and especially in giving an empirical solution to a very complicated problem in buckling of plates; that is, the buckling of plates behind the pin in a pin-connected tension member. The tests which are reported were made on plain plates of uniform width and the author does not claim that his conclusions will apply to members of non-uniform width, such as eye-bars. The writer wishes to call attention, however, to some tests on eye-bars that agree very closely with the results obtained by Mr. Johnston.

When the width of the plate is given, the thickness required to prevent "dishing" can be determined from Equations (5) and (6) as follows: Let $\frac{a}{b_e} = M$; and, $\frac{a}{t} = N$. Solving for t,

$$t = \frac{M}{N} b_e \qquad (61)$$

Dividing both sides of Equation (61) by D_h and letting $\frac{t}{D_h} = t_D$; and, $\frac{b_e}{D_h} = b_D$:

$$t_D = \frac{M}{N} b_D \qquad (62)$$

Calculations show that the factor, $\frac{M}{N}$, has an approximate value of $0.3 \sqrt{b_D}$; therefore $t_D = 0.3\, b_D \sqrt{b_D}$; and,

$$t = 0.3\, b_e \sqrt{\frac{b_e}{D_h}} \qquad (63a)$$

Note.—The paper by Bruce G. Johnston, Assoc. M. Am. Soc. C. E., was published in March, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[9] Designing Engr., Am. Bridge Co., New York, N. Y.

[9a] Received by the Secretary August 12, 1938.

Solving Equation (63*a*) for b_e,

$$b_e = 2.23\, t \sqrt[3]{\frac{D_h}{t}} \qquad (63b)$$

The value of a is determined from Equation (5).

The use of Equations (63*a*), (63*b*), and (5) will result in a design which has equal resistance to failure by fracture at the side of a pin-hole, fracture below a pin-hole, and "dishing." Specimens that failed by fracture at the side of a pin-hole did so at an average stress in that section equal to, or slightly higher than, the ultimate tensile strength of the material as determined by standard tensile tests. A common specification is that the net section through a pin-hole shall be at least 40% greater than the net section of the member.[10] This shows that standard design requirements are on the side of safety.

In the "Summary," the author states, "* * * the results of this paper are not intended to apply directly to pin-connected plates having the shape of standard eye-bars unless tests on such eye-bars show agreement with the present results." Records of tests of standard eye-bars which broke in the head are extremely rare because of the fact that eye-bars are usually made to certain minimum requirements, one of which is that the diameter of the pin shall be not less than seven-eighths the width of the bar.

Occasionally, however, eye-bars are made, which vary from standard practice, and they sometimes fail in the head. The latest case of this kind, to the writer's knowledge, occurred in 1931, when a 7-in. by 1-in. eye-bar was tested to destruction. The two heads were 16.5 in. in diameter and the pin-holes were 7 in. and 5.25 in. in diameter, respectively. The eye-bar failed by "dishing" and fracture in the head having the smaller pin-hole.

An analysis of this eye-bar head according to Equations (2), (3), and (4) indicates that failure should take place by "dishing." Solving Equation (4), S_{bt} = 93.8 kips.

This means that a 16.5-in. rectangular plate with the pin-holes in the two ends should have failed by "dishing" back of the 5.25-in. pin when the bearing stress between the pin and plate reached the foregoing value. The total tension on the plate would then be 492 450 lb. The eye-bar in question failed by "dishing" and fracture back of the 5.25-in. pin under a load of 492 000 lb (reading to the nearest kip), indicating agreement between this eye-bar test and the author's work. This is further verified by the fact that another eye-bar, which was a duplicate of the one noted, had been tested a few months earlier, and the failure had occurred in the body of the bar. This appears contradictory, but when it is noted that this eye-bar failed under a tension of 470 kips it is seen that the material failed just before the head of the eye-bar was about to fail by "dishing." These tests indicate that the author's results will apply to standard eye-bars.

If standard practice had been followed when making these eye-bars the minimum size of the pin would have been $6\frac{1}{8}$ in. in diameter (seven-eighths of 7 in.). A re-test of two more 7-in. by 1-in. eye-bars was made, but instead of using a 5.25-in. pin at one end the pin was made 6.25 in. in diameter in order to deter-

[10] A. R. E. A. Specification for Steel Railway Bridges, 1935.

mine whether this increase would have any influence on the result. Both these eye-bars broke in the body of the bar, as would be expected from the author's results (a solution of Equation (4), using a 6.25-in. pin, shows that it would require a total tension of 520 kips to cause this head to fail by "dishing," whereas the eye-bars failed under tensions of 474 kips and 470 kips, respectively).

The author refers (in the "Introduction") to "forged" eye-bars. The writer wishes to emphasize that, although the two heads are produced by upsetting (or forging), the main body of the eye-bar is a plain rolled plate, or bar. For example, an eye-bar 65 ft from center to center of end pins has nearly 60 ft of rolled material with 4 ft to 5 ft at each end that has been forged. When a "forging" is specified, the material is manufactured to A.S.T.M. Specification A18, whereas A.S.T.M. Specification A7 gives complete requirements for annealed eye-bars. There is a considerable difference between these two materials. The latter specification is generally used for eye-bars, whereas the former never is; and when the term, "forged," is introduced, it leads to confusion.

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DISCUSSIONS

AERODYNAMICS OF THE PERISPHERE AND TRYLON AT WORLD'S FAIR

Discussion

By Messrs. Elliott G. Reid, and Karl Arnstein

Elliott G. Reid,[5] Esq. (by letter).[5a]—It is the writer's opinion that several phases of the work reported in this paper should not be allowed to pass unchallenged.

The theory presented is not only inapplicable but highly misleading. To be sure, the distribution of pressure over a large part of the surface of a sphere which moves through an unlimited mass of air is satisfactorily predicted by the methods of classical hydrodynamics. Therefore, determination of the velocity potential for the motion of a sphere parallel to a plane boundary might be expected to enable qualitative determination of the boundary interference effects. However, the mathematical complexity of this problem is so formidable that Lamb[6] gives only "the first steps in the approximation" and, of course, not even Neumann's[7] rigorous solution covers the practical problem which is further complicated by interference effects arising from the foundation structure of the Perisphere. These facts would seem to justify, if not to compel, the omission of all but qualitative theoretical treatment.

Nevertheless, the authors actually present a substantial reproduction of Lamb's approximate solution—which is entirely inapplicable when the sphere is close to the boundary surface. It appears that the significance of the substitution of c^3 for r^3 has escaped them because they conclude (following Equation (18)) that "the velocity, theoretically * * * is the same at symmetrical points above and below the sphere, when in the presence of the ground," and that "the stagnation points are not changed by the ground board." Inspection of

Note.—The paper by Messrs. Alexander Klemin, Everett B. Schaeffer, and J. G. Beerer, Jr., was published in May, 1938, *Proceedings*. This discussion is printed in *Proceedings*, in order that the views expressed may be brought before all members for further discussion of the paper.

[5] Prof. of Aerodynamics, Dept. of Aeronautics, Stanford Univ., Stanford University, Calif.

[5a] Received by the Secretary July 1, 1938.

[6] "Hydrodynamics," by Horace Lamb, Cambridge, 1906, Third Edition, pp. 124–125.

[7] "Hydrodynamische Untersuchungen," by C. Neumann, Leipzig, 1883.

the potential flow pattern for this case[8] demonstrates not only that both conclusions are unfounded but that the experimental results (Fig. 13(*a*)) are in qualitative agreement with hydrodynamic theory.

The significance of the results of force and pressure distribution tests on the Perisphere is highly questionable because the authors' interpretation of Fig. 4 is not justifiable. They err first by assuming the numerical values to be correct. The important portion of the curve (that is, that for which $R > 3 \times 10^5$) is substantially that determined by Wieselsberger[9] who supported his spheres approximately as illustrated by Fig. 5. The importance of the interference effects which characterize such installations was first demonstrated by the writer[10] in 1923; subsequent investigations by Wieselsberger[11] and Flachsbart[12] confirm his findings and thus invalidate that part of the curve upon which the authors predicate their extrapolation of model test results to full scale. According to the most reliable information available,[13] the value of C_D at $R_e = 750\,000$ is about one-half that shown by Fig. 4. Even more important, however, is the authors' erroneous assumption (also based on Fig. 4) that an unchanging flow pattern exists at all values of R_e greater than 1 400 000. The fallacy is exposed by Jacobs' tests[13] which show that C_D increases continuously while R_e varies from 1 400 000 to 2 600 000. (It is noteworthy that in the case of the full-scale Perisphere the value of R_e would be 140 000 000 at a wind speed of 70 miles per hr.) Since neither theory nor experiment justifies such procedure, use of the unaltered model pressure data for "the most painstaking load integrations" seems likely to prove love's labor lost.

Discussion of the pressure distribution tests would be incomplete without reference to the explanation which appears in the last two sentences under "Pressure Distribution Tests on Sphere: Sphere in Free Air." The authors appear to have explained a non-existent phenomenon, that is, in Fig. 12, the maximum suction developed does not exceed the calculated value.

Turning now to the Trylon, if that were to be an isolated structure and if the variation of wind speed with height were unimportant, the model test results might provide a rational basis for estimating the total wind loads on the full-scale structure. However, since the interference of the adjacent Perisphere would undoubtedly modify—and, with some wind directions, would probably augment—these forces, the omission of interference tests makes the validity of the recorded results somewhat dubious. No occasion for surprise is evident in the result, $C_D = 1.433$; narrow rectangular plates have long been known to experience relatively large resistances. Finally, since information

[8] "Applied Hydro- and Aeromechanics," by L. Prandtl and O. G. Tietjens, New York, 1934, Fig. 58, p. 109.

[9] "Further Information on the Laws of Fluid Resistance," by C. Wieselsberger, National Advisory Committee for Aeronautics, *Technical Note No. 121,* 1922.

[10] "The Resistance of Spheres in Wind Tunnels and in Air," by D. L. Bacon and E. G. Reid, National Advisory Committee for Aeronautics, *Technical Report No. 185,* 1923.

[11] "Ueber die Verbesserung der Stromung in Windkanalen," by C. Wieselsberger, read at the 108th Session of the Japanese Soc. of Mech. Engrs., March 19, 1925.

[12] "Recent Researches on the Air Resistance of Spheres," by O. Flachsbart, National Advisory Committee for Aeronautics, *Technical Memorandum No. 475,* 1928.

[13] "Sphere Drag Tests in the Variable Density Wind Tunnel," by E. N. Jacobs, National Advisory Committee for Aeronautics, *Technical Note No. 312,* 1929.

concerning the distribution of wind loads was sought, it seems regrettable that tests were not made to determine the severity of the pressure gradients near the edges of the Trylon.

KARL ARNSTEIN,[14] ESQ. (by letter).[14a]—The wind tunnel tests and theoretical consideration of models of the Perisphere and Trylon that the authors are making add more light to the problem of designing structures of unusual nature to withstand aerodynamic loads. These tests are interesting to the writer because they are somewhat parallel to problems encountered in designing the large airship hangar at Akron, Ohio, for which the senior author conducted wind tunnel tests on a hangar model which were later confirmed to a very reasonable degree by full-scale tests made at a later date on the hangar itself. These full-scale tests and related design considerations have been published.[15]

There is one conclusion made by the authors which should be scrutinized more closely. In Conclusion (20) it is suggested that the static pressure inside the sphere should be about equal to the outside static pressure. In large buildings such as the Perisphere or an airship hangar, ample ventilation must be provided to accommodate changes in temperature and barometric pressure and for other considerations. For any static loading condition the internal static pressure of the building must equal the local pressure at the ventilator or other openings, such as doors, which, in turn, must correspond to the local external pressure at that point. Since the external pressure pattern may change greatly for any fixed point on the surface of the building, depending on the direction of the wind, it is difficult to imagine the internal pressure remaining at any constant value.

In the case of the airship hangar it was assumed that the internal pressure could vary from the greatest ventilating suction to the greatest over-pressure to be expected from any local opening.

[14] Chf. Engr., Goodyear-Zeppelin Corp., Akron, Ohio.

[14a] Received by the Secretary July 12, 1938.

[15] "Wind Pressures on the Akron Airship Dock," by K. Arnstein and W. Klemperer, *Journal of the Aeronautical Sciences*, January, 1936.

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DISCUSSIONS

OBSERVED EFFECTS OF GEOMETRIC DISTORTION IN HYDRAULIC MODELS

Discussion

By Herbert W. Ehrgott, Esq.

Herbert W. Ehrgott,[4] Esq. (by letter).[4a]—The general attitude toward models expressed in this paper may be described as an optimistic skepticism, a wise attitude for all hydraulic experimenters to adopt. When working for a "crank," it is politic to make a close study of his foibles, and to discover the limits beyond which it is not safe to go. No one will question the statement that hydraulic models—particularly open-channel models—are "cranky." The author has undertaken to evaluate the queer quirks of distorted models, while persuading them to yield valuable information despite their shortcomings.

In his discussion of Studies Nos. 2, 3, and 5, Table 1, Lieutenant Nichols shows that the so-called Law of Compatibility is of little value in cases of submerged dikes in open-channel models. This conclusion is correct, and is understandable, when it is considered that the law of compatibility (Equation (1)) was derived for the case in which: (*a*) Model and prototype roughnesses are equal ($n_r = 1$); (*b*) ratio of hydraulic radii is equal to depth ratio ($R_r = d_r$); and, (*c*) there is no supplementary slope $\left(S_r = \dfrac{d_r}{L_r}\right)$. Of these three assumptions only the third is true in the three studies cited. The hydraulic radius ratio in distorted models is never equal to the depth ratio, and varies from section to section and from stage to stage within each cross-section. The hydraulic roughness ratio in open-channel models can seldom be made equal to unity, as is apparent from Table 3 in which are given all roughness coefficient data for small channels, available to the writer.

Note.—The paper by Kenneth D. Nichols, Jun. Am. Soc. C. E., was published in June, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[4] Capt., Corps of Engrs., U. S. Army, Care of Dist. Engr., Binghamton, N. Y.

[4a] Received by the Secretary July 30, 1938.

TABLE 3.—OBSERVED VALUES OF MANNING'S ROUGHNESS COEFFICIENT n FOR SMALL MODELS

Item No.	Cross-Section		Alignment	Bed Material			Number of observations	Roughness coefficient, n
	Shape	Variation between sections		Type	Mean grain diameter, D_g, in millimeters	Movement		
1	Trapezoidal[a]	Uniform	Straight	Clay; smoothed and tamped[g]	....	None	7	0.0105
2	Rectangular[b]	Uniform	Straight	Smooth cement	....	....	95	0.0105
3	Rectangular[b]	Uniform	Straight	Fine sands; smooth bed	<0.6	Weak	52	0.0107
4	Natural	Slight	Curved[e]	Smooth cement	....	....	1	0.0130
5	Semi-circular	Uniform	Straight	Smooth cement	....	....	1	0.0132
6	Trapezoidal[a]	Uniform	Straight	Tamped clay	....	None	9	0.0135
7	Trapezoidal[a]	Uniform	Curved[e]	Tamped clay	....	....	15	0.0145
8	Natural[c]	Uniform	Straight	Sand	....	....	3	0.0146
9	Natural	Average	Curved[e]	No fines[n] (M = 0.75)	1.06	Waves	6	0.0148
10	Trapezoidal[a]	Uniform	Straight	Tamped clayey earth[h]	....	....	3	0.0155
11	Rectangular[b]	Uniform	Straight	Sand 9;[n] (M = 0.57)[i]	4.0	General	50	0.0161
12	Trapezoidal[a]	Uniform	Straight	Tamped clay[j] and rounded river stones, 10 to 12 cm[k]	....	None	4	0.0182
13	Natural	Moderate	Curved[e]	Haydite; towhead section	1.04	General; waves	6	0.0201
14	Natural[d]	Extreme	Curved[f]	Smooth concrete	....	....	4	0.0203
15	Trapezoidal[a]	Uniform	Straight	Gravelly earth	....	None	9	0.0205
16	Natural	Average	Curved[e]	Slag coal (ρ_g = 1.36)	1.08	Waves; general	17	0.0210
17	Rectangular	Uniform	Straight	Heavily stuccoed concrete	....	....	5	0.0214
18	Natural	Average	Curved[e]	Heavily stuccoed concrete[l]	....	....	6	0.0230
19	Natural	Average	Curved[e]	Fine sands	<0.6	Waves and riffles	55	0.0230 to 0.0260
20	Rectangular[b]	Uniform	Straight	Fine sands[m]	<0.6	Waves and riffles	....	0.013 to 0.026

[a] Compound trapezoidal section for proposed Canal d'Alsace. [b] Tilting flume at U. S. Waterways Experiment Station. [c] Stream allowed to scour to equilibrium within fixed banks. [d] Helena-Donaldsonville Model before roughening. [e] Moderately. [f] Extremely tortuous. [g] With extreme care. [h] Including 30% ovoidal calcareous particles which protruded as the clay was washed away. [i] Smooth and scouring bed; no riffles. [j] In the deep section. [k] In the shallow section. [l] Or very much roughened. [m] *Paper 17*, U. S. Waterways Experiment Station. [n] M = Kramer's uniformity modulus.

Referring to Item No. 15, Table 3, the gravelly earth had the following composition:

Type	Percentage
Arable earth	50
Sand	30
Fine gravel (3 to 10 mm)	10
Gravel (10 to 30 mm)	10
Total	100

In Item No. 20, the value of n decreases with increasing grain size, but not in direct proportion; for example:

Mean grain diameter, D_g, in millimeters	Roughness coefficient, n
0.205	0.026
0.35	0.021
0.56	0.013

The writer is inclined to be skeptical of the value of any quantitative results to be obtained in model studies of open-channel models involving submerged works, in which the channel is distorted, while the submerged sills, dikes, or dams remain undistorted. Such problems involve both Froude's and Reynolds' Laws of Similarity, acting together in undetermined ratios (Eisner's "Zusammenwirkungsgrenz"). Only a geometrically similar model of this type may be counted upon to give results that are transferable to Nature. This is not to say that model studies such as Study No. 3, Table 1, are without value. Quite the contrary. The action of different types of submerged works, as compared with each other, may be revealed clearly by such a study; and a general knowledge of the effects of a moderate degree of vertical distortion indicates that such relative action will probably be closely similar to the corresponding action in the prototype. It is to be hoped that if and when the sills are built in the St. Clair River, a re-check will be made on models to answer the question of transferability of results in problems of this ambiguous type.

In connection with Study No. 6, Table 1, the writer has made a study of the variation of the hydraulic radius ratio from cross-section to cross-section and from stage to stage. Forty typical river cross-sections were plotted to various distortions, and the ratios of hydraulic radii for each section and for each distortion were calculated. It was found that:

(1) The hydraulic radius distortion varies approximately parabolically with the depth distortion;

(2) The hydraulic radius distortion varies with the ratio, $\frac{\text{Width}}{\text{Maximum depth}}$; and,

(3) The width-maximum depth ratio varies with stage, in a quite uniform manner, regardless of exact shape of cross-section.

There was some indication that shape of cross-section had some importance, but the indications were so slight and erratic that it was felt that inclusion of this factor would constitute an unjustifiable refinement.

The author gives a corrected "law of compatibility," into which none of the three assumptions made in the case of Equation (1) enters. This formula (Equation (4)) may be clarified, and its application simplified, if it is rewritten:

$$\left(\frac{d_r}{L_r}\right)\frac{1}{S_r\, L_r^{0.33}} = \left(\frac{R_r}{L_r}\right)^{1.33}\frac{1}{n^2} \qquad (5)$$

In this form the three adopted scales built, or to be built, into the model appear on the left-hand side, whereas the resulting discrepancy from dynamic similarity, due to differing hydraulic radius distortions for each stage and cross-section, are given on the right-hand side, together with the degree of roughness distortion to correct for the discrepancy. Equation (4) written in this form (see Equation (5)) makes it perfectly clear why adjustment of a fixed bed model, such as the Helena-Donaldsonville Model (Study No. 6, Table 1), involves "cut-and-try" variation of the roughness throughout the model, and at various stages. Theoretically, the exact roughness to be applied for each section and each stage could be computed from Equation (5) (assuming that

Manning's equation is strictly applicable to the model, which it is not); actually, it is simpler to apply approximately the correct average roughness and then smooth or roughen the model locally to "iron out" the numerous humps and sags. Equation (4) would seem to indicate that, within the limits imposed by: (*a*) The streaming-shooting limit; (*b*) the laminar-turbulent limit; (*c*) the hydraulic roughness limit (n_m cannot be made smaller than 0.009, nor greater than 0.026, without actually constricting the cross-section); (*d*) the limit imposed by the angle of repose of bed material; and, (*e*) bed movement limit (tractive force limit); any combination of vertical, slope, and hydraulic roughness distortions which makes the left-hand member equal to unity, may be used.

Since the depth distortion, once chosen, is usually invariable, and since the hydraulic radius distortion for any particular section and stage is fully determined by the shape of the cross-section and the vertical distortion, it follows that if Froude's law is to be approximated, a model with constant roughness (at different stages) must be operated with varying slope distortion. Conversely, if a model is to be operated with constant slope-ratio, the roughness ratio must be made to vary with stage so as to keep Equation (4) in balance. In practice, of course, these considerations impose difficulties, often insuperable, so that the model cannot be brought to operate in compliance with Froude's law. The usual practice is to operate with constant roughness and slope ratio, and a discharge scale which varies with stage. (One model study completed at the United States Waterways Experiment Station, in 1938, was made with a variable slope scale, with excellent results.)

In Conclusion (3) of his description of the Helena-Donaldsonville model, Lieutenant Nichols gives the discharge scale, and states that it is greater than that required by Froude's law. If the reference is to Equation (3) it should be explained that this is not Froude's law, but an equation of compatibility between Froude's law and Manning's formula; Froude's law proper does not apply to open-channel models. If a different formula for C were used, which is applicable equally to both model and prototype, it is possible that the discharge scale would be found to check closely with that required by theory. By plotting the value of Chezy's C for a large number of observations taken from models of all sizes and descriptions, as well as from small, medium, and large canals, flumes, and rivers, against the corresponding values of the product, $V R$, the writer has developed the formula,

$$C = \frac{1.81}{n^{0.9375}} (V\,R)^{0.1175} \qquad (6a)$$

or,

$$V = \frac{1.96\,R^{0.700}\,S^{0.567}}{n^{1.062}} \qquad (6b)$$

which appears to fit models as well as prototypes. It would be interesting if the author would compute the value of Q_r for the Helena-Donaldsonville model, using Equation (6*b*) for comparison with the empirically determined value of $\frac{1}{1\,500\,000}$.

Under Study No. 13, Table 1, the author cites a case in which the same model was tested with two different depths, to determine the effect of distortion in a sharp bend. His first conclusion is that although the tractive force available in the two models was equal, greater bed movement occurred at the greater depth (smaller slope). This is extremely interesting as indicating that a true criterion of tractive force should include the factor of depth to a greater degree than in DuBoys' law—a conclusion which the writer has reached independently by a study of the tractive force investigations of the U. S. Waterways Experiment Station. The tractive force, according to DuBoys' law, is expressed by the product, $\rho\, d_x\, S$; in which d_x is the depth of flow at the particular point in question, and S is the general water-surface slope of the stream, the resultant of all the dynamic forces acting throughout the stream in that vicinity. DuBoys' expression for tractive force does not consider velocities, except as they are tied to d_x (through R), and S. Bed velocity is generally acknowledged to be of first importance in the transportation of bed materials, but practical means of measuring bed velocities have not been developed. Since they are determined by the velocity distribution, and the hydraulic resistance to flow near the boundaries, it would seem that a true criterion of tractive force should be intimately linked with Chezy's formula for mean velocity. A preliminary analysis of the flume traction studies at the Waterways Experiment Station shows that an equation of the general form:

$$G = \frac{K\, C^3}{\rho_g} (d\, S - d_0\, S_0)^{1.5} \ldots\ldots\ldots\ldots (7)$$

adequately represents the relationship between the solid flow and fluid flow in a small, straight, rectangular channel with smooth sides. In Equation (7): G is the solid flow, in cubic feet per foot width of flume per hour; K is an empirical constant representing the effect of shape and size of bed material, and coefficient of friction between the material and the bed (includes the factors, g and ρ_1); C is the Chezy roughness coefficient; ρ_g is the (wetted) specific gravity of the bed material; $d\, S$ is the depth-slope product for the particular hydraulic conditions under consideration; and, $d_0\, S_0$ is the corresponding product for conditions obtaining at inception of "general" movement.

It will be noted that according to Equation (7) the quantity of material moved will increase as the cube of the friction coefficient. If Manning's formula for C is used, $G \approx \frac{R^{0.5}}{n^3}$. The result is the same as that observed in the results of the author's Study No. 13, Table 1. Of course, the results of flume traction studies cannot be applied directly to natural open-channel models, or to full-sized streams; but it is to be expected that as knowledge of the mechanics of bed-load movement expands, some formula of the general form of Equation (7) will be found to apply. The author's observations on the effects of distortion on velocity distribution are very pertinent. It is here that the crux of the entire question of evaluation of distortion effects is to be found. The vertical velocity curves and the corresponding isovel plots are the resultant of all the complex dynamic effects of sinuosity of channel, variation in size and

shape of cross-section, absolute roughness, type and quantity of bed-load movement, etc. If a relationship could be established between the isovel plot, the bottom velocities, and Chezy's friction coefficient, C, one may anticipate a rational design of open-channel models. Herein lies a fruitful field of investigation.

The writer would like to plead with all engineers who may have occasion to make stream measurements, to make every effort to report at the same time all pertinent information as to: (*a*) Water temperatures; (*b*) rate of change of surface slopes; (*c*) configuration of the reach, both longitudinal and lateral; (*d*) variation in shape and size of cross-section along the stream axis; (*e*) mean grain size of bed material, or careful description of channel roughness; (*f*) type of bed-material movement (smooth, riffling, sand waves, super-wave smooth, anti-dune), together with average height, wave length, and speed of sand waves; (*g*) quantity of bed load; and, (*h*) kineticity of flow (this requires taking two velocity cross-sections and the slope between them). The accumulation of a widely distributed body of such data should lead to a much more useful table of n-values, and eventually to a definitive solution of the entire problem of open-channel flow.

Study No. 7, Table 1, is one of the most interesting model studies reported to date. Professors A. H. Gibson and J. Allen (see author's bibliography reference (12))[5] made an exhaustive study of the suitability of various bed materials for this particular model, which involved open-channel flow (unidirectional), tidal flow in a funnel-shaped estuary, and tidal bore. Any conclusions which they draw should be applied to other models only with the greatest caution, since the two vertical distortions used were far greater than has been generally accepted as a safe limit for vertical distortion. Despite such excessive distortions, extremely good verification was obtained, particularly as to height of sand banks and depth of pot-holes. Lieutenant Nichols stresses the importance of a careful verification; in the opinion of the writer this cannot be over-emphasized. The reliance to be placed in any results obtained with new dispositions built into the model depends entirely upon the accuracy with which the model was able to reproduce known conditions or known changes in Nature. In the study of the Severn Estuary about one-half the bed materials tested (which included sands, emeries, and pumices) failed to reproduce the correct position of the thalweg just below a sharp bend. Some of the materials that produced good results in the $\frac{1}{200}$ -model failed in this respect when used in the $\frac{1}{100}$ -model; on the other hand, one sand, which gave the incorrect thalweg in the $\frac{1}{200}$ -model gave the correct position in the model with smaller vertical distortion. It is evident that some undetermined relationship between the size and specific gravity of the bed material, and the tractive forces developed by the flowing water (dependent upon the vertical and slope distortions) is effective in such a model, and that a fine balance must exist between the solid

[5] *Proceedings*, Am. Soc. C. E., June, 1938, p. 1101.

and fluid flows if similarity of bed-load movement is to be attained. It is significant that for the three materials which produce the best results, the values of the product, $d_g (\rho_g - \rho_1)$, were nearly equal.

The author is to be congratulated upon his paper, which avoids all matters of conjecture and deals only with actually observed effects of geometric distortion. Hydraulic models are here to stay, and within the limits imposed by the particular laboratory, and funds available, distortions are unavoidable; it is a question of fundamental importance, therefore, to determine what allowances are to be made for each such distortion in the interpretation of the measured model results. The paper is a valuable contribution to current knowledge in this field, so far as vertical and slope distortions are concerned. Much remains to be written, particularly in the field of bed material distortion.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

A THEORY OF SILT TRANSPORTATION

Discussion

By Joe W. Johnson, Jun. Am. Soc. C. E.

Joe W. Johnson,[13] Jun. Am. Soc. C. E. (by letter).[13a]—The author extends his previous theory of silt transportation and presents supporting data which, in most cases, show excellent agreement between observed and calculated values of silt load. The theory set forth presents a method which, with certain limitations, appears to permit the silt load of a stream to be computed from easily observed factors. The method should be a valuable aid in many studies concerned with the transportation of sediment.

The various examples given in the paper cover, rather completely, the various conditions that may prevail in practice. Other data which represent conditions slightly different from those presented by the author are given in Table 6. These data were computed from observations made in Italy by Mario Giandotti[14] on the Po River and Francesco Sensidoni[15] on the Reno River, and, in Texas, by O. A. Faris,[16] M. Am. Soc. C. E., on the Brazos River. Unfortunately, only the data of Mr. Faris include a mechanical analysis of the suspended material, thereby permitting colloids to be excluded from consideration.

The observations on the Po River are interesting in that the two gagings were taken in the autumn and in the spring at practically the same stage; yet the total silt load in the latter gaging was about four times that in the former. As mentioned by Professor Giandotti, in the spring gaging the stream carried a large quantity of fine material—material which the author would class as colloids. Referring to Table 6(*a*) it is noted that with the exception of one vertical the calculated value of silt content for the autumn

Note.—The paper by W. M. Griffith, Esq., was published in May, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[13] Asst. Hydr. Engr., U. S. Dept. of Agri., SCS, Sedimentation Studies, Washington, D. C.

[13a] Received by the Secretary August 4, 1938.

[14] "Les Déplacements du Lit du Po," by Mario Giandotti, Le Service Hydrographique Italien, XV Congrès International de Navigation, Venice, September, 1931, p. 297.

[15] "Rilievi e Determinazioni sul Trasporto Solido del Fiume Reno a Malalbergo," by Francesco Sensidoni, *Annali* dei Lavori Pubblici, Anno LXXVI–Fascicolo, N. 1, January, 1938, A. XVI, Rome, Italy.

[16] "The Silt Load of Texas Streams," by O. A. Faris, *Technical Bulletin No. 382*, U. S. Dept. of Agri., September, 1933.

TABLE 6.—ANALYSIS OF SILT DISTRIBUTION

Distance between station and bank, in feet	Total depth, d, in feet	Velocity, V, in feet per second	Silt Content, in Parts per 10 000 (Colloids Included): Arithmetic mean	Silt Content, in Parts per 10 000 (Colloids Included): Graphic mean	Value of $\frac{7.5\,v}{d^{0.57}}$
(1)	(2)	(3)	(4)	(5)	(7)
(*a*) Po River near Roncocorrente, Italy, December 9, 1924 (Width of Water Surface, 754.4 Feet; and Discharge, 61 510 Cubic Feet per Second)					
32.8	11.5	2.17	2.8	2.5	4.1
98.4	12.1	2.87	2.9	2.7	5.2
164.0	14.1	3.05	2.3	2.2	5.1
229.6	17.2	3.14	2.6	2.3	4.7
295.2	20.3	3.61	5.1	3.3	4.9
360.8	24.3	3.68	4.1	2.9	4.5
426.4	27.9	3.99	6.4	4.5	4.5
492.0	31.7	4.27	6.9	4.7	4.5
557.6	33.3	4.35	7.3	4.4	4.4
623.2	33.1	4.63	4.7	3.1	4.7
688.8	33.1	4.36	3.4	2.4	4.5
(*b*) Po River near Roncocorrente, Italy, March 4, 1925 (Width of Water Surface, 767.5 Feet; and, Discharge, 75 850 Cubic Feet per Second)					
78.7	14.8	2.97	12.0	10.6	4.8
144.3	17.7	3.39	14.1	11.8	5.0
209.9	21.2	3.42	13.8	12.6	4.5
275.5	23.6	3.69	11.0	9.2	4.6
341.1	24.9	4.23	12.3	9.9	5.1
406.7	28.5	4.56	14.1	10.1	5.1
472.3	31.5	4.61	16.4	12.2	4.8
537.9	34.4	5.20	17.8	13.0	5.2
603.5	34.1	5.38	17.3	13.3	5.4
669.1	33.1	4.18	12.2	8.8	4.3
734.7	31.1	3.43	7.7	6.2	3.6
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Distance between station and bank, in feet	Total detph, d, in feet	Velocity, V, in feet per second	Silt Content, in Parts per 10 000: Arithmetic mean (colloids included)	Silt Content, in Parts per 10 000: Graphic mean (colloids included)	Silt Content, in Parts per 10 000: Proportion retained by Sieve No. 200; graphic mean	Value of $\frac{7.5\,v}{d^{0.57}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
(*c*) River Reno, near Malalbergo, Italy. (Width of Water Surface, 142.8 Feet)						
4.9	5.1	0.49	7.78	7.86	....	1.45
13.1	9.8	2.36	7.54	7.55	....	4.81
32.8	8.6	3.12	8.24	7.77	....	6.88
42.6	9.3	3.14	8.12	7.56	....	6.63
52.5	9.5	3.23	8.38	8.06	....	6.71
72.2	8.9	3.17	8.18	7.42	....	6.83
82.0	9.8	3.23	7.49	7.13	....	6.86
91.8	10.0	3.20	6.78	6.75	....	6.45
111.5	10.0	3.05	7.44	6.98	....	6.17
121.4	9.7	3.14	6.58	6.34	....	6.45
131.2	10.3	2.15	6.02	5.82	....	4.27
(*d*) Brazos River, at Rosenberg, Tex., April 16, 1929						
63	18.2	4.39	....	80.1	7.9	6.3
98	20.0	4.42	....	80.9	7.1	6.0
138	21.3	4.65	....	79.9	6.8	6.1
178	24.3	5.98	....	75.4	4.6	7.3
218	23.5	6.77	....	73.7	3.6	8.4
258	25.0	6.44	....	73.1	3.4	7.7
298	12.3	5.76	....	72.1	3.1	10.3
338	3.4	5.13	....	68.5	2.9	19.1
(*e*) Brazos River, at Rosenberg, Tex., May 31, 1929						
68	41.5	5.03	....	72.0	8.1	4.54
(*f*) Brazos River, at Rosenberg, Tex., June 1, 1929						
68	42.2	5.03	....	56.9	8.6	4.47

gaging is greater than the observed silt content, although the observed value is for total silt content and, of course, includes colloids. On the other hand, in the spring gaging (Table 6(*b*)) the calculated silt contents are without exception much smaller than the observed silt content, a fact due no doubt to a large quantity of colloidal matter.

Table 6(*c*) gives an analysis of silt distribution in the Reno River, in Italy. In most verticals, very close agreement between observed and calculated values of silt content are noted, although the observed silt load, as in the Po River, includes colloids.

Tables 6(*d*), 6(*e*), and 6(*f*) give calculated and observed values of silt content for the Brazos River, in Texas. In addition to the observed silt content which was retained by a No. 200 sieve, the silt content including colloids is also given. Agreement between observed and calculated mean silt content is good in the verticals near one stream bank but becomes poor as the opposite bank is approached. Such a condition is probably due to a local distortion in the velocity distribution and will be discussed subsequently.

A factor of more practical importance than whether there is good agreement between calculated and observed mean silt content in the various verticals appears to be whether the total calculated silt load of the stream agrees with the total observed load. Discharges and the observed and calculated silt loads, determined by ordinary methods, for the various canals and rivers for which data were summarized in Tables 1, 2, and 6, are shown in Table 7(*a*). Data from flume studies[17] conducted at the U. S. Waterways Experiment Station, Vicksburg, Miss., are included in Table 7(*b*). As mentioned previously the observed silt loads in the Po and Reno Rivers include colloids, and the difference between observed and calculated values are relatively large; however, except for the March, 1925, gaging in the Po, the Imperial Canals, and Brazos River data do not show agreement appreciably better than the Italian gagings.

TABLE 7.—SUMMARY OF OBSERVED AND COMPUTED SILT LOAD IN VARIOUS RIVERS AND CANALS

(*a*) DATA FROM CANALS AND RIVERS					(*b*) FLUME TESTS			
Canal or river	Discharge Q, in cubic feet per second	Silt Load, in Tons per Day			Run	Discharge Q, in cubic feet per second	Observed silt load, in tons per day	Table No. in U. S. Waterways Experiment Station *Paper No. 17*
		Observed	Computed	Table No. in this paper				
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)
Dahlia Canal	27	122	74	2	24	1.5	0.30	32
Central Main Canal	486	1 005	950	1	25	2.2	0.99	32
Brazos River	31 000	34 850	64 850	6(*d*)*	16	1.7	0.46	33
Reno River	3 665	6 830	5 680	6(*c*)	17	2.2	0.74	33
Po River, December 9, 1924	61 510	51 500	72 130	6(*a*)	13	1.8	0.58	34
Po River, March 4, 1925	75 850	204 770	90 785	6(*b*)	14	2.2	1.01	34

* Also Tables 6(*e*) and 6(*f*).

An interesting result may be observed by plotting the data in Table 7, logarithmically, so as to obtain a plot of points representing the silt load calculated according to the author's theory. The Brazos River data failed to

[17] "Studies of River Bed Materials and Their Movement, with Special Reference to the Lower Mississippi River," *Paper No. 17*, U. S. Waterways Experiment Station, 1935.

plot according to that of the other streams, possibly because of the relatively high value of 1.4 for the velocity-head coefficient,[18] α, at this cross-section. Such a high value for α in an open channel is usually found only where the velocity distribution is greatly distorted by obstructions, bends, etc.

For the given data, which cover a wide range of discharge and location of streams, the silt load of loose, granular material (consisting of material larger than $\frac{1}{500}$ in. in diameter and excluding colloids) expressed, in tons per day, is given by the equation,

$$L = 4.0\, Q^{0.89} \qquad (14)$$

in which Q = discharge, in cubic feet per second. That the silt load (including colloids) of a stream may be many times the load expressed by Equation (14) is evident upon examination of Tables 6(*d*), 6(*e*), and 6(*f*), wherein it is observed that 90% of the suspended silt in the Brazos River consisted of particles small enough to pass the No. 200 sieve and thereby can be classed as colloids. Due to the fact that vertical eddies are perhaps not important in the transportation of very fine material, it is doubtful whether a simple relationship between discharge and the silt load of a stream (including colloids) can be formulated. A viewpoint on this subject, as stated by C. P. Vetter,[19] M. Am. Soc. C. E., is:

"As long as the discharge and the slope of the river are constant the intensity of the boundary layer, and the concentration of suspended matter maintained by it, is undoubtedly also constant. The fine material, not available in the bed of the stream for many miles up stream from the point of observation, must originate elsewhere on the river or on its tributaries and it cannot be expected that any fixed relationship should exist between the discharge and the load of the finer material. Bearing in mind that the term, 'saturation,' should probably be used with caution in this connection, the phenomenon can, nevertheless, be illustrated by saying that the stream is saturated with the material freely available in the bed but under-saturated with material finer than that found in the bed.

"Yet another conclusion may be drawn from the above reasoning. If the part of the silt load which lies within the range of sizes abundantly available in the bed bears a fixed relationship to the discharge there is no reason to believe that this relationship will be materially changed if the finer part of the load is partly or entirely eliminated."

In the latter conclusion it might also be noted with reference to the Brazos River data that the load of very fine material may be ten times that of the "loose granular" material and still the load-discharge relationship is not changed provided the abnormal velocity distribution as discussed herein is local in character and responsible for the relatively high value of the calculated load.

The ultimate solution of the problem of transportation of suspended silt appears to be from the approach of fundamental mechanics according to a

[18] "Velocity-Head Correction for Hydraulic Flow," by Morrough P. O'Brien, Assoc. M. Am. Soc. C. E., and Joe W. Johnson, Jun. Am. Soc. C. E., *Engineering News-Record*, Vol. 113, No. 7, August 16, 1934, p. 214.

[19] "Why Desilting Works for the All-American Canal?" by C. P. Vetter, *Engineering News-Record*, Vol. 118, No. 8, March 4, 1937, p. 321.

theory such as that developed by W. Schmidt,[20] enlarged by Professor O'Brien,[21] and applied by J. E. Christiansen,[22] Assoc. M. Am. Soc. C. E. Although the author's theory of silt transportation has little scientific foundation, the important factor is that it apparently "works" and does so under a large variety of conditions. In fact, it describes silt transportation, within certain limits, approximately as well as the Chezy formula describes the flow of water in open channels.

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[20] "Die Massenaustauch in freien Luft und verwandte Erscheinungen," von W. Schmidt, H. Grand, Hamburg, 1925.

[21] "Review of the Theory of Turbulent Flow and Its Relation to Sediment Transportation," by Morrough P. O'Brien, *Transactions*, Am. Geophysical Union, Hydrology Section, 1933.

[22] "Distribution of Silt in Open Channels," by J. E. Christiansen, *Transactions*, Am. Geophysical Union, Hydrology Section, 1935, p. 478.

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DISCUSSIONS

GRIT CHAMBER MODEL TESTS FOR DETROIT, MICHIGAN, SEWAGE TREATMENT PROJECT

Discussion

BY W. E. HOWLAND, ASSOC. M. AM. SOC. C. E.

W. E. HOWLAND,[14] Assoc. M. Am. Soc. C. E. (by letter).[14a]—The perfection of a technique for model testing of sedimentation basins is of great importance, and the studies reported in this paper should be of considerable value for this purpose.

In trying to account for the lack of agreement in results of the tests of percentage removal of fine sand obtained in the different models, several explanations may be given. The author suggests that the reduced turbulence in the smaller model (due presumably to a reduced Reynold's number and a nearer approach to the stream-line condition of flow) may account for the observed high percentage removal of the finer sand by this model. This may be a valid explanation notwithstanding the data presented by L. G. Straub,[15] Assoc. M. Am. Soc. C. E., which shows that the so-called (and mis-named) efficiency of sedimentation above a certain limiting value of the Reynold's number is independent of that function. This "efficiency" (defined as the ratio of the flowing-through time to the detention period probably first measured in sedimentation basins by C. H. Capen,[16] M. Am. Soc. C. E., in 1927), may be judged on theoretical grounds to have a doubtful effect upon the percentage removal of sediment in the basin—as has already been argued.[17]

One other matter that deserves further emphasis is the so-called "transporting power of the stream." In a recent article,[18] Mr. W. M. Griffith considers this matter at length and gives a criterion for the silt transporting

NOTE.—The paper by George E. Hubbell, Assoc. M. Am. Soc. C. E., was published in December, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1938, by G. M. Ridenour, Assoc. M. Am. Soc. C. E.; and April, 1938, by Thomas R. Camp, M. Am. Soc. C. E.

[14] Associate Prof., San. Eng., School of Civ. Eng., Purdue Univ., West Lafayette, Ind.

[14a] Received by the Secretary June 6, 1938.

[15] "Transportation of Sediment in Suspension," by Lorenz G. Straub, *Civil Engineering*, May, 1936, p. 321.

[16] "Study of Sewage Settling Tank Design," by C. H. Capen, *Engineering News-Record*, November 24, 1927, Vol. 99, p. 833.

[17] *Engineering News-Record*, February 16, 1928, Vol. 100, p. 290.

[18] "A Theory of Silt Transportation," by W. M. Griffith, *Proceedings*, Am. Soc. C. E., May, 1938, p. 859.

power of a given stream as $\frac{v}{d^{0.57}}$, in which v is the mean velocity and d the mean depth of the stream. Of course, one cannot be sure that this criterion is valid for the conditions met in a grit chamber since it was developed empirically for different conditions and it may not be at all applicable to a small scale model. Nevertheless, its consideration may throw some light upon the behavior of grit chambers. It would not be surprising to find some kind of an inverse relation between the percentage removal of sediment in a settling basin and the "transporting power" of the stream in the basin if, as in the case of these models, the values of $\frac{a}{t}$, as required by Hazen's theory of sedimentation,[19] are the same in all cases. (The letter symbol, a, is the mean time that a particle is in the basin, and t is the time theoretically required for the particle to settle from top to bottom of the basin.)

The following values of this function are determined for the various chambers considered in this study:

Prototype, $1 \div 16^{0.57}$	= 0.206
Smallest scale model, $0.258 \div 1.067^{0.57}$	= 0.247
Grand Rapids model, $0.9 \div 6^{0.57}$	= 0.324
Dearborn model, $0.9 \div 3^{0.57}$	= 0.480

It should be noted that no study has been made of the concentration of sand in the various tests. Without considering such effects these calculations indicate that the foregoing order of enumeration should also be the order of magnitude of the percentage removal of sediment in the basins. There is thus a certain measure of qualitative agreement between the foregoing statement and the observed percentage removal of sand in the model basins—the smallest model removed the greatest percentage of sand even when the sand used was half as large as in the larger models. Whether or not there is any value in this consideration the author can decide.

The writer would like to inquire if there are any data available showing the possible effect, if any, of the quantity of sand added upon the observed percentage removal for any given basin.

In conclusion, the writer would like to point to the value of running a similar test on the percentage removal of 0.2-mm sand in the prototype when built. Since the two sets of models used were designed on different principles, a final check on the prototype should help to answer such questions as these: Should sedimentation models, designed for identical values of $\frac{a}{t}$, have the same Froude number as the prototypes, or should, instead, the Reynold's number be made as nearly the same as possible as in the prototypes? Should the "transporting power," as defined herein, receive attention in the design of such models?

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[19] "On Sedimentation," by the late Allen Hazen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LIII, December, 1904, p. 45.

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DISCUSSIONS

STRUCTURAL BEHAVIOR OF BATTLE-DECK FLOOR SYSTEMS

Discussion

BY JONATHAN JONES, M. AM. SOC. C. E.

JONATHAN JONES,[8] M. AM. SOC. C. E. (by letter).[8a]—It would be quite futile to undertake, either by reasoning or experiment, a general solution for the case of a floor plate stiffened by ribs. Enough is known of the mathematical complexity of the problem, and of the many variables entering in, to guarantee that any such generalization, if obtainable, would be too formidable to have much practical value.

About all that is practically worth doing is to select, for the purpose in mind, an approximate arrangement of parts and sizes that promises to be successful and economical, and then, by a combination of experiment and reasoning, to develop some general rules of design that may be applied within a narrow field. This process may appear crude, but it may also be highly useful. It may not be rigorous mathematics, but it can be good engineering. This piece of research was conducted in just such a fashion, and it has produced results and recommendations of great value in its intended range, but which should not be extrapolated far therefrom.

It was the desire of the Committee of the American Institute of Steel Construction, that initiated this research, to discover the constructional advantages of the "battle-deck floor," in steel, which it believed had been obscured by the lack of a basis for design. In particular, it believed that plates of excessive thickness were being used, to the great economic disadvantage of this type of floor, simply for lack of any better method of design than a guess at a distribution, and then a continuous beam formula and the usual working stresses.

This Committee, therefore, guided the research into just those arrangements of parts in which it believed this type of construction would be most successful.

NOTE.—The paper by Inge Lyse, M. Am. Soc. C. E., and Ingvald E. Madsen, Jun. Am. Soc. C. E., was published in January, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1938, by Messrs. H. N. Hill, and R. L. Moore.

[8] Chf. Engr., Fabricated Steel Constr., Bethlehem Steel Co., Bethlehem, Pa.; Chairman, Committee on Technical Research, Am. Inst. of Steel Constr.

[8a] Received by the Secretary July 6, 1938.

Thus, it opened up the beam spacing at the expense of added weight, to minimize manufacturing costs; it discarded transverse distributors for the same reason. All this it has reported adequately elsewhere.[9] The successive models, and the final full-sized panel, were designed by the Committee, as more data became available, to come closer and closer to an economical and reasonable design.

The results have been extremely satisfactory. Within the purposely narrow range, simple rules of design have been developed by approximate analysis of the test results. Upon these rules the sponsor has compiled a booklet[9] giving recommended practice as to the arrangement of parts, beam spacing, beam sizes, plate thicknesses, etc., which give certainty of success.

Not mentioned by the authors, but important to the designer, is the choice of unit stresses by the Sponsor Institute for its recommended practice. For tension in the bottom flange of its beams, it recommends 20 000 lb per sq in., but in its tables it uses 18 000 in deference to present bridge specifications. For plate stress, it recommends 28 000 lb per sq in.

Thus, in Table 3, for the full-sized panel, it will be noted that 27 900 lb per sq in. was the conscious objective of the final design, although only 26 000 was actually measured. At first sight it may seem to be taking great liberties to propose, for regular design, a unit stress as high as 28 000 lb per sq in. As a matter of fact, it will be seen from Fig. 13 that it is truly conservative. All that this 28 000 means is a "spot" of highly stressed material backed up by a wide area of under-stressed plate; danger is impossible under such conditions. It is perhaps analogous to the, say, 40 000 lb per sq in. that exists at the edge of a hole in a tension member, but in no way is it analogous to the 16 000 or 18 000 lb per sq in., for which that member was designed.

The behavior of these floors under over-load, and their recovery on release (as described by the authors under "Test Data and Relations: Tests of Full-Sized Floor Panels"), convinced all observers that the design rules set up by A. I. S. C. will produce a floor that simply cannot be made to fail under any condition of highway service. The authors are to be congratulated upon the manner in which they fitted their technical skill into a problem which was narrowed not by, but for, them.

It is interesting to observe[10] that German engineers are also experimenting in an endeavor to reduce the weight of their highway bridge floors. As might be expected, however, they are working with designs of great complexity in order to effect a maximum saving of material; their transverse stiffeners, four-sided supports, stiffened buckle-plates, etc., represent a degree of fabrication which, in general, American designers must avoid.

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[9] "The Battledeck Floor for Highway Bridges," Am. Inst. of Steel Construction, New York, N. Y., 1938.

[10] *Die Bautechnik,* June 3, 1938, p. 306 ff.

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DISCUSSIONS

DESIGN OF PILE FOUNDATIONS

Discussion

BY JOHN M. COAN, JR., JUN. AM. SOC. C. E.

JOHN M. COAN, JR.,[26] JUN. AM. SOC. C. E. (by letter).[26a]—In focusing the attention of many engineers on the idea of the "dummy pile," the author is to be commended. It is recognized as a convenient tool in that it enables the designer to predict the pile moments under certain conditions of restraint, whereas, many methods used to-day neglect or, at most, approximate these moments.

In so far as practical applications are concerned, the theory is incomplete in that certain factors pertinent to intelligent design have not been included. For example, the inability to determine, with any degree of accuracy, the location of the point of pile restraint in the foundation and the failure to take into consideration the characteristics and confinement of the surrounding soil make it advisable to design with a liberal margin of safety. Such an analysis as Mr. Vetter presents might be applied to towers, pedestals, or similarly supported bodies with laterally unsupported elements or, more specifically, to pile piers passing through alluvial or practically non-resistant materials to resistant strata.

The author's treatment of the dummy-pile method is restricted to what amounts to two-dimensional arrangements in which the piles are parallel with, and symmetrical with respect to, a vertical plane of symmetry, and in which the resultant of the dead and applied loads lies in the plane of symmetry. If the plane of loading remains coincident with the plane of symmetry, such an analysis might be extended to include the case of a three-dimensional system in which the piles are symmetrical with respect to a vertical plane of symmetry, but not parallel to it. In this case the individual piles may be projected into the plane of symmetry, analyzed, and the results thus obtained corrected by the ratios of the true to the projected lengths. Furthermore, in the event of an arrangement having no vertical plane of symmetry, if the piles are of such sec-

NOTE.—The paper by C. P. Vetter, M. Am. Soc. C. E., was published in February, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1938, by Messrs. Hibbert M. Hill, and Odd Albert; and June, 1938, by Messrs. August E. Niederhoff, A. A. Eremin, and Jacob Feld.

[26] Stress Analyst, Glenn L. Martin Co., Middle River, Md.

[26a] Received by the Secretary August 1, 1938.

tion that each has equal moments of inertia about two mutually perpendicular axes, and if the resultant of the applied loads lies in a vertical plane which contains the centroid of the pile group, the piles will deflect in planes whose horizontal traces are parallel to the plane of loading. Such a condition may be analyzed as if a vertical plane of symmetry existed and coincided with the plane of loading.

The writer has made an extensive model analysis [27] of such a case and, although the pile arrangement studied was unique, it is considered worth mentioning because the axial loads and end moments obtained from a dummy-pile analysis agree reasonably well with the experimental results.

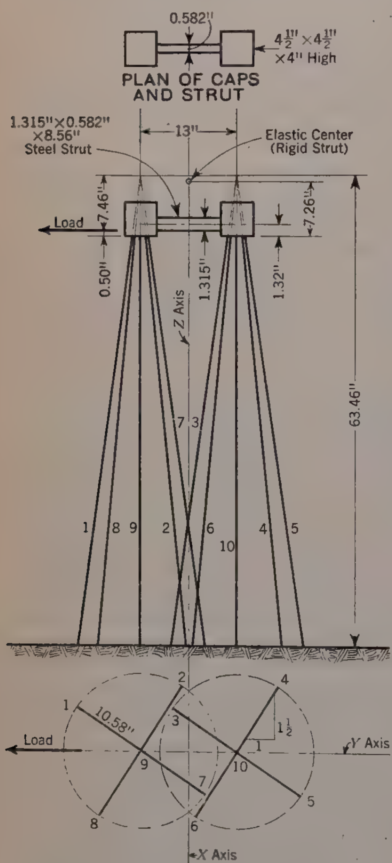

FIG. 16.—PLAN AND ELEVATION OF MODEL

The model (Fig. 16) was one-fortieth scale and consisted of two groups of five symmetrically arranged tubular steel piles, each having an outside diameter of 0.757 in. and a wall thickness of 0.030 in. The piles were fixed in concrete at their bases, and in mortar caps at their upper extremities. These caps were connected by a non-rigid rectangular strut which permitted the transfer of loads and caused the action of the two groups to be interdependent. The points at which the piles were fixed corresponded to the arbitrarily located points of restraint mentioned by Mr. Vetter under the heading, "General Considerations." No attempt was made to simulate the resistance to lateral displacement that the alluvial materials, through which the prototype piles are to be driven, would offer. In view of the elasticity of the strut, it was necessary to correct the results obtained from a dummy-pile analysis because they had been determined under the assumption that the supported structure was infinitely rigid. This correction was made by cutting the strut, finding the unbalanced moment on each pier cap, and then distributing this moment to the strut and to the piles in that group in proportion to their relative stiffnesses. (The batter of the piles permitted only a negligible side-sway for which it was unnecessary to correct the distributed moments.) The result of this elastic strut correction was an inappreciable

[27] "Model and Theoretical Analyses of a Tubular Pile Bridge Pier," by John M. Coan, Jr.; an unpublished essay submitted as partial requirement for the degree of Master of Civil Engineering, at The Johns Hopkins Univ., Baltimore, Md., May, 1938.

change in the axial loads and an average increase in the maximum pile moments of about 20 per cent.

Three loading conditions—dead + live + impact, ice, and wind—were analyzed and, in the case herein illustrated, a 120-lb ice load, corresponding to a 200 000-lb load on the prototype pier, was applied as shown. Table 4 gives a comparison of the axial loads and maximum bending moments as determined experimentally and as computed by the dummy-pile method and corrected for the elastic strut.

TABLE 4.—Effects of a 120-Pound Ice Load Action Along the Y-Axis

Description	Pile Nos.									
	1	2	3	4	5	6	7	8	9	10
Axial Load, in Pounds:										
Measured........	−141.1	108.2	−168.2	93.7	135.0	−110.9	163.3	−89.2	8.4	− 7.1
Computed.......	−135.2	111.9	−159.4	87.3	135.2	−111.9	159.4	−87.3	12.3	−12.3
Maximum Moment,* in Inch-Pounds:										
Measured........	12.7	11.3	11.1	11.5	10.7	10.4	8.4	13.3	9.5	9.2
Computed.......	10.4	10.5	9.2	9.4	9.2	9.3	10.4	10.5	10.5	9.4

* The pile moments are either positive or negative.

The average difference in axial loads, except the lightly loaded verticals, is just over 3% and the average variation in the maximum moments is within 16 per cent. In this case, due to portal action, the bending stresses are considerably less than one-third the total stresses and the differences between the measured and the computed bending moments are reduced in importance. Under the condition in which equal loads were applied at the centroid of each group and parallel to the X-axis, the displacements of the pile groups were independent of the strut, the end moments were of a much greater magnitude, and a closer agreement between the analytical and experimental moment determinations was found, the average difference being about 1 per cent.

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DISCUSSIONS

NATURAL PERIODS OF UNIFORM CANTILEVER BEAMS

Discussion

BY K. BERT HIRASHIMA, ESQ.

K. BERT HIRASHIMA,[14] ESQ. (by letter).[14a]—This interesting paper by Professor Jacobsen should prove an important addition to the literature of the subject treated. Especially valuable are the tables and graphs illustrating the theory.

The normal modes of transverse vibrations of a rigidly built-in cantilever, resulting from a consideration of shear only and flexure only, are well known. Taking the frequency equation for each of these two elementary cases, the vibrations that occur in the Y-direction are given by Equations (7) and (10). It will seem that Equation (7) (shear only considered) is comparatively simple and easy to solve. The solution of Equation (10) (flexure only) is more complicated; but it can be solved by a method of successive approximations.[15]

However, as shown in the paper, neither of these comparatively simple equations gives the correct result for the general case in which neither shear nor flexure is negligible. Thanks to Table 3, however, the periods of the various modes for this general case can readily be obtained by simple multiplication from the solution of either equation. Equation (7) will probably be preferable for reasons of simplicity.

The author's analysis of the effect on the vibration periods of the elastic yielding of the ground is highly interesting; but the solution of the frequency equation for this case is very involved. However, Table 2 and the graphs in Figs. 6, 7, and 8, give a clear indication of what can be expected when the ground yields elastically. The ground deformation moduli, ϵ_v, ϵ_h, and ϵ_s, of course, can be determined by experimental methods. The ground rigidities, R_x, R_y, $R_{x\theta}$, $R_{y\theta}$, and R_ϕ, can then be determined in the manner described in the paper. Finally, with numerical values of the ground constants thus determined

NOTE.—The paper by Lydik S. Jacobsen, Esq., was published in March, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[14] Asst. Highway Planning Engr., Territorial Highway Dept., Honolulu, Hawaii.

[14a] Received by the Secretary July 12, 1938.

[15] "Applied Elasticity," by J. Prescott, p. 206.

the frequency equation can be solved for the values of the various periods. The disadvantage of the foregoing procedure is that it is long and tedious with the likelihood that errors will be introduced.

In practical engineering work, as differentiated from theoretical investigations, what is most desired is not 100% theoretical accuracy, but an accuracy sufficient for the nature and needs of the problem. With this standard limitation in mind it seems that a table similar to Table 3 could be constructed giving conversion factors for various types of soils and foundations. The period of the various modes for a cantilever "planted" in sandy soil, for example, can then be determined by simple multiplication from the solution of either Equation (7) or Equation (10); or, by reference to Tables 2 and 3 and Figs. 6, 7, and 8, the engineer could derive his own factors for converting the periods as obtained from a solution of Equation (7) or Equation (10) into the estimated true periods for the case of an elastically yielding ground. Such procedure requires sound judgment on the part of the engineer. The errors incident thereto are probably not more serious than the uncertainties inherent in the nature of the problem. On important work, investigation along the lines indicated in the paper is justified.

It may be remarked that the various frequency equations, involving as they do either trigonometric or hyperbolic functions, have an infinite number of roots. When a rod (or, in the present case, a cantilever) is set in motion by being bent to one side and then released, it is not likely to begin vibrating in any one of the normal modes, unless the curve into which the cantilever is first bent was the same curve as the cantilever assumes in one of the extreme positions of that particular normal mode. However, it is known that the subsequent motion is composed of a number of superposed normal modes of which the lower modes are the more important.

The further discussion of the superposition of normal modes and the development in terms of Fourier's series is beyond the scope of the paper. The interested reader is referred to the various special treatises such as Byerly's "Fourier's Series and Spherical Harmonics"; Lord Rayleigh's "Theory of Sound"; Prescott's "Applied Elasticity"; and Love's "Mathematical Theory of Elasticity."

Corrections for *Transactions:* In Equation (16), change "0.228^2" to read "0.288^2"; in the numerical coefficients of Equations (24) and (26) move the decimal points one place to the left; in the numerator of Equation (53*a*), change "*b*" to "*a*"; and, in the numerator of Equation (53*b*), change "*a*" to "*b*."